Models for Homeless Housing

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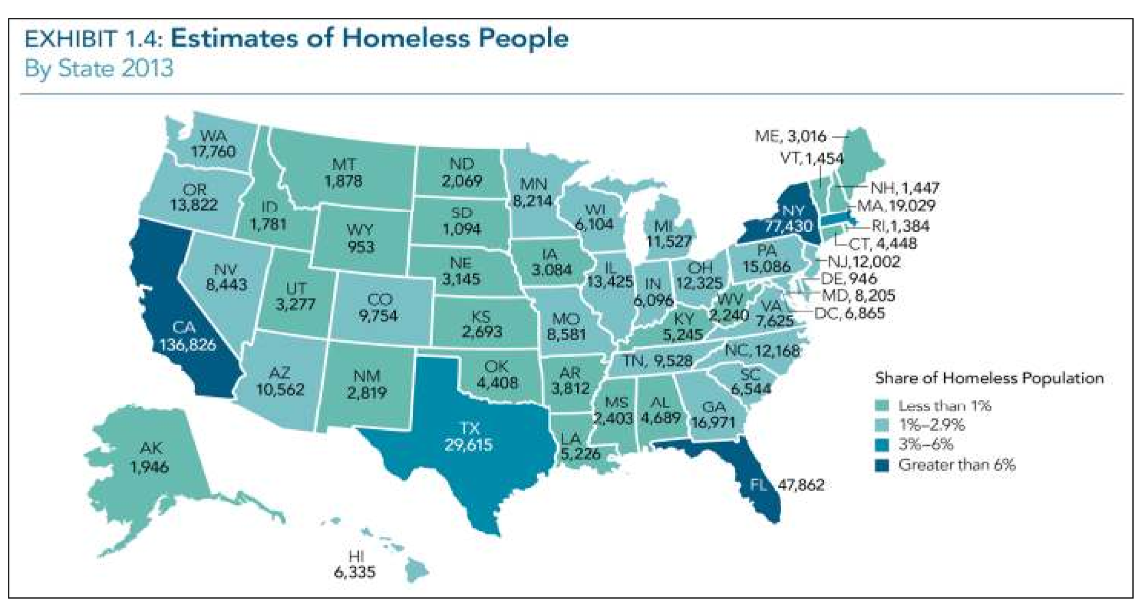
Kellogg Honors College Capstone Project

Background

In the United States, 610,042 people were homeless on a given night in January 2013, and 1/4 of these were children. About 1/3 of this population did not live in a shelter. With many people living with chronic diseases, mental illnesses, and isolation, there has been recent government attention (since 2010) and local initiatives to remedy this issue. The Annual Homeless Assessment Report to Congress has been issued since 2006. With the Insan Foundation, I also participated in the 100,000 Homes Campaign in April 2013 to find Pomona homeless individuals, interview them regarding the services they required, and potentially secure permanent homes for the most vulnerable. This experience and others inspired me to construct mathematical models of homeless people as they moved between housing structures.



Modeling Methods

Modeling the movement of populations is a common application of mathematics and statistics to biology. Populations that are studied can range from viruses to communities of people.

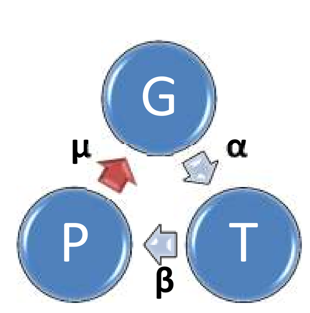
Deterministic models are systems in which all the data is known beforehand, so you can predict any outcome if you know where the system starts.
(ex: fixed interest rate)

Stochastic models include chance, so you only know how the variables are related to one other, and you can make generalizations about the future. You can get different answers at the same point because of randomness.
(ex: stock market)

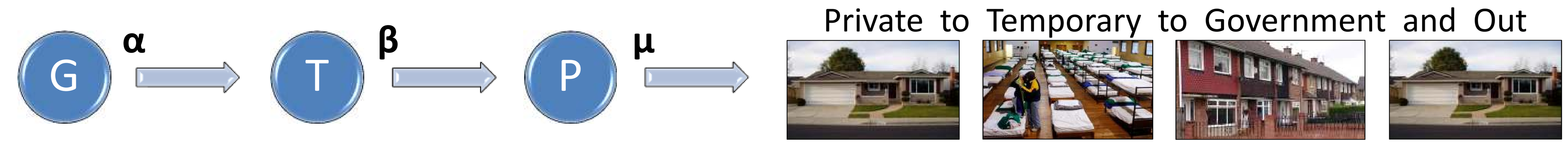
Using **partial differential equations, Kolmogorov equations,** and **probability generating functions** (pgfs), we first examined a simplified model with temporary (T) and government (P) housing. Then we considered a complex model with T, P, and G (private) units, with solutions for both the deterministic and stochastic cases.

Further Directions

Future work includes creating a more inclusive model that expands upon the complex one presented. If the individuals leaving government units (P) went directly into the private sector (G), they could become homeless again. Though mathematically rigorous, especially for the stochastic model, it would be interesting to see how this change affects the steady states and pgfs.



State Diagram



Private to Temporary to Government and Out

Deterministic Model

From the 2011 AHAR, we estimated the following annual rates:

α = 0.004406 (rate at which Americans enter homeless shelters)

β = 0.981 (rate at which homeless leave emergency shelters)

μ = 0.864 (rate at which homeless leave transitional housing)

Initial Conditions: Go = 30, To = 5, Po = 5

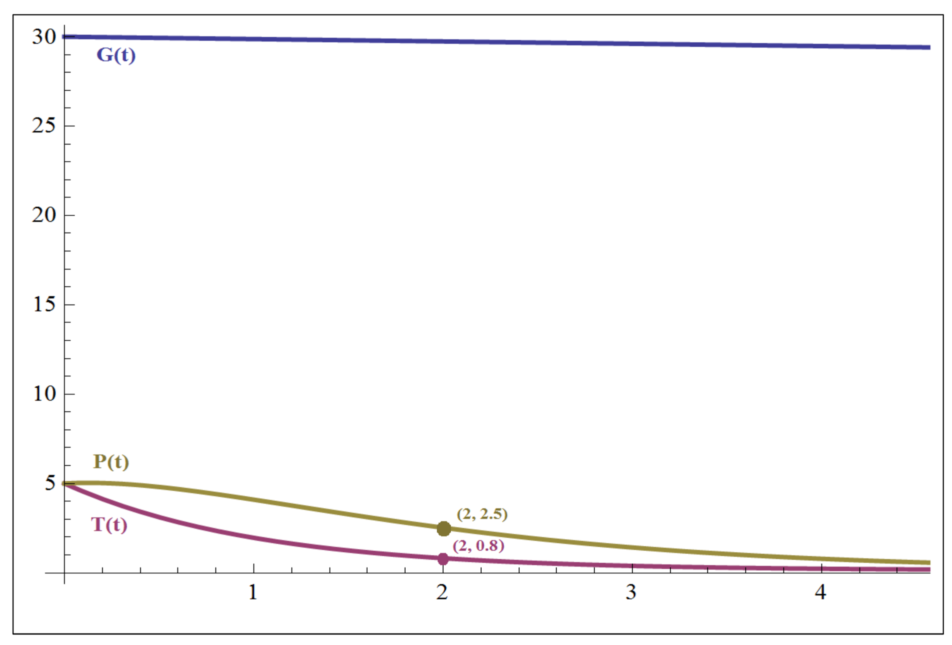
Note: The means of both models match!

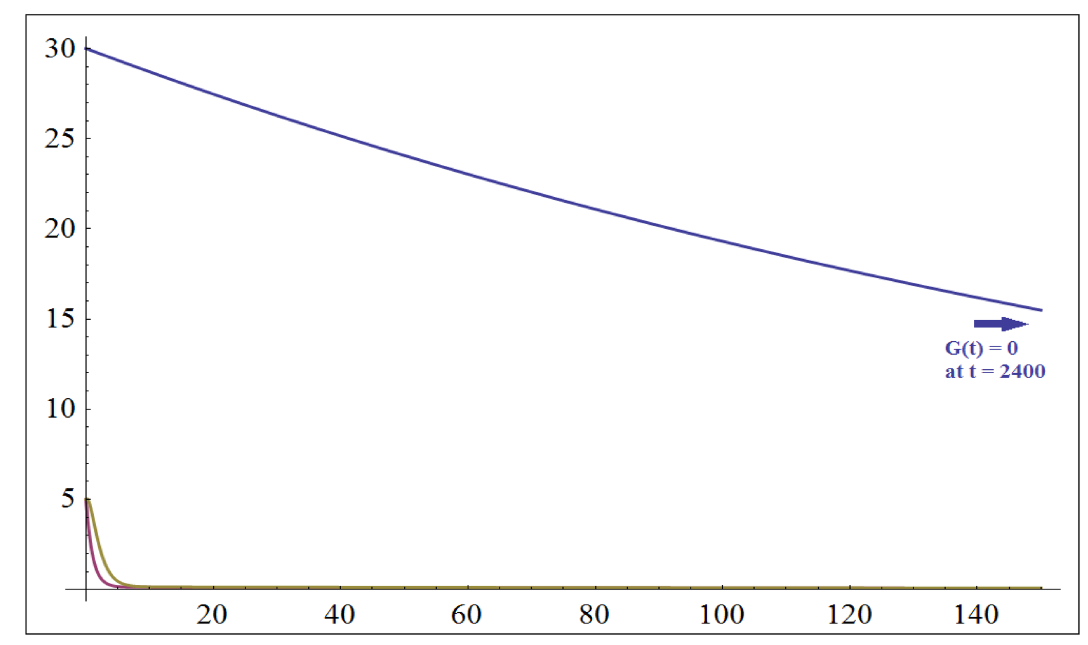
$$\frac{dG}{dt} = -\alpha G \qquad \frac{dT}{dt} = \alpha G - \beta T \qquad \frac{dP}{dt} = \beta T - \mu P$$

$$G(t) = G_0 e^{-\alpha t}$$

$$T(t) = T_0 e^{-\beta t} + \frac{\alpha G_0}{\beta - \alpha}\left(e^{-\alpha t} - e^{-\beta t}\right)$$

$$P(t) = P_0 e^{-\mu t} + \frac{\beta T_0}{\mu - \beta}\left(e^{-\beta t} - e^{-\mu t}\right) + \frac{\alpha\beta G_0}{\beta - \alpha}\left(\frac{e^{-\alpha t} - e^{-\mu t}}{\mu - \alpha} - \frac{e^{-\beta t} - e^{-\mu t}}{\mu - \beta}\right)$$





Stochastic Model

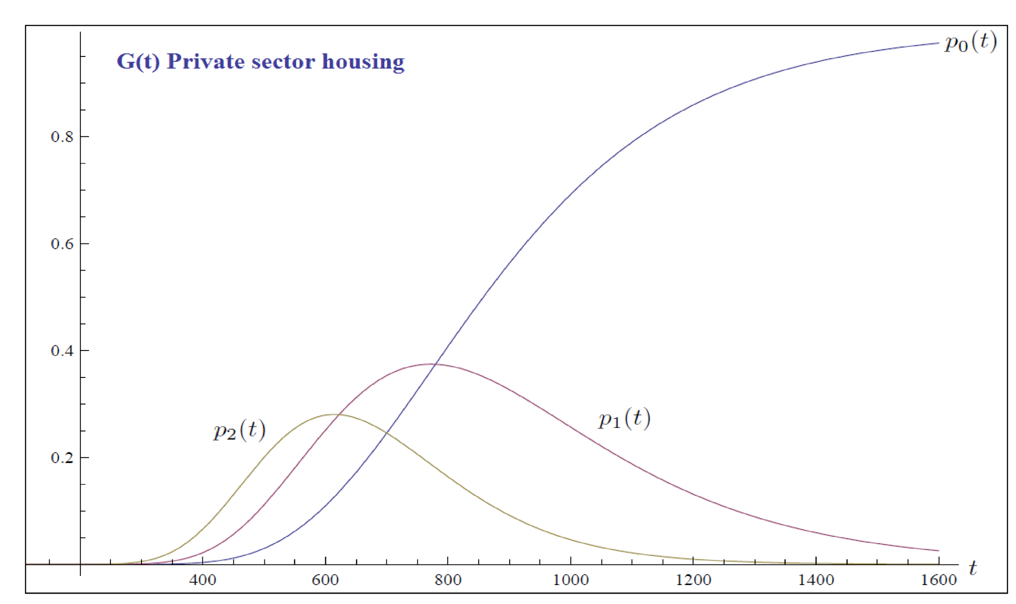
$$p_{i,j,k}(t) = P\{P(t) = i, T(t) = j, G(t) = k \mid P(0) = P_0, T(0) = T_0, G(0) = G_0\}$$

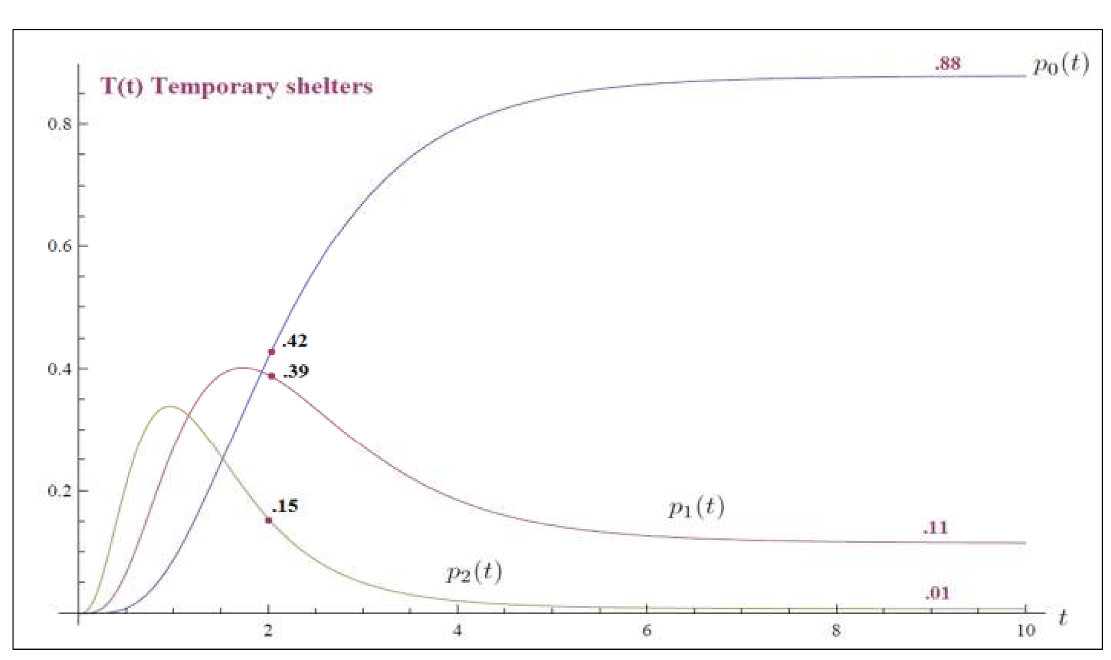
$$\frac{dp_{i,j,k}}{dt} = -(\mu i + \beta j + \alpha k)p_{i,j,k} + \mu(i+1)p_{i+1,j,k} + \beta(j+1)p_{i-1,j+1,k} + \alpha(k+1)p_{i,j-1,k+1}$$

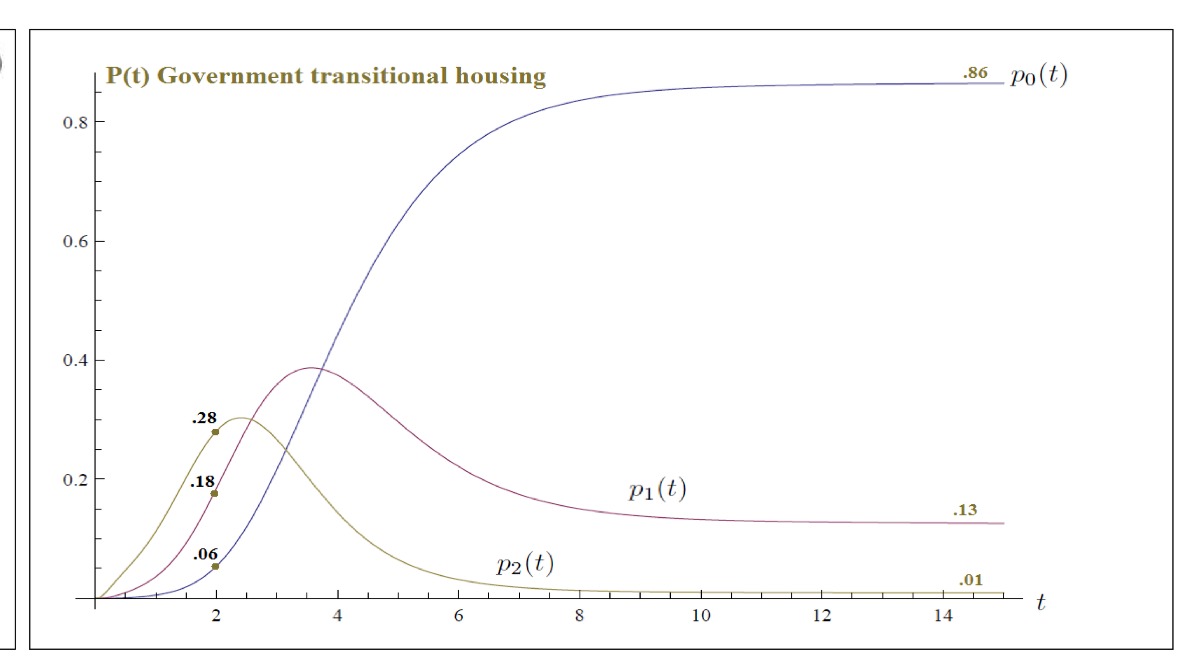
$$\phi(1,1,w,t) = \left((w-1)e^{-\alpha t} + 1\right)^{G_0}$$

$$\phi(1,v,1,t) = \left((v-1)e^{-\beta t} + 1\right)^{T_0}\left(\frac{\alpha(v-1)}{\beta - \alpha}\left(e^{-\alpha t} - e^{-\beta t}\right)\right)^{G_0}$$

$$\phi(u,1,1,t) = \left(ue^{-\mu t} + 1 - e^{-\mu t}\right)^{P_0}\left(1 + \frac{\beta(u-1)}{\mu - \beta}\left(e^{-\beta t} - e^{-\mu t}\right)\right)^{T_0} \times \left((u-1)\left[\frac{\alpha\beta}{(\beta-\alpha)(\mu-\alpha)}(e^{-\alpha t} - e^{-\mu t}) - \frac{\alpha\beta}{(\beta-\mu)(\mu-\beta)}(e^{-\beta t} - e^{-\mu t})\right]\right)^{G_0}$$







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References

- "Mathematical modelling of homeless populations," L.G. Byatt-Smith, A.A. Lacey, D.F. Parker, D. Simpson, W.R. Smith, and J.A.D. Wattis. 2003.
- "Counting the homeless in Los Angeles County," R. Berk, B. Kriegler, and D. Ylvisaker. 2008.
- The 2013 Annual Homeless Assessment Report to Congress
- The 2011 Annual Homeless Assessment Report to Congress