**Important Inertia Function Facts**

**Fact#1:** All products of inertia involving the normal coordinate to a plane of mirror image symmetry are identically zero.

**proof:**

\[ P_1 : \ m_{P_1} = m \quad \text{and} \quad (x_{P_1}, y_{P_1}, z_{P_1}) = (a, b, +c) \]
\[ P_2 : \ m_{P_2} = m \quad \text{and} \quad (x_{P_2}, y_{P_2}, z_{P_2}) = (a, b, -c) \]

\[
I_{zx} = I_{xz} = \sum_{P \in \mathcal{B}} m_P (x_P z_P)
= m_P (x_P z_P) + m_P (x_P z_P) + \{ \text{similar terms for other mirror image pairs} \}
= m(a c) + m[a(-c)] + \{ \text{similar terms for other mirror image pairs} \}
I_{zx} = I_{xz} = 0
\]

\[
I_{zy} = I_{yz} = \sum_{P \in \mathcal{B}} m_P (y_P z_P)
= m_P (y_P z_P) + m_P (y_P z_P) + \{ \text{similar terms for other mirror image pairs} \}
= m(b c) + m[b(-c)] + \{ \text{similar terms for other mirror image pairs} \}
I_{zy} = I_{yz} = 0
\]

**Fact#2:** If a material body \( \mathcal{B} \) is mirror image symmetric with respect to two (or more) coordinate planes, then all three of its products of inertia are zero and the mass moment representation matrix (inertia matrix) is diagonal, i.e.

\[
\begin{bmatrix}
I_x & 0 & 0 \\
0 & I_y & 0 \\
0 & 0 & I_z
\end{bmatrix}
\]

**Terminology:** A set of Cartesian coordinates relative to which a material body \( \mathcal{B} \) has a diagonal inertia matrix (as above) are said to be principal coordinates for \( \mathcal{B} \).