# THE APPROXIMATE ANALYSIS OF STATICALLY INDETERMINATE SINGLE-STORY PORTAL FRAMES UNDER UNIFORMLY DISTRIBUTED LOADS

CalPolyPomona
College of Engineering


Khanh Nguyen Phuong Doan, Department of Civil Engineering
Mentor: Felipe J. Perez, Ph.D., P.E., Department of Civil Engineering

Kellogg Honors College Capstone Project
College of Engineering, California State Polytechnic University, Pomona

11th Annual Student Research, Scholarship & Creative Activities (RSCA) Conference
March 4th, 2023

KELLOGG HONORS COLLEGE — A Community of Scholars

## I. About the Approximate Analysis

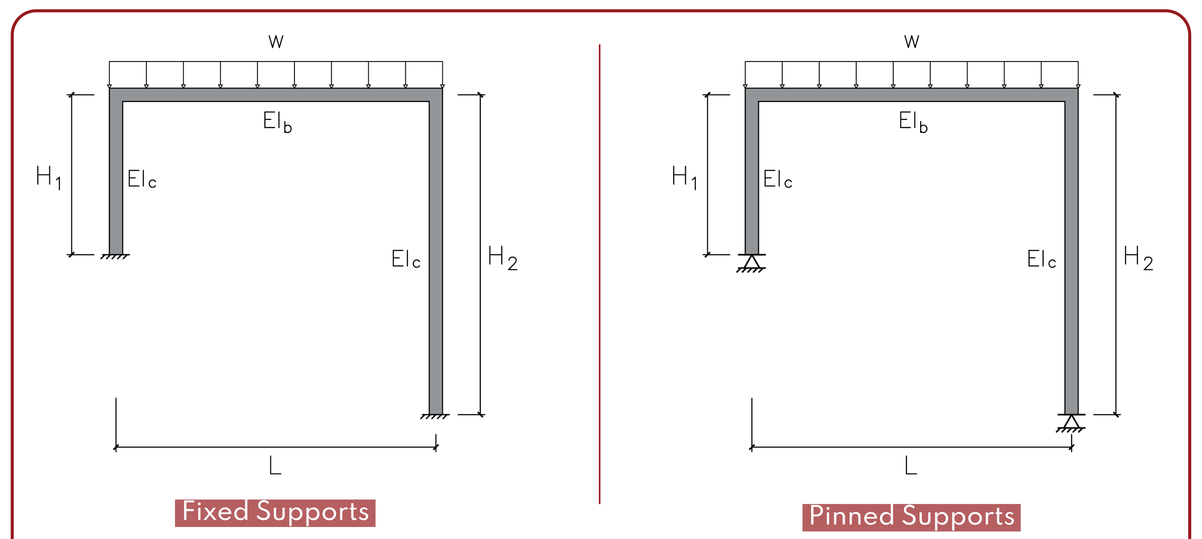
The basis for an Approximate Analysis of a statically indeterminate portal frame resisting uniformly-distributed gravity loads is to represent points of inflection (i.e., points where the moment is zero) with hinges. In doing so, the statically indeterminate frame is converted to a statically determinate frame which is easy to analyze using principles of static equilibrium.

## II. Significance of the Research

Results from this study offer recommendations for the proper selection of hinge placement so that an Approximate Analysis yields results comparable to a more sophisticated analysis of indeterminate frames.

## III. Configurations: Parameters & Boundary Conditions

The portal frame analyzed is a single-story steel frame subjected to a uniformly distributed gravity load. The beam is statically indeterminate to the third degree.




$H_1$ - left column height (ft)
$H_2$ - right column height (ft)
L - beam length (ft)
E - modulus of elasticity ( $\frac{kips}{in^2}$ or ksi)
$I_b$ - moment of inertia of beam ($in^4$)
$I_c$ - moment of inertia of columns ($in^4$)
w - uniformly distributed load ( $\frac{kips}{ft}$ )

**Constants:**

$\omega$ = 2 kips / ft
E = $E_{steel}$ = 29,000 ksi
$H_1$ = 12 feet
$I_{column}$ = 600 $in^4$

Aside from the variations in the support conditions, recommendations of hinges are also based on a set of parametric studies involving variations in column heights, beam span and moments of inertia:

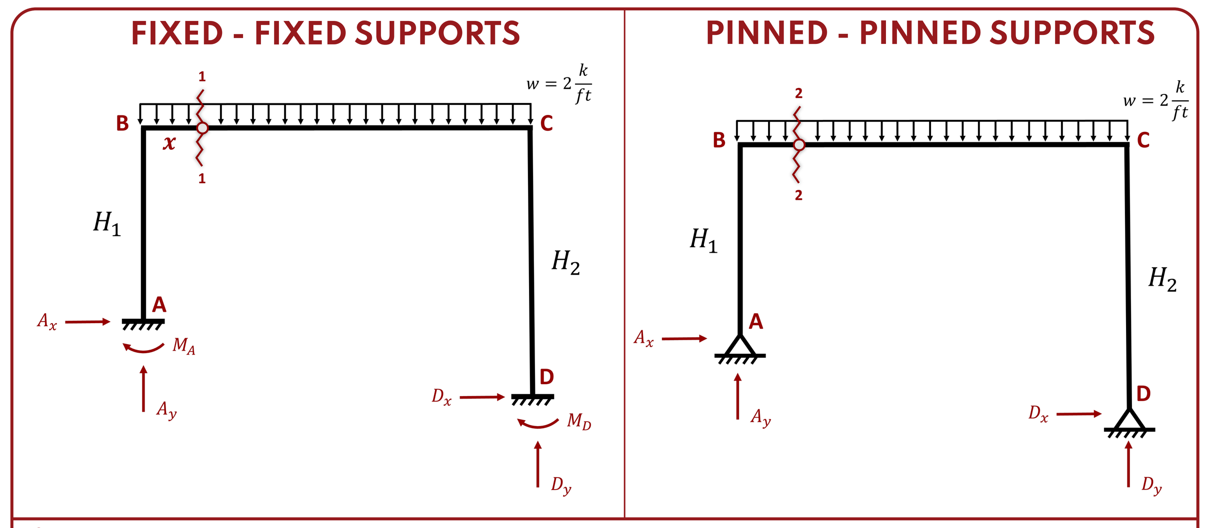
α - moments of inertia factor
$I_b = \alpha \cdot I_c$, where $\alpha$ = {1.0 , 1.25 , 1.50 , 1.75 , 2.0 }

β - beam lenght factorT
$L = \beta \cdot H_2$, where $\beta$ = {1.0 , 1.1 , 1.2 , 1.3 , 1.4 , 1.5 }

γ - column heights factos
$H_2 = \gamma \cdot H_1$, where $\gamma$ = {1.0 , 1.25 , 1.50 , 1.75 , 2.0 }

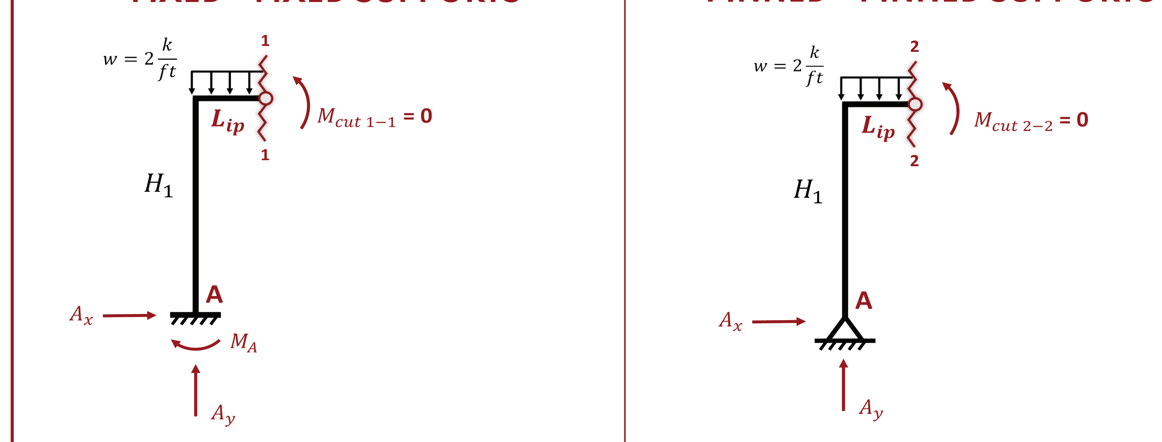
## IV. Approach

FIXED - FIXED SUPPORTS | PINNED - PINNED SUPPORTS




i. Cut through the beam at an arbitrary inflection point location ($L_{ip}$), where moment $M(L_{ip})$ = 0 k · ft

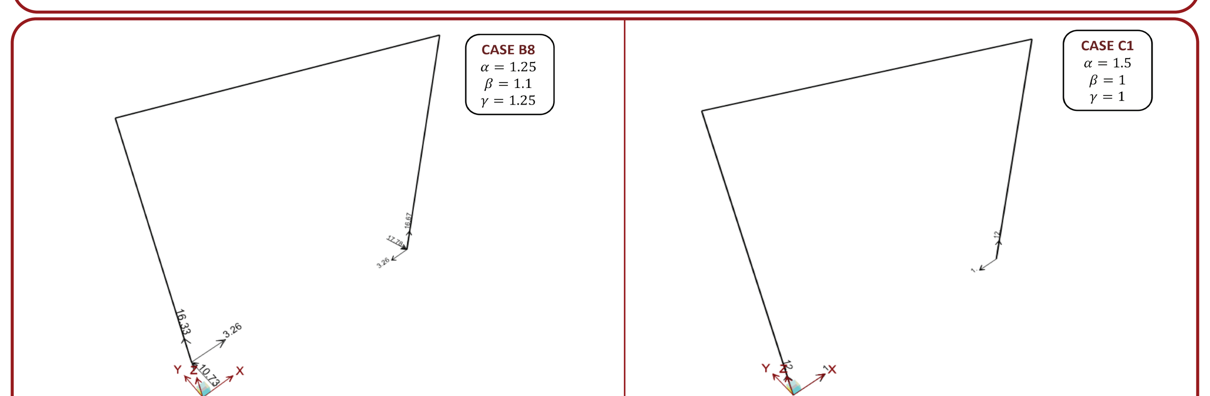
FIXED - FIXED SUPPORTS | PINNED - PINNED SUPPORTS




ii. Draw Free-Body Diagram (FBD)

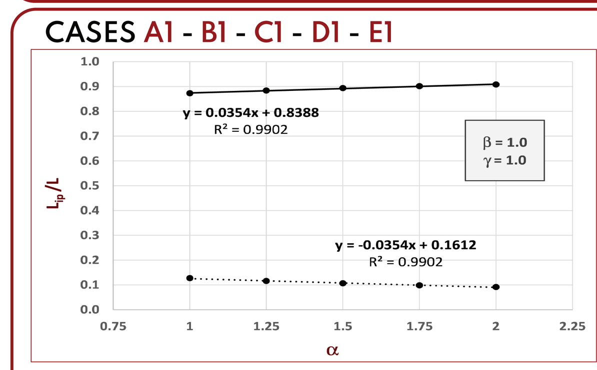
$$L_{ip} = \frac{A_y \pm \sqrt{(A_y)^2 - 4(12A_x - M_A)}}{2} \qquad L_{ip} = \frac{A_y \pm \sqrt{(A_y)^2 - 4(12A_x)}}{2}$$

iii. Express $L_{ip}$ as a function of the support reactions at joint A, resulting from the moment equilibrium about the cut

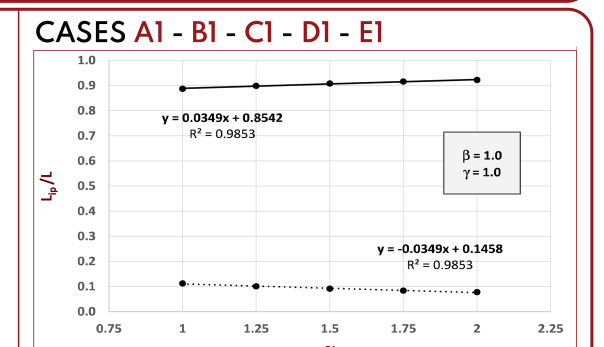



iv. Extract support reactions from SAP2000 and determine $L_{ip}$ values

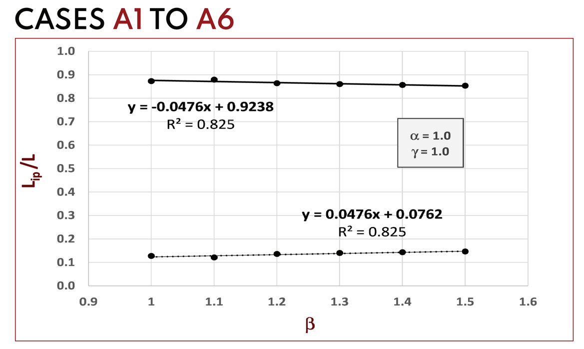
CASES A1 - B1 - C1 - D1 - E1



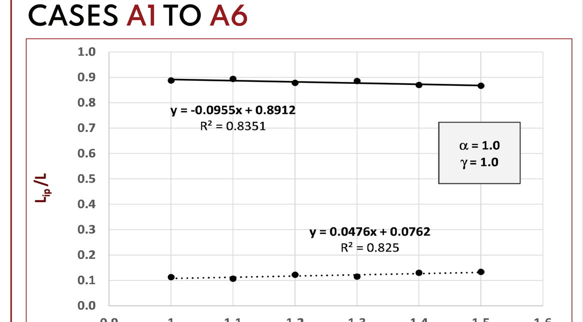

CASES A1 - B1 - C1 - D1 - E1



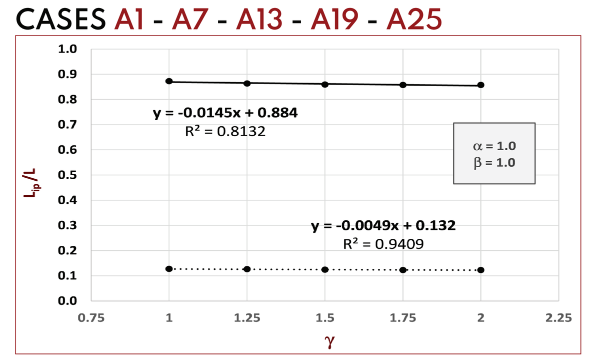

CASES A1 TO A6



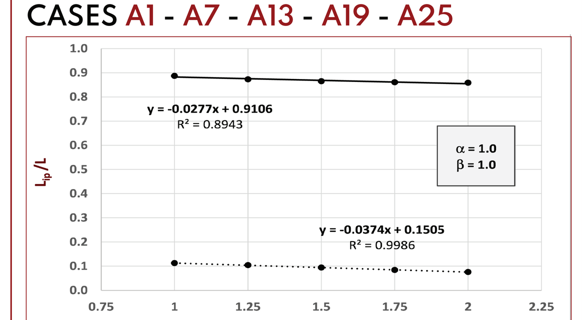

CASES A1 TO A6




CASES A1 - A7 - A13 - A19 - A25




CASES A1 - A7 - A13 - A19 - A25



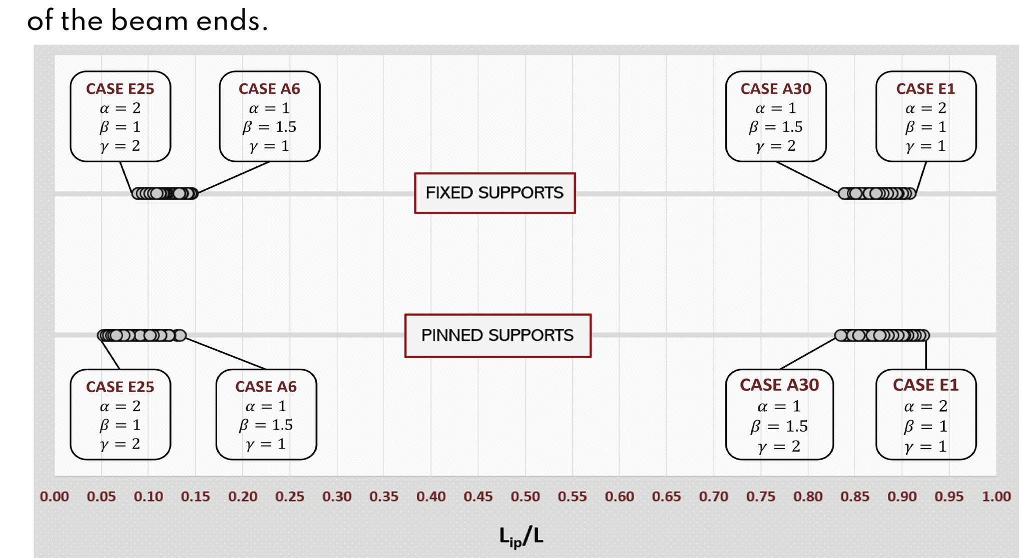

v. Plot trendlines showing the effect of individual parameters on the variations in inflection point location relative to the left end of the beam $\frac{L_{ip}}{L}$

## V. Principal Findings

- The adequate placement of hinges is at approximately 12% and 11% of the beam span, each measured from the beam ends for fixed-fixed and pinned-pinned cases, respectively.
- The greatest variation in inflection point locations for frames with fixed supports results when changes in beam length are made. The greatest variation in left and right inflection point locations for frames with pinned supports results when changes in relative column heights and changes in beam length are made, respectively.

| | | Fixed - Fixed condition | | Pinned - Pinned condition | |
|---|---|---|---|---|---|
| | | Left inflection point | Right inflection point | Left inflection point | Right inflection point |
| Average location of the inflection points on the beam span, relative to the beam ends | | 11.6% | 12.9% | 8.7% | 12.2% |
| Behavior of left and right inflection points in response to the increase in: | moments of inertia ratio (α) | Travel towards the respective columns | | Travel towards the respective columns | |
| | beam length ratio (β) | Travel towards the center of the beam | | Travel towards the center of the beam | |
| | column heights ratio (γ) | Both travel towards the left end of the beam | | Both travel towards the left end of the beam | |
| Sensitivity of inflection points (average rate of change of the trendlines) in response to the variation in: | moments of inertia ratio (α) | 3.27% | 3.41% | 2.79% | 3.76% |
| | beam length ratio (β) | 3.90% | 4.68% | 3.67% | 4.96% |
| | column heights ratio (γ) | 0.44% | 1.47% | 4.50% | 2.66% |

- For the fixed-fixed condition, inflection points range between 9% to 15% of the beam ends.
- For the pinned-pinned condition, inflection points range between 5% to 17% of the beam ends.

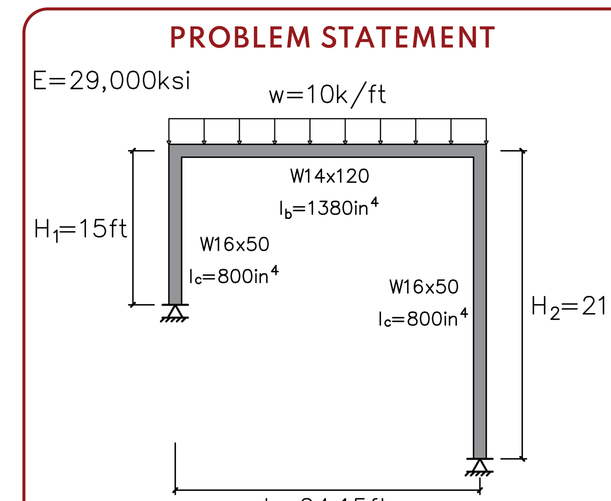



Inflection points distribution along the beam

## VI. The Approximate Analysis

Using the suggestions for the placement of hinges conducted in this research

PROBLEM STATEMENT

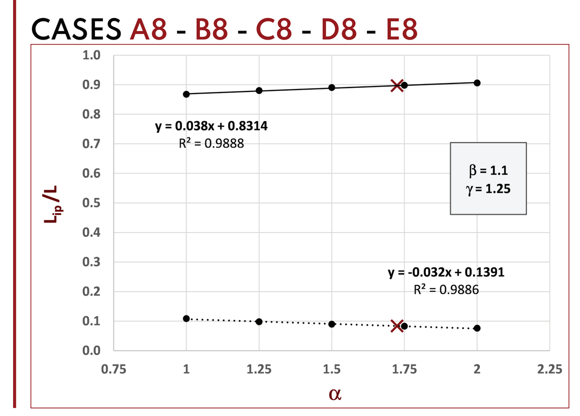



1. Specify the values for α , β, and γ

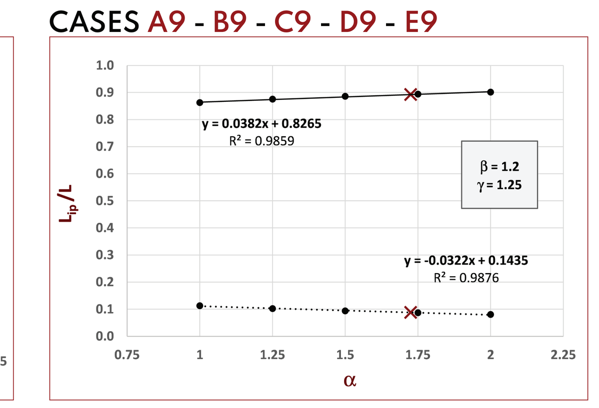
| | | |
|---|---|---|
| $\alpha$ | $= \frac{I_b}{I_c} = \frac{1{,}380\ in^4}{800\ in^4}$ | $= 1.725$ |
| $\beta$ | $= \frac{L}{H_2} = \frac{24.15\ ft}{21\ ft}$ | $= 1.15$ |
| $\gamma$ | $= \frac{H_2}{H_1} = \frac{21\ ft}{15\ ft}$ | $= 1.40$ |

2. Select the appropriate charts and determine $\frac{L_{ip}}{L}$ (left and right) from the equations of the trendlines.

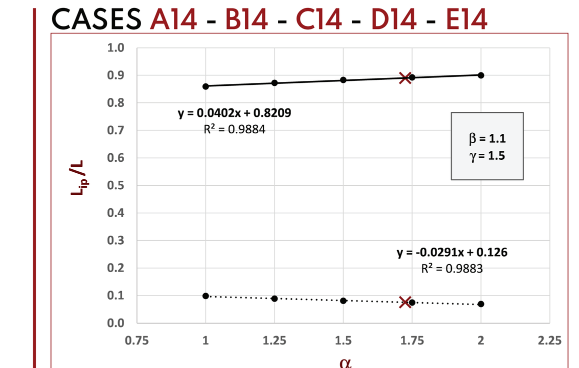
CASES A8 - B8 - C8 - D8 - E8



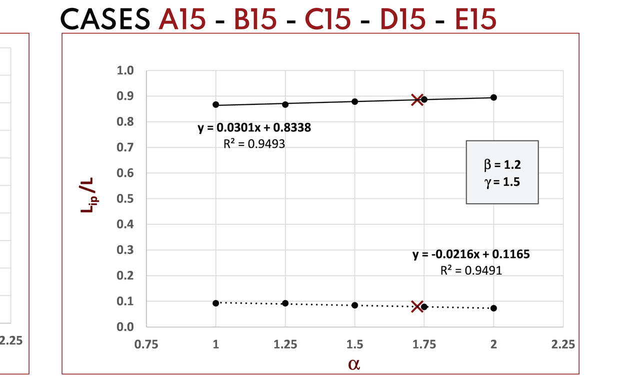

CASES A9 - B9 - C9 - D9 - E9




CASES A14 - B14 - C14 - D14 - E14



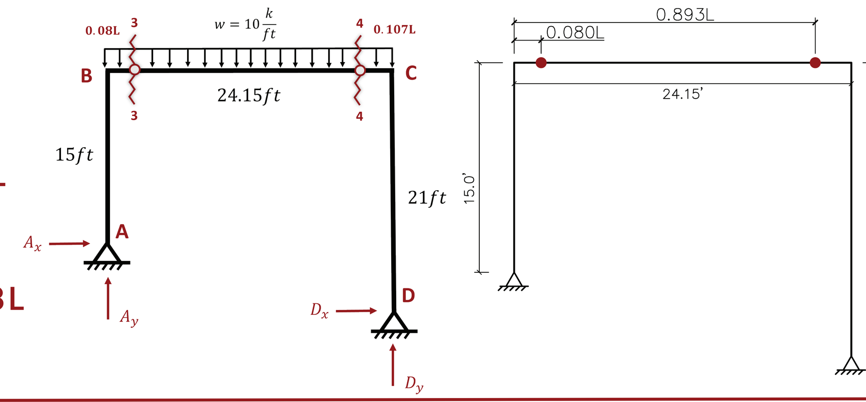

CASES A15 - B15 - C15 - D15 - E15



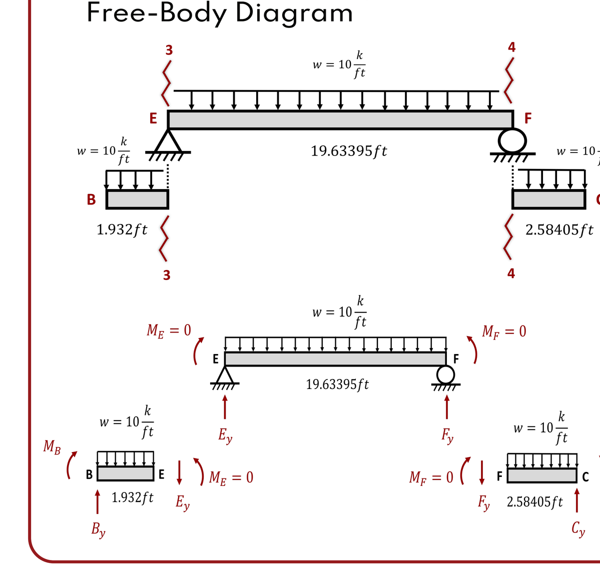

By interpolations:

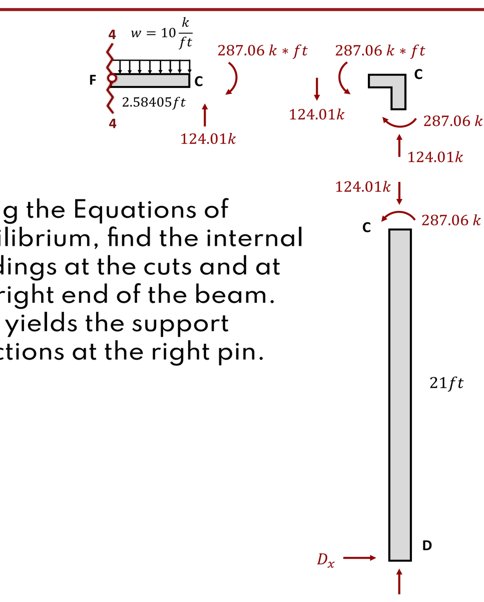
$\left(\frac{L_{ip}}{L}\right)_{left}$ = 0.080L

$\left(\frac{L_{ip}}{L}\right)_{right}$ = 0.0893L

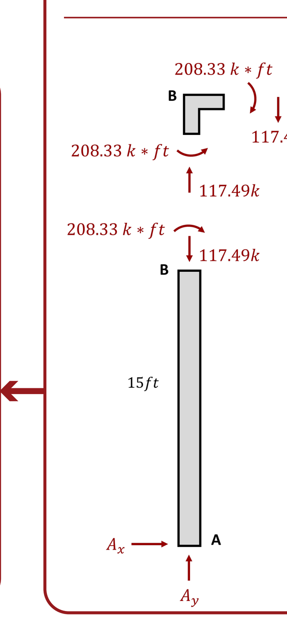



3. Make the cuts through the beam at the located hinges in Step 2. Draw a Free-Body Diagram




4. Using the Equations of Equilibrium, find the internal loadings at the cuts and at the right end of the beam. This yields the support reactions at the right pin.




Using the Equations of Equilibrium, find the internal loadings at the cuts and at the left end of the beam. This yields the support reactions at the left pin.

5. Compare hand-calculation to SAP2000's support reactions.

| | Approximate Analysis | SAP2000 | Percent difference |
|---|---|---|---|
| $A_x$ | 19.89 (kips) → | 19.93 kips → | 0.29% |
| $A_y$ | 117.49 (kips) ↑ | 117.29 kips ↑ | 0.17% |
| $D_x$ | 13.67 (kips) ← | 13.93 kips ← | 1.88% |
| $D_y$ | 124.01 (kips) ↑ | 124.21 kips ↑ | 0.16% |

## VII. Conclusion

Based on the parameter variations, this work provides the equations that can be used to estimate the inflection point locations with a high level of certainty (below 2% error), resulting in a simple and effective analysis of an indeterminate portal frame.

## VIII. Acknowledgements


I would like to express my sincere appreciation to Dr. Perez, as well as other Professors (Dr. Lomiento, etc.) who continue to believe in me and my work.
I would like to thank Dr. Garcia and Mr. Choi for bringing about an academically-challenging environment and an opportunity to engage myself in research.
Thank you to the Computers & Structures, Inc., (CSI), for donating the SAP2000 software to college students.
And to my Mom, Dad, brother, and my friends who always root for my ongoing overseas journey, thank you.


## IX. References


- Hibbeler R. C. *Structural Analysis*. Tenth ed. Pearson Education 2018.
- American Institute of Steel Construction. *Steel Construction Manual*. Fifteenth ed. American Institute of Steel Construction 2017.
- Computers & Structures, Inc. *CSI Analysis Reference Manual*. Nineteenth ed. Computers & Structures, Inc. 2016