Seismic Base Isolation: An Energy Formulation

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INTRODUCTION

Earthquakes are one of the largest forces a building must withstand within its lifetime. Over the past 6 years, there were over 800,000 earthquake-related fatalities worldwide.[1] The Federal Emergency Management Agency states that it would take California approximately $3.3 billion to recover from a major earthquake.[2] There are current industry solutions that are used for earthquake mitigation such as shear walls, braced frames, and moment-frame systems. However, theses systems' purpose is to uphold the building long enough so occupants can evacuate. After a major earthquake, these buildings are no longer useable and must be demolished to re-build new ones. However, there is a new earthquake mitigation method emerging in the industry called Base Isolation Systems.

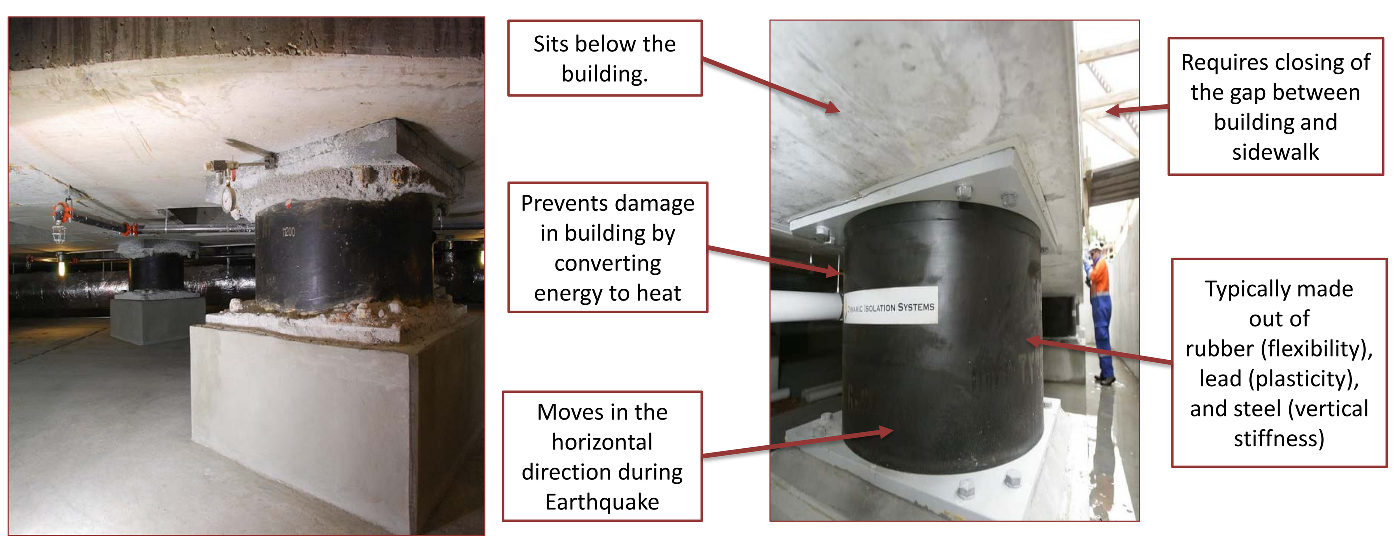


Figure 1: Base Isolation System-Ohio State Capitol

A Base Isolation System sits below a building and prevents damage in the building by converting the earthquake energy to heat. It moves in the horizontal direction during an earthquake, and it usually made out rubber, lead and steel. To design this system, it requires the consideration of multiple design parameters such as the building type, shape, earthquake magnitude, etc. In addition, it requires individual simulation for each building. Due to its complexity, this system is not widespread.

The objective of this research is to create a simple design tool that structural engineers can use to design a Base Isolation System by limiting the design parameter to energy absorption. Using the single-degree freedom behavior of a building and Dr. James Kelly's Linear Theory of Seismic Isolation[3], a single equation can be formulated to determine the required stiffness of the Base Isolation System.

Linear Theory of Seismic Isolation[3]

According to Dr. James Kelly of UC Berkley, a base-isolated building has a separate mass, stiffness, and displacement of the base and the building. With these characteristics, a building has two modes of vibration during an earthquake. In the 1st Mode, φ_1, the base moves a unit value (1) while the building moves a fraction relative to the base, denoted by ϵ (frequency ratio of the building and base). In the 2nd Mode, φ_2 the base moves a a unit value (1) while the building displaces greatly relative to the base. See Figure 2 for the displacement of a base-isolated building. Dr. Kelly quantified the participation ratio of the 1st Mode of Vibration. The participation factor of the 1st Mode can be determined using the L_1 equation. The target is to activate the 1st Mode due to its small building displacement.

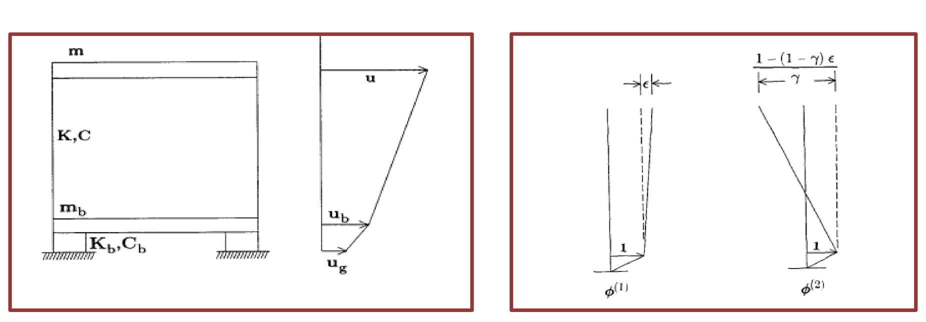


Figure 2: Linear Theory of Seismic Isolation

1st Mode of Vibration

$$L_1 = 1 - \gamma\epsilon$$

ω_b = frequency of the base
ω_s = frequency of the structure

m = mass of base
M = mass of base + building

Frequency Ratio

$$\epsilon = \left(\frac{\omega_b}{\omega_s}\right)^2$$

Mass Ratio

$$\gamma = \frac{m}{M}$$

K = *stiffness of building*
K_b = *stiffness of base*

U = *movement of building*
U_b = *movement of base*
U_g = *movement of ground*

Energy Theory of Seismic Isolation

During an earthquake, a building behaves like a spring. It has a natural circular frequency, period of vibration, and energy within the system. Using the spring energy equation, an energy equation was derived for both the base and the building using their individual stiffness, K, and displacement according to the linear theory. From these two equation, a formula was derived to determine the percent of energy absorbed in the base which relates the mass ratio and frequency ratio of a base-isolated building.

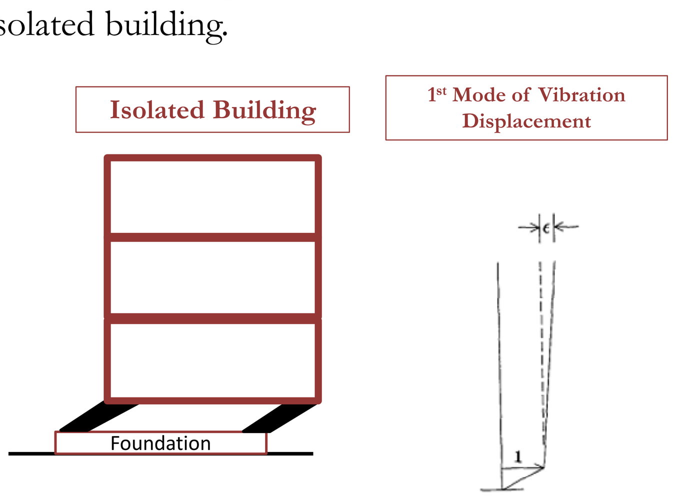


Figure 3: Energy Theory of Seismic Isolation

Recall that a building behaves like a spring, so the energy of the system is

$$W = \frac{Ku^2}{2}$$

Amount of Energy absorbed in base

$$W_b = \frac{k_b}{2}$$

Amount of Energy absorbed in building

$$W_s = \frac{k_s(\epsilon)^2}{2}$$

Total Energy in System:

$$W_T = W_b + W_s$$

Percent of Energy in base:

$$\%W_b = \frac{W_b}{W_b + W_s}$$

Percent of Energy in base:

$$\%W_b = \frac{W_b}{W_b + W_s}$$

Energy Equation

$$\%W_b = \frac{1}{1+\gamma\epsilon}$$

RESULTS

Through the base energy absorption equation, a range of ϵ were calculated to achieve an energy absorption of 90%, 95%, and 99%. These ϵ values were determined for a range of 1 story building to a 10 story building. A graph plotting ϵ vs. % energy absorbed is displayed in Figure 4.

Energy Equation

$$\%W_b = \frac{1}{1+\gamma\epsilon}$$

Target:
Achieve 90%, 95%, and 99% energy absorption by varying ϵ (Frequency Ratio)

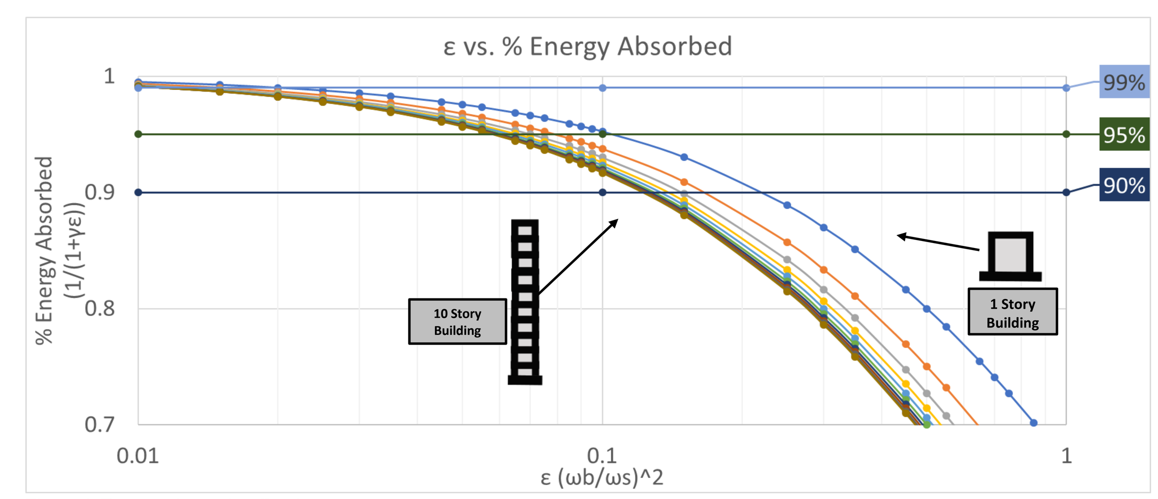


Figure 4: Energy Absorption Graph

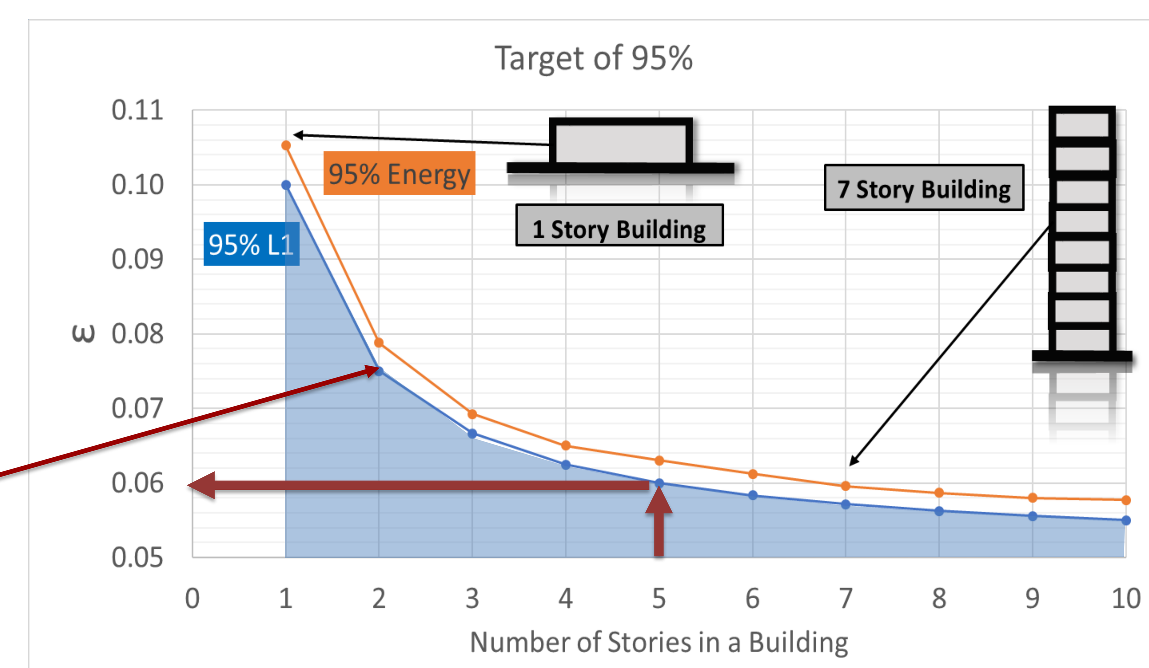


Figure 5: 95% Graph

$\epsilon_{95\%}$ of L_1 Governs

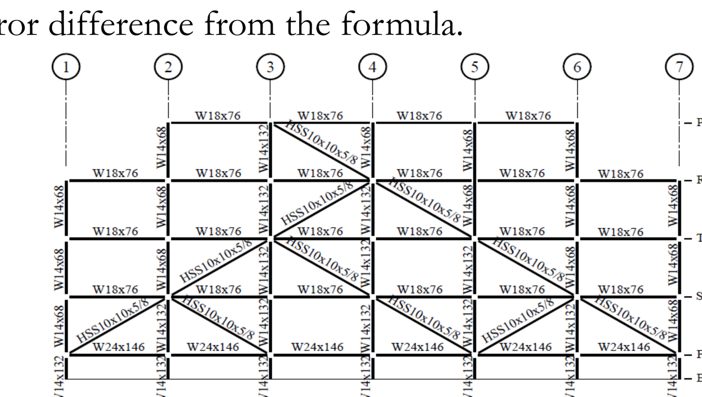
Reaching a minimum of $\epsilon_{95\%}$ of L_1 guarantees a 95% energy absorption.

Figure 5 displays a graph of the 95% energy absorption and the 1st Mode of Vibration on a plot of building stories vs. ϵ. Activating 95% of the 1st Mode of Vibration using its ϵ value will guarantee a 95% energy absorption.

Design Energy Equation

The final design equation was derived using the percent energy equation as well as the frequency ratio equation. This equation is the design tool that structural engineers can use to design a Base Isolation System. It relates the percent energy absorbed, the mass ratio, and the stiffness of the building to determine the stiffness of the base. This formula was applied to the design of the base system of this four-story building below to verify its accuracy. Using the stiffness of the building and its mass ratio to achieve a target of 95% energy absorption, a base stiffness of 2,484 kip/foot was calculated. Inputting this value into a structural modeling program, SAP 2000, and simulating an earthquake, it was determined that the base absorbed 94% of the earthquake energy; a 1% error difference from the formula.

$$\%W_b = \frac{1}{1+\gamma\epsilon} \quad K_b = \frac{\epsilon K_s}{\gamma} \quad K_b = \frac{\frac{1}{\%W_b} - 1}{\gamma^2} K_s$$



$\gamma = 0.77$

$K_s = 28{,}174 \frac{kip}{ft}$

Target: 95%

Analytical Approach

$$K_b = \frac{\frac{1}{\%W_b} - 1}{\gamma^2} K_s$$

$$K_b = 2{,}484 \, {}^{kip}/_{ft}$$

SAP2000 Check

94% Energy Absorption

Figure 6: Formula Application

Future Work

This research can be further progressed by validating this equation for different building types such as ones with unique floor and elevation plans. In addition, incorporating this formula into the design of other infrastructure such as bridges or power lines can be another path this research can take. Another route would be the creation of a graphical method such as design charts to compliment this formula.

REFERENCES

1 "Earthquake Statistics." *U.S. Geological Survey*, earthquake.usgs.gov/earthquakes/browse/stats.php.
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3 Kelly, Jame M. "Base Isolation: Linear Theory and Design." *Earthquake Spectra*, vol. 6, no. 2, 1990, pp. 223–244.