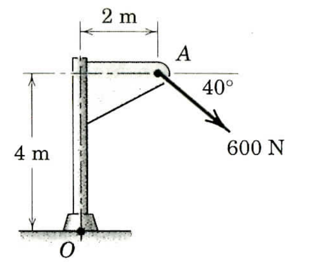
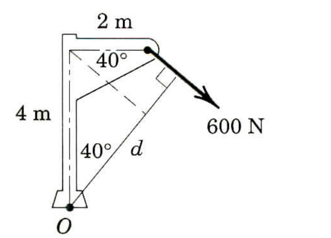
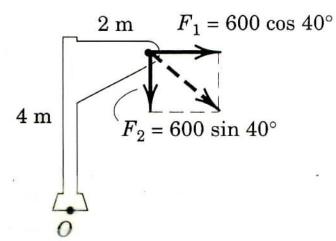
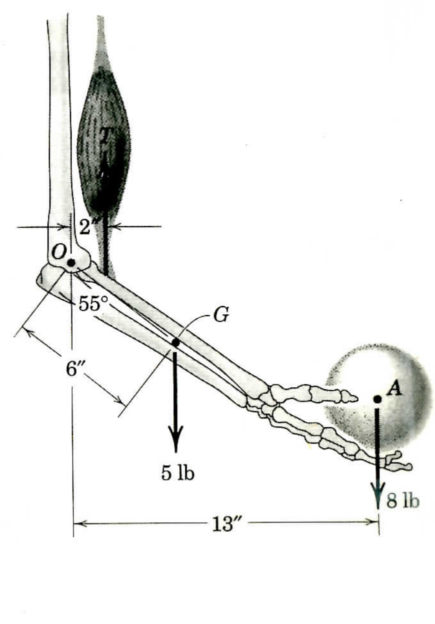
Question 1:
Determine the Moment of 600-N force about point O:
a-Using Force-Distance Method.
b-Using Cross Product Method.
c-Find shortest distance from point O to the line of action of 600-N force.





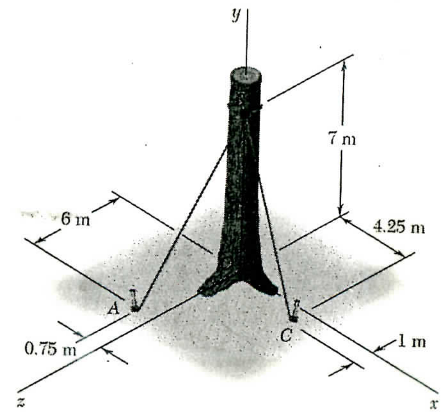


Question 2:
Determine Moment of forces acting on the arm about point O, the elbow.
What is-the tension required in the arm's muscle?



Question :
Before the trunk of a large tree is felled, cables AB and AC are attached as shown. Knowing that the tension in cables AB and AC are 555-N and 660-N, respectively, Determine:

a-the moment about O of the resultant force exerted on the tree at B.
b-the angle between cables AB and AC.
c-the perpendicular distance from point O to cable BC.

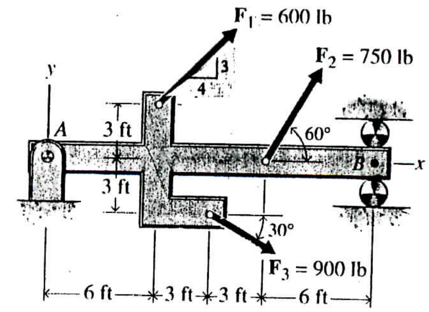


ME 214 SEC 05
Date: 10-14-'04

Sample Quizzes

NAME:____________________________ SID #____________________

Question 1: (20 Points)
Three forces are applied to a bracket as shown.
a- Determine the moments of these forces about point **B** (Not A) using **FORCE-DISTANCE** method.
b- Find moments of forces $\mathbf{F}_1$ and $\mathbf{F}_2$ about point **A** using **CROSS PRODUCT** method.

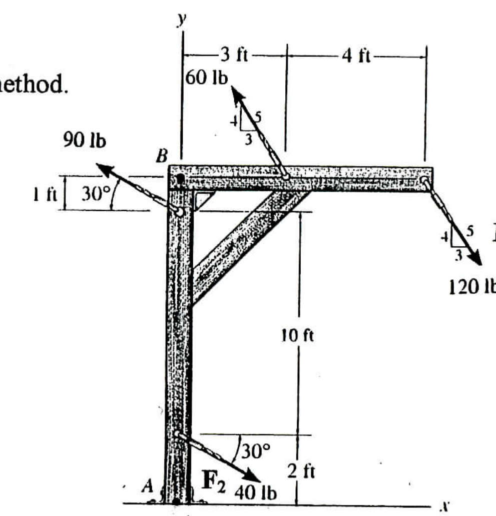


ME 214 SEC 06
Date: 10-14-'04

Quiz # 3

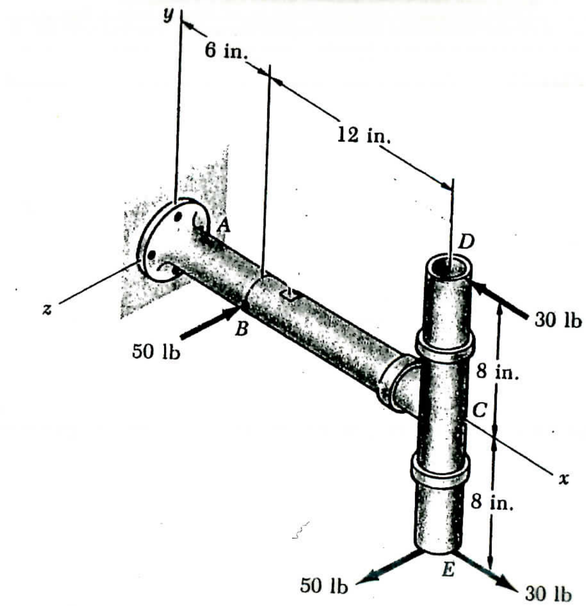
NAME:____________________________ SID #____________________

Question 1: (20 Points)
Four forces are applied to a bracket as shown.
a- Determine the moments of these forces about point **A** using **FORCE-DISTANCE** method.
b- Find moments of forces $\mathbf{F}_1$ and $\mathbf{F}_2$ about point **B** using **CROSS PRODUCT** method.

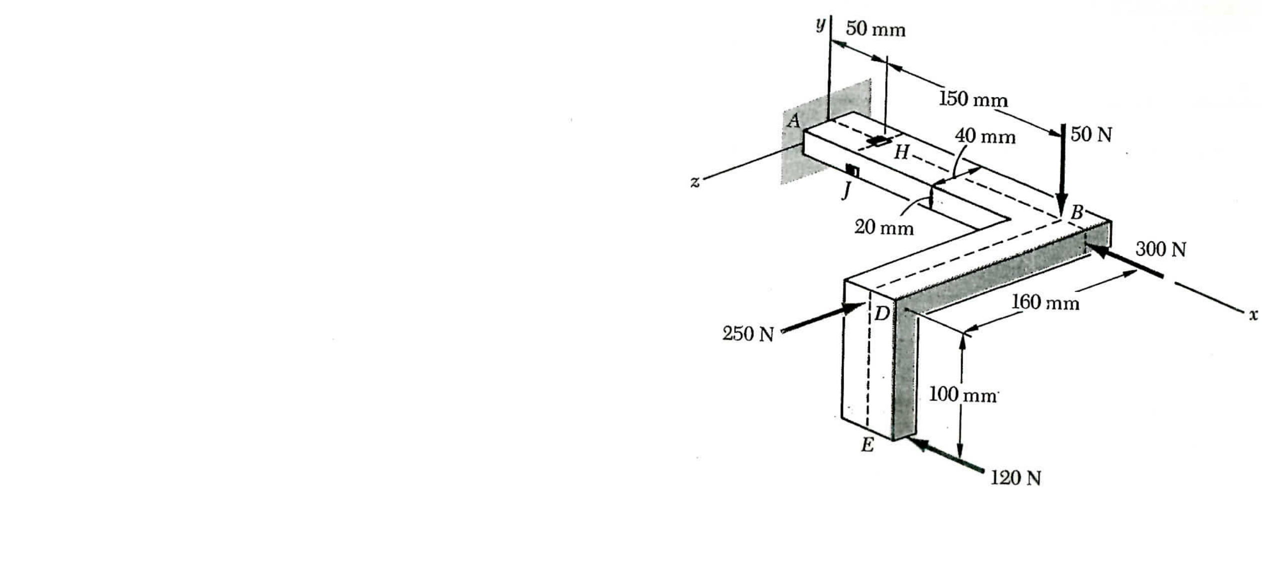


Use **Force Distance** method to find the moment of each force shown about x, y, and z axes. Then find total moment of all forces about point A. Draw vector representation of this moment.

Question 1:

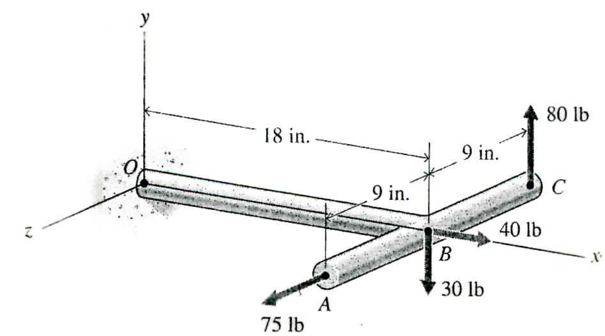


Question 2:

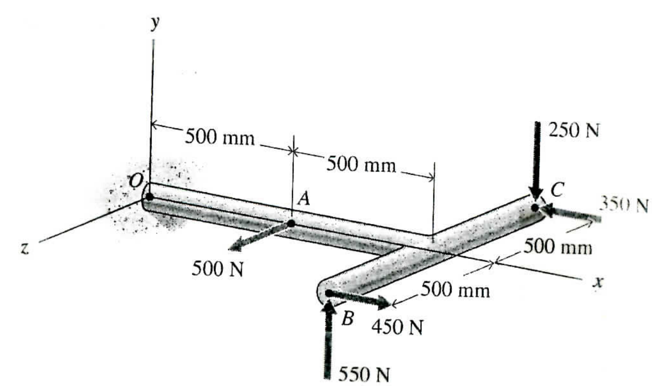


Use **Force Distance** method to find the moment of each force shown about x, y, and z axes. Then find total moment of all forces about point O. Draw vector representation of this moment.

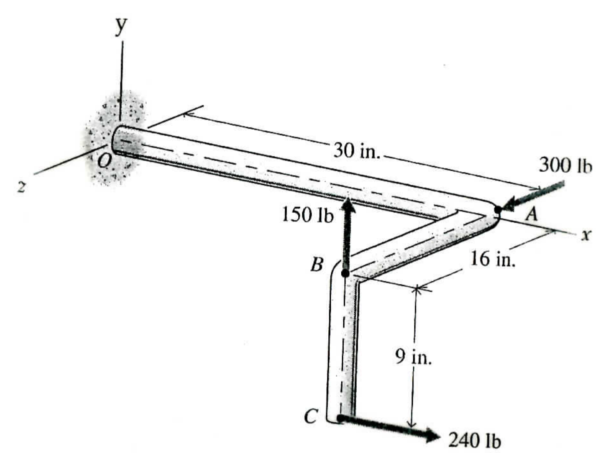
Question 3:



Question 4:



Question 5:



1.5 Vector Multiplication*

a. Dot (scalar) product

Figure 1.10 shows two vectors **A** and **B**, with θ being the angle between their positive directions. The *dot product* of **A** and **B** is defined as

$$\boxed{\mathbf{A} \cdot \mathbf{B} = AB\cos\theta \qquad (0 \leq \theta \leq 180^\circ)} \tag{1.19}$$

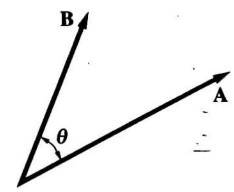


Fig. 1.10

Because the dot product is a scalar, it is also called the *scalar product.*

The dot product is positive if $\theta < 90^\circ$ and negative if $\theta > 90^\circ$. If **A** and **B** are parallel and have the same sense ($\theta = 0$), then $\mathbf{A} \cdot \mathbf{B} = AB$. For a vector dotted with itself, we see that $\mathbf{A} \cdot \mathbf{A} = A^2$. If **A** and **B** are parallel but have opposite sense ($\theta = 180^\circ$), then $\mathbf{A} \cdot \mathbf{B} = -AB$.

The following two properties of the dot product follow from its definition in Eq. (1.19).

- The dot product is commutative: $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$
- The dot product is distributive: $\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$

From the definition of the dot product, we also note that the base vectors of a rectangular coordinate system satisfy the following identities:

$$\begin{aligned} \mathbf{i} \cdot \mathbf{i} &= \mathbf{j} \cdot \mathbf{j} = \mathbf{k} \cdot \mathbf{k} = 1 \\ \mathbf{i} \cdot \mathbf{j} &= \mathbf{j} \cdot \mathbf{k} = \mathbf{k} \cdot \mathbf{i} = 0 \end{aligned} \tag{1.20}$$

When **A** and **B** are expressed in rectangular form, their dot product becomes

$$\mathbf{A} \cdot \mathbf{B} = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \cdot (B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k})$$

which, using the distributive property of the dot product and Eqs. (1.20), reduces to

$$\boxed{\mathbf{A} \cdot \mathbf{B} = A_xB_x + A_yB_y + A_zB_z} \tag{1.21}$$

Equation (1.21) is a powerful and relatively simple method for computing the dot product of two vectors that are given in rectangular form.

The following are two of the more important applications of the dot product.

Finding the Angle Between Two Vectors The angle θ between the two vectors **A** and **B** in Fig. 1.11 can be found from the definition of the dot product in Eq. (1.19), which can be rewritten as

$$\cos\theta = \frac{\mathbf{A} \cdot \mathbf{B}}{AB} = \frac{\mathbf{A}}{A} \cdot \frac{\mathbf{B}}{B}$$

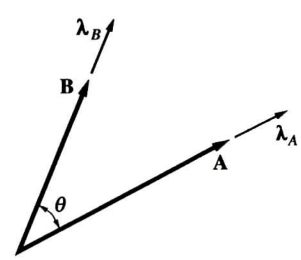


Fig. 1.11

*Note that division by a vector, such as 1/**A** or **B**/**A**, is not defined.

Letting $\boldsymbol{\lambda}_A = \mathbf{A}/A$ and $\boldsymbol{\lambda}_B = \mathbf{B}/B$ be the unit vectors that have the same directions as **A** and **B**, as shown in Fig. 1.11, the last equation becomes

$$\boxed{\cos\theta = \boldsymbol{\lambda}_A \cdot \boldsymbol{\lambda}_B} \tag{1.22}$$

If the unit vectors are written in rectangular form, this dot product is easily evaluated.

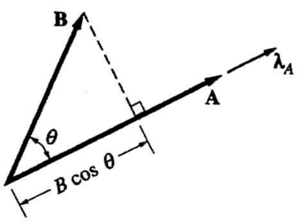


Fig. 1.12

Determining the Orthogonal Component of a Vector in a Given Direction If we project **B** onto **A** as in Fig. 1.12, the projected length $B\cos\theta$ is called the *orthogonal component of* **B** *in the direction of* **A**. Because θ is the angle between **A** and **B**, the definition of the dot product, $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$, yields

$$B\cos\theta = \frac{\mathbf{A}\cdot\mathbf{B}}{A} = \mathbf{B}\cdot\frac{\mathbf{A}}{A}$$

Because $\mathbf{A}/A = \boldsymbol{\lambda}_A$ (the unit vector in the direction of **A**), as shown in Fig. 1.12, the last equation becomes

$$\boxed{B\cos\theta = \mathbf{B}\cdot\boldsymbol{\lambda}_A} \tag{1.23}$$

Therefore,

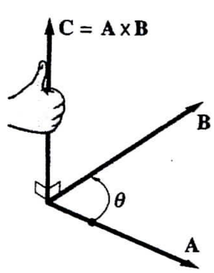
The orthogonal component of B in the direction of A equals $\mathbf{B}\cdot\boldsymbol{\lambda}_A$. (1.24)

The fact that the dot product is positive for $0 < \theta < 90°$ and negative for $90° < \theta < 180°$ means that Eq. (1.23) can be used to find any value of θ between 0° and 180°.

b. *Cross (vector) product*

The *cross product* **C** of two vectors **A** and **B**, denoted by

$$\boxed{\mathbf{C} = \mathbf{A}\times\mathbf{B}} \tag{1.25}$$



has the following characteristics (see Fig. 1.13):

- $C = AB\sin\theta$, where θ ($0 \le \theta \le 180°$) is the angle between the positive directions of **A** and **B**. (Note that C is always a positive number.)
- **C** is perpendicular to both **A** and **B**.
- The sense of **C** is determined by the right-hand rule, which states that when the fingers of your right hand are curled in the direction of the angle θ (directed from **A** toward **B**), your thumb points in the direction of **C**.*

It can be shown that the cross product is distributive; that is,

$$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{C})$$

However, the cross product is neither associative nor commutative. In other words,

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) \neq (\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$$
$$\mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A}$$

In fact, it can be deduced from the right-hand rule that $\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$.

From the definition of the cross product, we see that (1) if **A** and **B** are perpendicular ($\theta = 90°$), then $C = AB$; and (2) if **A** and **B** are parallel ($\theta = 0°$ or $180°$), then $C = 0$.

From the properties of the cross product, we deduce that the base vectors of a rectangular coordinate system satisfy the following identities:

$$\begin{array}{lll} \mathbf{i} \times \mathbf{i} = \mathbf{0} & \mathbf{j} \times \mathbf{j} = \mathbf{0} & \mathbf{k} \times \mathbf{k} = \mathbf{0} \\ \mathbf{i} \times \mathbf{j} = \mathbf{k} & \mathbf{j} \times \mathbf{k} = \mathbf{i} & \mathbf{k} \times \mathbf{i} = \mathbf{j} \end{array} \tag{1.26}$$

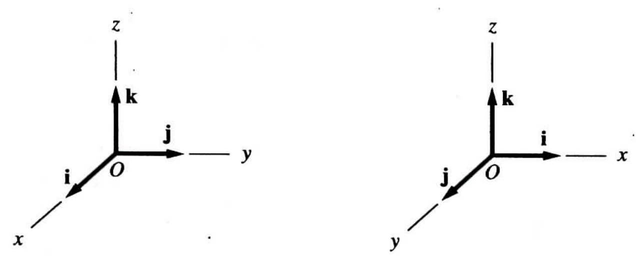
where the equations in the bottom row are valid in what is defined as a *right-handed* coordinate system. If the coordinate axes are labeled such that $\mathbf{i} \times \mathbf{j} = -\mathbf{k}$, $\mathbf{j} \times \mathbf{k} = -\mathbf{i}$, and $\mathbf{k} \times \mathbf{i} = -\mathbf{j}$, the system is said to be *left-handed.* Examples of both right- and left-handed coordinate systems are shown in Fig. 1.14.*

When **A** and **B** are expressed in rectangular form, their cross product becomes

$$\mathbf{A} \times \mathbf{B} = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \times (B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k})$$

Using the distributive property of the cross product and Eqs. (1.26), this equation reduces to

$$\boxed{\begin{aligned} \mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} \\ + (A_xB_y - A_yB_x)\mathbf{k} \end{aligned}} \tag{1.27}$$



Right-handed coordinate system ($\mathbf{i} \times \mathbf{j} = \mathbf{k}$, etc.)

Left-handed coordinate system ($\mathbf{i} \times \mathbf{j} = -\mathbf{k}$, etc.)

Fig. 1.14

*In this text, we assume that all rectangular coordinate systems are right-handed.

The identical expression is obtained when the rules for expanding a 3 × 3 determinant are applied to the following array of nine terms (because the terms are not all scalars, the array is not a true determinant):

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \tag{1.28}$$

You may use any method for determinant expansion, but you will find that the following technique, called *expansion by minors using the first row,* is very convenient.

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$
$$= a(ei - fh) - b(di - fg) + c(dh - eg)$$

Expanding Eq. (1.28) by this method, we find that the 2 × 2 determinants equal the **i**, **j**, and **k** components of the cross product.

c. Scalar triple product

Of the vector products that involve three or more vectors, the one that is most useful in statics is the scalar triple product. The *scalar triple product* arises when the cross product of two vectors is dotted with a third vector—for example, $\mathbf{A} \times \mathbf{B} \cdot \mathbf{C}$. When writing this product, it is not necessary to show the parentheses, because $\mathbf{A} \times \mathbf{B} \cdot \mathbf{C}$ can be interpreted only in one way—the cross product must be done first; otherwise the expression is meaningless.

Assuming that **A**, **B**, and **C** are expressed in rectangular form and recalling Eqs. (1.21) and (1.27), the scalar triple product becomes

$$\mathbf{A} \times \mathbf{B} \cdot \mathbf{C} = \left[(A_y B_z - A_z B_y)\mathbf{i} - (A_x B_z - A_z B_x)\mathbf{j} + (A_x B_y - A_y B_x)\mathbf{k}\right] \cdot (C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k})$$

Using the properties of the dot products of the rectangular base vectors, this expression simplifies to

$$\mathbf{A} \times \mathbf{B} \cdot \mathbf{C} = (A_y B_z - A_z B_y)C_x - (A_x B_z - A_z B_x)C_y + (A_x B_y - A_y B_x)C_z \tag{1.29}$$

Therefore, the scalar triple product can be written in the following determinant form, which is easy to remember:

$$\mathbf{A} \times \mathbf{B} \cdot \mathbf{C} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} \tag{1.30}$$

The following identities relating to the scalar triple product are useful:

$$\mathbf{A} \times \mathbf{B} \cdot \mathbf{C} = \mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = \mathbf{B} \cdot \mathbf{C} \times \mathbf{A} = \mathbf{C} \cdot \mathbf{A} \times \mathbf{B} \tag{1.31}$$

Observe that the value of the scalar triple product is not altered if the locations of the dot and cross are interchanged or if the positions of **A**, **B**, and **C** are changed—provided that the cyclic order *A–B–C* is maintained.