Symmetry Considerations

Mirror Image Symmetry

Both Products of Inertia involving the coordinate which is NORMAL to a cartesian coordinate plane of Mirror-Image-Symmetry (MIS) are ZERO - leaving only (possibly) that product associated the MIS plane itself.

The PRODUCT of INERTIA of a body with respect to a coordinate plane of Mirror-Image-Symmetry (MIS) is the ONLY ONE of the three Products which can have a non-zero value.

If MORE THAN ONE (i.e two or three) of the three Cartesian coordinate planes divide a body into MIS halves, then ALL three products of inertia are ZERO, and the associated Inertia Matrix reduces to the highly desirable “diagonal” form.

A Diagonal Inertia Matrix is the defining characteristic of a set of Principal Coordinates for a Body.

If $x$-$y$-$z$ is a set of Principal Coordinates (origin at pt. O) for a rigid Body, then

$$I_O(A) = I_x A_x \hat{i} + I_y A_y \hat{j} + I_z A_z \hat{k}$$
$$\vec{A} \cdot I_O(\vec{B}) = I_x A_x B_x + I_y A_y B_y + I_z A_z B_z$$
$$\vec{A} \times I_O(\vec{B}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ I_x B_x & I_y B_y & I_z B_z \end{vmatrix}$$

in terms of the mass moments of inertia of the body with respect to the three principal coordinate axes.

Note: It is an important mathematical fact (not proved here) that there exists at least one principal coordinate system at any specified origin, for any specified body, regardless of the apparent lack of symmetry of the body in question. Thus, the dynamical analysis of rigid bodies can, at least in principle, always be carried out in a principal coordinate system in which the Inertia Matrix for the body has the desirable diagonal form.
Mirror Image Symmetry

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