Generalised Ballot Theorem in a Combinatorial Analysis of $M/M^{[m]}/1$ Queue

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ABSTRACT: Neuts' Matrix Geometric Method makes use of the left-skip free characteristic of $M/G/1$ type Markov chains and determines the first passage distribution matrix G by solving a non-linear matrix equation. In this focus we focus on the k-step first passage problem. In particular, we identify three associated matrices, namely the matrix G_k, the conditional first passage probability matrix P_k, and the first passage count matrix T_k. The reformulation allows for combinatorial techniques. Specifically, we refer to an extension of Takacs' Ballot theorem. We note that the matrix P_k exhibits some ballot properties. In the case of $M/M^{[m]}/1$ queue, we establish the special structure of the count matrix T_k using lattice path arguments. Furthermore, we obtain a closed form expression for the G matrix where the first passage probabilities are expressed in terms of generalised hyper-geometric functions.

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