Hilbert Polynomial of Ladder Determinantal Ideals: A Perspective

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A problem concerning these is to find the Hilbert function of the ideal I_p (L) of all p-by-p minors in a ladder shaped (like Ferrer's diagram like) subset L of the m-by-n rectangle. Or, more generally, of the ideal I_p (a; Y) for any saturated subset Y of the rectangle. Note that when Y is an m-by-n rectangle, Abhyankar gives the Hilbert function of I_p (a; Y) = I_p (a). For a ladder L, an explicit formula for the Hilbert function of I_p (L), in the case of p = 2, was obtained by Kulkarni in 1985. In 1987, Abhyankar and Kulkarni showed that the ideal I_p (a; Y) is Hilbertian, i.e., its Hilbert function is a polynomial for all nonnegative integers. However, there remained the problem of finding an explicit formula for the Hilbert function of Ip (L) for any p. This has been solved in 2002 by Ghorpade. Also noteworthy is the work of Krattenthaler and Prohaska (1999, TAMS), which solved the problem for one-sided ladder determinantal ideals. There have been extensive results on the coefficients of the Hilbert polynomial related to counting of non-intersecting families of paths by Modak (1992) and Krattenthaler (1992). We will provide a perspective on this productive journey of collaboration between algebra and combinatorics.