Internal Forces

7.1 Internal Forces Developed in Structural Members

The design of any structural or mechanical member requires an investigation of the loading acting within the member in order to be sure the material can resist this loading. These internal loadings can be determined by using the *method of sections*. To illustrate this method, consider the "simply supported" beam shown in Fig. 7–1*a*, which is subjected to the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ and the *support reactions* $\mathbf{A}_x$, $\mathbf{A}_y$, and $\mathbf{B}_y$, Fig. 7–1*b*. If the *internal loadings* acting on the cross section at C are to be determined, then an

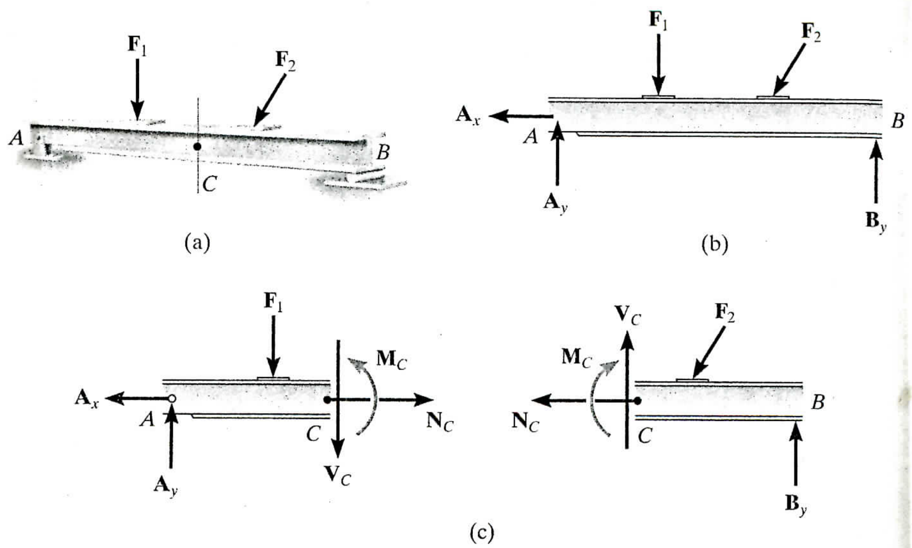


Fig. 7–1

imaginary section is passed through the beam, cutting it into two segments. By doing this the internal loadings at the section become *external* on the free-body diagram of each segment, Fig. 7–1*c*. Since both segments (*AC* and *CB*) were in equilibrium *before* the beam was sectioned, equilibrium of each segment is maintained provided rectangular force components $\mathbf{N}_C$ and $\mathbf{V}_C$ and a resultant couple moment $\mathbf{M}_C$ are developed at the section. Note that these loadings must be equal in magnitude and opposite in direction on each of the segments (Newton's third law). The magnitude of each of these loadings can now be determined by applying the three equations of equilibrium to either segment *AC* or *CB*. A *direct solution* for $\mathbf{N}_C$ is obtained by applying $\Sigma F_x = 0$; $\mathbf{V}_C$ is obtained directly from $\Sigma F_y = 0$; and $\mathbf{M}_C$ is determined by summing moments about point C, $\Sigma M_C = 0$, in order to eliminate the moments of the unknowns $\mathbf{N}_C$ and $\mathbf{V}_C$.

In mechanics, the force components $\mathbf{N}$, acting normal to the beam at the cut section, and $\mathbf{V}$, acting tangent to the section, are termed the *normal or axial force* and the *shear force*, respectively. The couple moment $\mathbf{M}$ is referred to as the *bending moment*, Fig. 7–2*a*. In three dimensions, a general internal force and couple moment resultant will act at the section. The *x, y, z* components of these loadings are shown in Fig. 7–2*b*. Here $\mathbf{N}_y$ is the *normal force*, and $\mathbf{V}_x$ and $\mathbf{V}_z$ are *shear force components*. $\mathbf{M}_y$ is a *torsional or twisting moment*, and $\mathbf{M}_x$ and $\mathbf{M}_z$ are *bending moment components*. For most applications, these *resultant loadings* will act at the geometric center or centroid (C) of the section's cross-sectional area. Although the magnitude for each loading generally will be different at various points along the axis of the member, the method of sections can always be used to determine their values.

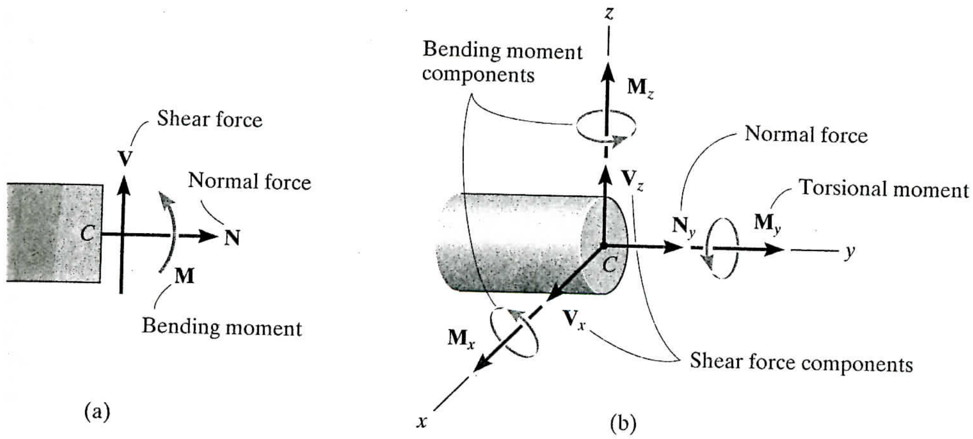


Fig. 7–2

Free-Body Diagrams. Trusses are composed of two-force members that only support normal loads. On the other hand, frames and machines are composed of *multiforce members*, and so each of these members will generally be subjected to internal normal, shear, and bending loadings. For example, consider the frame shown in Fig. 7–3*a*. If the blue section is passed through the frame to determine the internal loadings at points *H*, *G*, and *F*, the resulting free-body diagram of the top portion of this section is shown in Fig. 7–3*b*. At each point where a member is sectioned there is an unknown normal force, shear force, and bending moment. As a result, we cannot apply the *three* equations of equilibrium to this section in order to obtain these *nine unknowns*. Instead, to solve this problem we must *first dismember* the frame and determine the reactions at the connections of the members using the techniques of Sec. 6.6. Once this is done, *each member* may then be sectioned at its appropriate point, and the three equations of equilibrium can be applied to determine **N**, **V**, and **M**. For example, the free-body diagram of segment *DG*, Fig. 7–3*c*, can be used to determine the internal loadings at *G* provided the reactions of the pin, $\mathbf{D}_x$ and $\mathbf{D}_y$, are known.

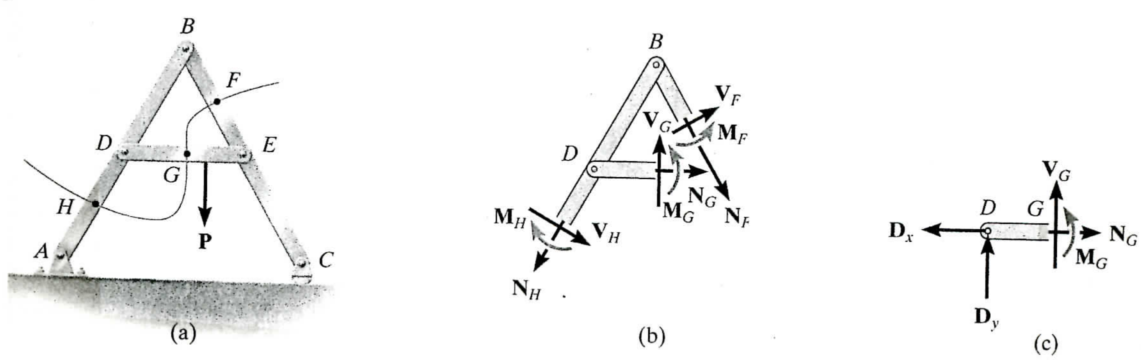
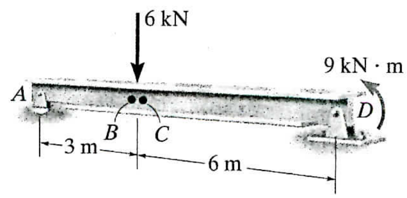


Fig. 7–3

EXAMPLE 7.3

The beam supports the loading shown in Fig. 7–6*a*. Determine the internal normal force, shear force, and bending moment acting just to the left, point *B*, and just to the right, point *C*, of the 6-kN force.



(a)

SOLUTION

Support Reactions. The free-body diagram of the beam is shown in Fig. 7–6*b*. When determining the *external reactions*, realize that the 9-kN · m couple moment is a free vector and therefore it can be placed *anywhere* on the free-body diagram of the entire beam. Here we will only determine $\mathbf{A}_y$, since segments *AB* and *AC* will be used for the analysis.

$$\circlearrowleft+\Sigma M_D = 0; \quad 9\text{ kN}\cdot\text{m} + (6\text{ kN})(6\text{ m}) - A_y(9\text{ m}) = 0$$

$$A_y = 5\text{ kN}$$

Free-Body Diagrams. The free-body diagrams of the left segments *AB* and *AC* of the beam are shown in Figs. 7–6*c* and 7–6*d*. In this case the 9-kN · m couple moment is *not included* on these diagrams since it must be kept in its *original position* until *after* the section is made and the appropriate body is isolated. In other words, the free-body diagrams of the left segments of the beam do not show the couple moment since this moment does not actually act on these segments.

Equations of Equilibrium.

Segment AB

$$\xrightarrow{+}\Sigma F_x = 0; \quad N_B = 0 \qquad \textit{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \quad 5\text{ kN} - V_B = 0 \qquad V_B = 5\text{ kN} \qquad \textit{Ans.}$$

$$\circlearrowleft+\Sigma M_B = 0; \quad -(5\text{ kN})(3\text{ m}) + M_B = 0 \qquad M_B = 15\text{ kN}\cdot\text{m} \qquad \textit{Ans.}$$

Segment AC

$$\xrightarrow{+}\Sigma F_x = 0; \quad N_C = 0 \qquad \textit{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \quad 5\text{ kN} - 6\text{ kN} + V_C = 0 \qquad V_C = 1\text{ kN} \qquad \textit{Ans.}$$

$$\circlearrowleft+\Sigma M_C = 0; \quad -(5\text{ kN})(3\text{ m}) + M_C = 0 \qquad M_C = 15\text{ kN}\cdot\text{m} \qquad \textit{Ans.}$$

Here the moment arm for the 5-kN force in both cases is approximately 3 m since *B* and *C* are "almost" coincident.

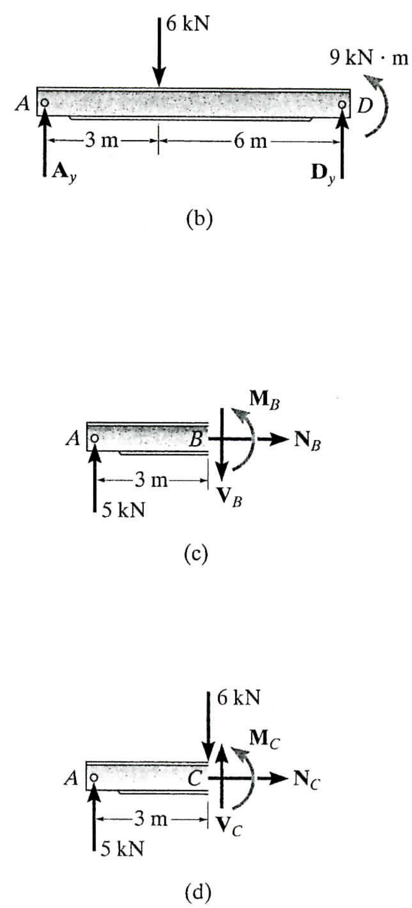
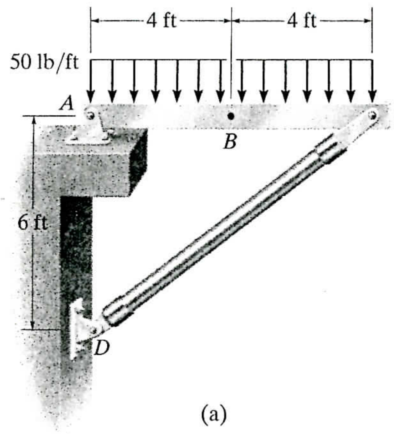


Fig. 7–6

EXAMPLE 7.4

Determine the internal normal force, shear force, and bending moment acting at point B of the two-member frame shown in Fig. 7–7a.



(a)

SOLUTION

Support Reactions. A free-body diagram of each member is shown in Fig. 7–7b. Since CD is a two-force member, the equations of equilibrium need to be applied only to member AC.

$$\circlearrowleft + \Sigma M_A = 0; \quad -400 \text{ lb } (4 \text{ ft}) + \left(\tfrac{3}{5}\right) F_{DC}(8 \text{ ft}) = 0 \qquad F_{DC} = 333.3 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad -A_x + \left(\tfrac{4}{5}\right)(333.3 \text{ lb}) = 0 \qquad A_x = 266.7 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 400 \text{ lb} + \tfrac{3}{5}(333.3 \text{ lb}) = 0 \qquad A_y = 200 \text{ lb}$$

Free-Body Diagrams. Passing an imaginary section perpendicular to the axis of member AC through point B yields the free-body diagrams of segments AB and BC shown in Fig. 7–7c. When constructing these diagrams it is important to keep the distributed loading exactly as it is until *after* the section is made. Only then can it be replaced by a single resultant force. Why? Also, notice that $\mathbf{N}_B$, $\mathbf{V}_B$, and $\mathbf{M}_B$ act with equal magnitude but opposite direction on each segment—Newton's third law.

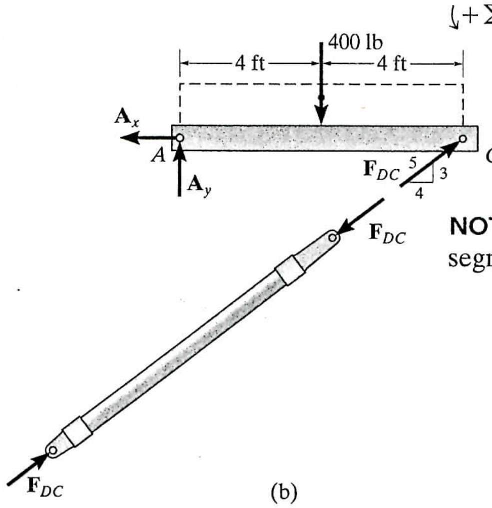
Equations of Equilibrium. Applying the equations of equilibrium to segment AB, we have

$$\xrightarrow{+} \Sigma F_x = 0; \quad N_B - 266.7 \text{ lb} = 0 \qquad N_B = 267 \text{ lb} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad 200 \text{ lb} - 200 \text{ lb} - V_B = 0 \qquad V_B = 0 \qquad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_B = 0; \quad M_B - 200 \text{ lb } (4 \text{ ft}) + 200 \text{ lb } (2 \text{ ft}) = 0$$

$$M_B = 400 \text{ lb} \cdot \text{ft} \qquad \textit{Ans.}$$



(b)

NOTE: As an exercise, try to obtain these same results using segment BC.

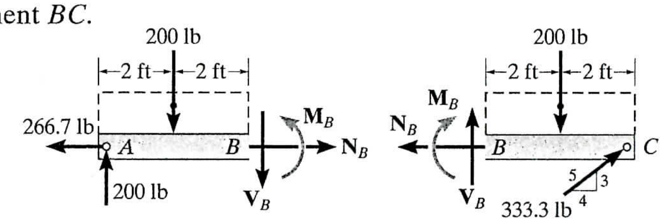


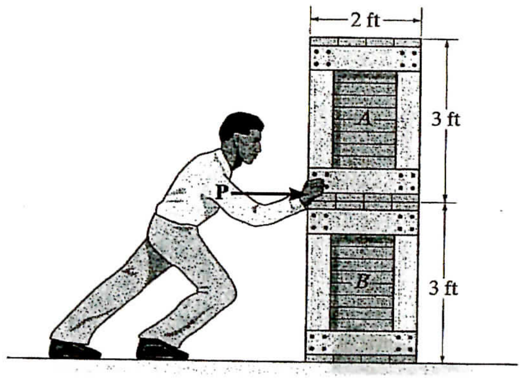
Fig. 7–7

(c)

Friction Questions:

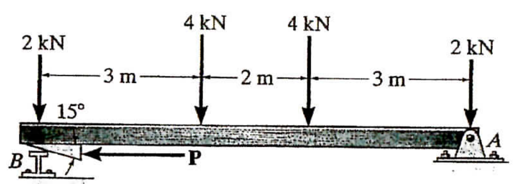
Question 1:

8–37. The man having a weight of 150 lb pushes horizontally on the bottom of crate A, which is stacked on top of crate B. Each crate has a weight of 100 lb. If the coefficient of static friction between each crate is $\mu_s = 0.8$ and between the bottom crate, his shoes, and the floor is $\mu_s' = 0.3$, determine if he can cause impending motion.



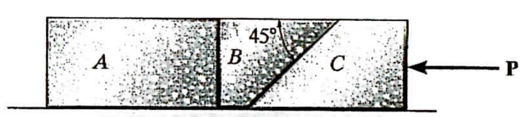
Question 2:

8–63. The wedge is used to level the floor of a building. For the floor loading shown, determine the horizontal force P that must be applied to move the wedge forward. The coefficient of static friction between the wedge and the two surfaces of contact is $\mu_s = 0.25$. Neglect the size and weight of the wedge and the thickness of the beam.



Question 3:

*8–64. The three stone blocks have weights of $W_A = 600$ lb, $W_B = 150$ lb, and $W_C = 500$ lb. Determine the smallest horizontal force P that must be applied to block C in order to move this block. The coefficient of static friction between the blocks is $\mu_s = 0.3$, and between the floor and each block $\mu_s' = 0.5$.



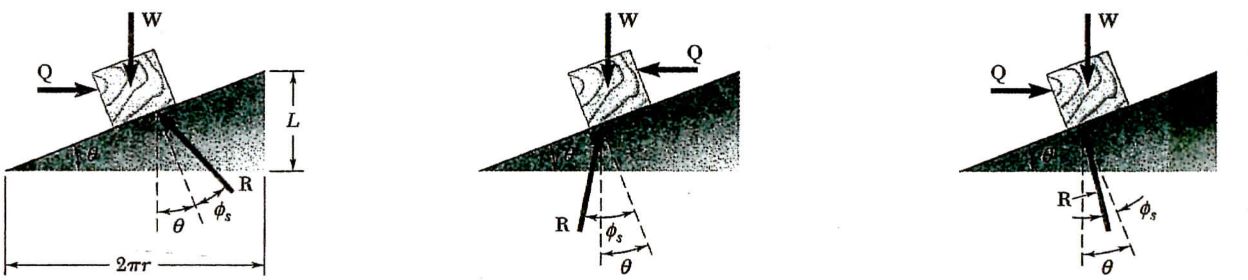
If:

α angle between **R** and the vertical axis,

θ angle between inclined surface and the horizontal axis,

and $\Phi = \tan^{-1}\mu_s$

then:

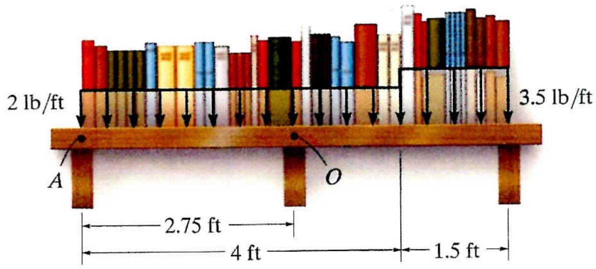


(*a*) Impending motion upward (*b*) Impending motion downward with $\phi_s > \theta$ (*c*) Impending motion downward with $\phi_s < \theta$

$\alpha = \Phi + \theta$ $\alpha = \Phi - \theta$ $\alpha = \theta - \Phi$

Question 1:

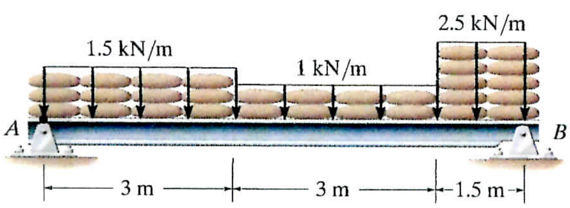
4–139. The loading on the bookshelf is distributed as shown. Determine the magnitude of the equivalent resultant location, measured from point *O*.



Prob. 4–139

Question 2:

4–146. The beam supports the distributed load caused by the sandbags. Determine the resultant force on the beam and specify its location measured from point *A*.



Prob. 4–146

Question 3: ***Center of Gravity of a Composite Body***

Problem Statement The base of the composite machine part shown in Fig. E8.13a has specific weight $\gamma = 78\ \text{kN/m}^3$. The remainder of the part has specific weight $\gamma = 26\ \text{kN/m}^3$. Determine the center of gravity of the part, with respect to the xyz axes shown.

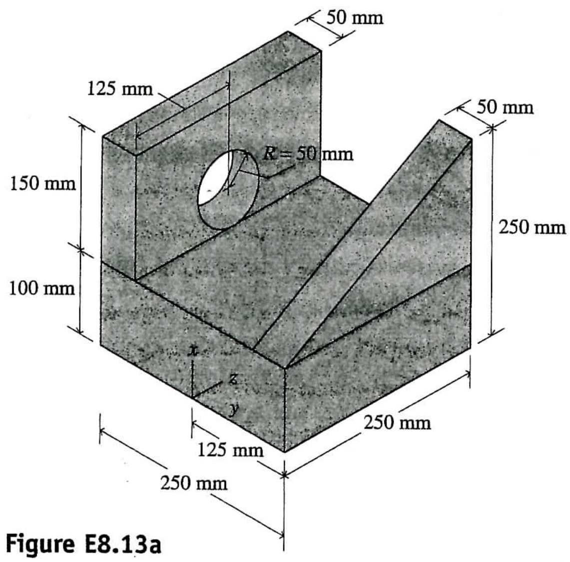


Figure E8.13a

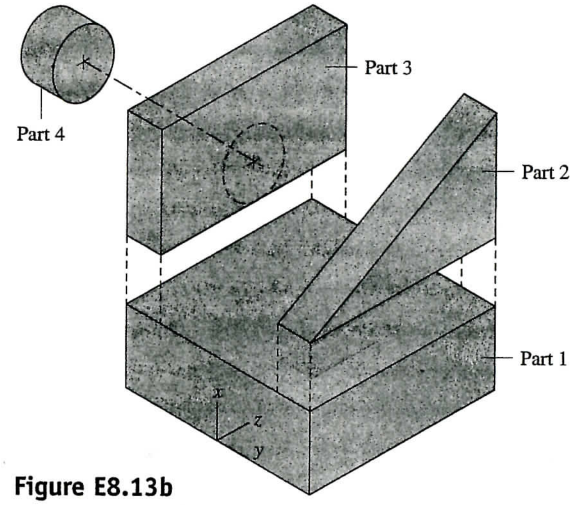


Figure E8.13b

Solution The assembly consists of the base (part 1), the triangular prism (part 2), and the vertical rectangular slab without the hole (part 3). A circular disk forming the hole (part 4) must be removed from part 3 (see Fig. E8.13b). Since the disk is removed, its weight is listed as negative in Table E8.13, which presents the results of calculations.

Using Eq. (8.1) and the sums from Table E8.13, we obtain coordinates of the center of gravity as

$$\bar{x} = \frac{35.04}{550.42} = 63.66\ \text{mm}$$

$$\bar{y} = \frac{-1.42}{550.42} = -2.58\ \text{mm}$$

$$\bar{z} = \frac{69.81}{550.42} = 126.83\ \text{mm}$$

Table E8.13
Center of gravity of a composite body

PART	VOLUME [10^6 mm^3]	WEIGHT [N]	CENTROID DISTANCES [mm]			MOMENTS OF THE WEIGHTS [N·m]		
i	V_i	$w_i = \gamma_i V_i$	x_i	y_i	z_i	$w_i x_i$	$w_i y_i$	$w_i z_i$
1	6.2500	487.50	50	0	125.00	24.38	0.00	60.94
2	0.9375	24.38	150	100	166.67	3.66	2.44	4.06
3	1.8750	48.75	175	−100	125.00	8.53	−4.88	6.09
4	−0.3927	−10.21	150	−100	125.00	−1.53	1.02	−1.28
Sums	—	550.42	—	—	—	35.04	−1.42	69.81