Simple fixed axis rotation (coordinate-based/matrix approach)

When a rigid body or structure rotates about a fixed axis, then ANY point along the body's axis of rotation qualifies as an ANOTHER point. Thus, the body's IFD can be constructed by placing the body's inertial force

\[ iF = M\Omega \hat{\omega} \]  @ pt. 0

at any such pt. 0 along the axis of rotation, with the 0-based inertial couple

\[ iC_0 = \Gamma_0\hat{\omega} + \omega \times \Gamma_0\hat{\omega} \]

By selecting a Cartesian (x,y,z) system with its origin at pt. 0 and its z-axis aligned along the rotation axis, it follows that

\[ \hat{\omega} = \Omega \hat{k} \quad \text{&} \quad \hat{\omega} = \Omega \hat{k} \]

and

\[ iC_0 = (-I_{yz} \hat{\Omega} + I_{yx} \Omega^2)\hat{i} + (-I_{yz} \hat{\Omega} - I_{yx} \Omega^2)\hat{j} + I_z \Omega \hat{k} \]

from the inertial matrix expansions (p.7 of ECO)

[Diagram of IFD with\( \hat{\omega} \) and\( \Gamma_0 \hat{\omega} \) labelled]

Note: \( \Gamma_0 = \Gamma_{xy} \hat{i} + \Gamma_{yz} \hat{j} \)

Since it would necessarily have simple circular motion about the z-axis.
A Rigid Body/Structure experiencing this sort of motion is said to be **STATICALLY** Balanced if its mass center $G$ is located on (along) the rotation axis, in which case

$$\vec{F} = M\vec{a}_G = \vec{0}$$

leaving only the 3-components of Inertial couple.

A statically balanced assembly is further designated as being **DYNAMICALLY** Balanced whenever the $(xz)$ & $(yz)$ products of inertia are zero, i.e.

$$I_{xz} = I_{yz} = \Omega.$$ 

Thus, the TFO for a **DYNAMICALLY** Balanced Rigid Body reduces to:

![Diagram](image)

This one remaining component of inertial couple is, of course, related to the torque (about the fixed axis of rotation) that must be provided in order to change (speed up or slow down) the angular rotation speed $\Omega$. Most important, however, is the fact that the bearings which sustain this Bodies fixed-axis will not be required to produce any lateral (xy-plane) rotating support reactions.
Steady, Fixed (z) Axis Rotation of Statically Balanced Shafts

\[
\begin{align*}
\vec{\omega} &= \Omega \hat{\mathbf{k}} \quad (\Omega = \text{const}) \\
\vec{\alpha} &= 0 \\
\vec{\alpha}_G &= 0 \Rightarrow \vec{i}F = M \vec{G}_C = 0
\end{align*}
\]

\[
iF = 0 \\
iG_C = I_{yz} \Omega^2 \hat{\mathbf{z}} - I_{xz} \Omega^2 \hat{\mathbf{x}}
\]

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\[
\text{ST} \quad \overrightarrow{EG} \\
I_{xz} \Omega^2 \\
I_{yz} \Omega^2
\]

\[
\begin{align*}
\frac{h F_y}{h F_x} &= \frac{I_{yz} \Omega^2}{I_{xz} \Omega^2} \\
R_b &= V \sqrt{F_x^2 + F_y^2} = \sqrt{\frac{I_{xz}^2 + I_{yz}^2}{h^2}} \Omega^2
\end{align*}
\]

Static Equivalence: Dynamic Bearing Reactions which rotate w/ the shaft!