Chapter 2

2.4 Energy Balance

If work is positive when the system performs work on the surroundings, the energy balance on the system is written as

\[ \Delta E = E_2 - E_1 = \Delta KE + \Delta PE + \Delta U = Q - W \quad (2.4-1a) \]

In this equation \( E_2 \) and \( E_1 \) are the final and initial energy of the system, \( Q \) is the heat transferred to the system, and \( W \) is the work performed by the system.

If work is positive when the system receives work from the surroundings, the energy balance on the system is written as

\[ \Delta E = E_2 - E_1 = \Delta KE + \Delta PE + \Delta U = Q + W \quad (2.4-1b) \]

In this equation \( W \) is the work received by the system. No matter what is the sign convention for work, the system energy will increase when work is performed on the system. As an example of the sign convention for work, let consider the expansion work when the system volume change by \( dV \). If work is considered to be positive when performed by the system then

\[ \delta W = pdV \quad (2.4-2a) \]

When the volume increases \( dV > 0 \), the gas or system performs work on the surroundings and \( \delta W \) is positive. If If work is considered to be positive when performed by the surroundings on the system then

\[ \delta W = -pdV \quad (2.4-2b) \]

When the volume decreases \( dV < 0 \), the surroundings perform work on the gas and \( \delta W \) is positive. The differential form of the energy balance with positive work performed by the system is

\[ dE = \delta Q - \delta W = \delta Q - pdV \quad (2.4-3a) \]
The differential form of the energy balance with positive work performed by the surroundings is

\[ dE = dQ + dW = \delta Q - pdV \]  \hspace{1cm} (2.4-3b)

The final result of the energy balance must be independent of the sign convention for work. The instantaneous time rate form of the energy balance is

\[ \frac{dE}{dt} = \frac{dKE}{dt} + \frac{dPE}{dt} + \frac{dU}{dt} = \dot{Q} + \dot{W} \]  \hspace{1cm} (2.4-4)

**Example 2.4-1**

Air is contained in a vertical-cylinder assembly fitted with an electrical resistor. The atmosphere exerts a pressure of of 14.7 lbf/in\(^2\) on top of the piston, which has a mass of 100 lb and a face area of 1 ft\(^2\). Electric current passes through the resistor, and the volume of the air slowly increases by 1.6 ft\(^3\) while its pressure remains constant. The mass of the air is 0.6 lb, and its specific internal energy increases by 18 Btu/lb. The air and piston are at rest initially and finally. The piston-cylinder is a ceramic composite and thus a good insulator. Friction between the piston and cylinder wall can be ignored, and the local acceleration of gravity is \( g = 32.0 \text{ ft/s}^2 \). Determine the heat transfer from the resistor to the air, in Btu, for a system consisting of (a) the air alone, (b) the air and the piston.\(^9\)

\[ \begin{align*}
\rho_{\text{atm}} &= 14.7 \text{ lbf/in}^2 \\
 m_{\text{piston}} &= 100 \text{ lb} \\
 A_{\text{piston}} &= 1 \text{ ft}^2 \\
 V_2 - V_1 &= 1.6 \text{ ft}^3 \\
 \Delta u_{\text{air}} &= 18 \text{ Btu/lb}
\end{align*} \]

**Solution**

(a) Taking the air as the system, the energy balance becomes

\[ \Delta KE + \Delta PE + \Delta U = Q + W \]

Since \( \Delta KE = 0 \), \( \Delta PE = 0 \), solving for \( Q \) we have

\[ Q = \Delta U - W \]

The work performed by the air is given by

\[ W = - \int_{V_1}^{V_2} pdV = -p(V_2 - V_1) \]

The gas pressure on the piston can be determined from

\[ pA_{\text{piston}} = m_{\text{piston}}g + \rho_{\text{atm}}A_{\text{piston}} \]

\[ p = \frac{m_{\text{piston}}}{A_{\text{piston}}} + p_{\text{atm}} \]

\[ p = \frac{(100 \text{ lb})(32.0 \text{ ft/s}^2)}{1 \text{ ft}^2} - \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} - \frac{1 \text{ ft}^2}{144 \text{ in}^2} + 14.7 \frac{\text{ lbf}}{\text{ in}^2} = 15.4 \frac{\text{ lbf}}{\text{ in}^2} \]

Thus the work is

\[ W = -p(V_2 - V_1) = -15.4 \frac{\text{ lbf}}{\text{ in}^2} (1.6 \text{ ft}^3) \frac{144 \text{ in}^2}{1 \text{ ft}^2} = -3,548.2 \text{ ft} \cdot \text{lbf} \]

\[ W = -3,548.2 \text{ ft} \cdot \text{lbf} \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} = -4.56 \text{ Btu} \]

The heat transfer is

\[ Q = \Delta U - W = m_{\text{air}} \Delta u_{\text{air}} - W \]

\[ Q = (0.6 \text{ lb})(18 \text{ Btu/lb}) + 4.56 \text{ Btu} = 15.4 \text{ Btu} \]

(b) Taking the air and the piston as the system, the energy balance becomes

\[ (\Delta KE + \Delta PE + \Delta U)_{\text{air}} + (\Delta KE + \Delta PE + \Delta U)_{\text{piston}} = Q + W \]

Since \( \Delta KE_{\text{air}} = 0 \), \( \Delta PE_{\text{air}} \approx 0 \), and \( \Delta KE_{\text{piston}} = 0 \) solving for \( Q \) we have

\[ Q = \Delta U_{\text{air}} + \Delta PE_{\text{piston}} - W \]

The work is done by the upper surface of the piston where \( p = p_{\text{atm}} \):

\[ W = -\int_{V_1}^{V_2} p dV = -p_{\text{atm}}(V_2 - V_1) \]

\[ W = -14.7 \frac{\text{ lbf}}{\text{ in}^2} (1.6 \text{ ft}^3) \frac{144 \text{ in}^2}{1 \text{ ft}^2} \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} = -4.35 \text{ Btu} \]

The elevation change \( \Delta z \) can be evaluated from the volume change of the air.

\[ \Delta z = \frac{V_2 - V_1}{A_{\text{piston}}} = \frac{1.6 \text{ ft}^3}{1 \text{ ft}^2} = 1.6 \text{ ft} \]

The potential energy change of the piston is

\[ \Delta PE_{\text{piston}} = m_{\text{piston}} g \Delta z \]
\[ \Delta P_{\text{piston}} = (100 \text{ lb})(32.0 \text{ ft/s}^2)(1.6 \text{ ft}) \frac{1 \text{ lbf}}{32.2 \text{ lb} \cdot \text{ft/s}^2} \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} = 0.2 \text{ Btu} \]

\[ Q = m_{\text{air}} \Delta u_{\text{air}} + \Delta P_{\text{piston}} - W \]

\[ Q = (0.6 \text{ lb})(18 \text{ Btu/lb}) + 0.2 \text{ Btu} + 4.35 \text{ Btu} = 15.4 \text{ Btu} \]

**Example 2.4-2**

A small well insulated cylinder and piston assembly contains an ideal gas at 10.13 bar and 294.3 K. A mechanical lock prevents the piston from moving. The length of the cylinder containing the gas is 0.305 m and the piston cross sectional area is \(1.858 \times 10^{-2} \text{ m}^2\).

![Diagram of piston moves from A to B](image)

The piston, which weights 226 kg, is tightly fitted and when allowed to move, there are indications that considerable friction is present. When the mechanical lock is released, the piston moves in the cylinder until it impacts and is engaged by another mechanical stop; at this point, the gas volume has just double. The heat capacity of the ideal gas is 20.93 J/mol\(\cdot\)K, independent of temperature and pressure. Consider the heat capacity of the piston and cylinder walls to be negligible. As an engineer, can you estimate the temperature and pressure of the gas after such an expansion? Clearly state any assumption. Repeat the calculations if the cylinder were rotated 90° and 180° before tripping the mechanical lock.

**Solution**

Let system = gas in the cylinder

**Case 1:** piston rises

\[ \Delta E_{\text{surroundings}} = Q + W \]

\[ Q = 0. \]

Work done on the surroundings is given by

\[ W = P_a(V_f - V_i) + m_p g \Delta z, \text{ where } \Delta z = 0.305 \text{ m} \]

\[ V_i = \text{initial gas volume} = (0.305)(1.858 \times 10^{-2}) = 5.6669 \times 10^{-3} \text{ m}^3 \]

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\( V_f = \text{final gas volume} = 2 \times 5.6669 \times 10^{-3} \, \text{m}^3 = 1.1334 \times 10^{-2} \, \text{m}^3 \)

\( W = (1.013 \times 10^5)(5.6669 \times 10^{-3}) + (226)(9.81)(0.305) = 1.250 \, \text{kJ} \)

\( \Delta E_{\text{surroundings}} = - \Delta E_{\text{system}} = 1.250 \, \text{kJ} = - \Delta U_{\text{system}} = - N_g C_v (T_{gf} - T_{gi}) \)

\( N_g = \text{moles of gas} = \frac{P_g V_{gi}}{R T_{gi}} = \frac{(10.13 \times 10^5)(5.6669 \times 10^{-3})}{(8.314)(294.3)} = 2.3461 \, \text{mol} \)

\( T_{gf} = T_{gi} - 1.250 \, J/(N_g C_v) = 294.3 - 1.250/(2.3461 \times 20.93) = 268.8 \, \text{K} \)

\( P_{gf} = \frac{N_g R T_{gf}}{V_{gf}} = \frac{(2.3461)(8.314)(268.8)}{1.1334 \times 10^{-2}} = 462,681 \, \text{Pa} \)

**Case 2:** piston moves horizontally (cylinder was rotated 90°)

\( \Delta E_{\text{surroundings}} = Q + W \)

\( Q = 0. \)

Work done on the surroundings is given by

\( W = P_a (V_f - V_i) - m_p g \Delta z, \) where \( \Delta z = 0.305 \, \text{m} \)

\( W = (1.013 \times 10^5)(5.6669 \times 10^{-3}) + (226)(9.81)(0.305) = 574 \, \text{J} \)

\( \Delta E_{\text{surroundings}} = - \Delta E_{\text{system}} = 574 \, \text{J} = - \Delta U_{\text{system}} = - N_g C_v (T_{gf} - T_{gi}) \)

\( T_{gf} = T_{gi} - 574 \, J/(N_g C_v) = 294.3 - 1.250/(2.3461 \times 20.93) = 282.6 \, \text{K} \)

\( P_{gf} = \frac{N_g R T_{gf}}{V_{gf}} = \frac{(2.3461)(8.314)(282.6)}{1.1334 \times 10^{-2}} = 486,380 \, \text{Pa} \)

**Case 3:** piston moves down (cylinder was rotated 180°)

\( \Delta E_{\text{surroundings}} = Q + W \)

\( Q = 0. \)

Work done on the surroundings is given by

\( W = P_a (V_f - V_i) - m_p g \Delta z, \) where \( \Delta z = 0.305 \, \text{m} \)

\( W = (1.013 \times 10^5)(5.6669 \times 10^{-3}) - (226)(9.81)(0.305) = -102 \, \text{J} \)
\[ \Delta E_{\text{surroundings}} = - \Delta E_{\text{system}} = 1.250 \text{ kJ} = - \Delta U_{\text{system}} = - N_g C_v (T_{gf} - T_{gi}) \]

\[ T_{gf} = T_{gi} + 102 J/(N_g C_v) = 294.3 + 1.250/(2.3461 \times 20.93) = 296.4 \text{ K} \]

\[ P_{gf} = \frac{N_g R T_{gf}}{V_{gf}} = \frac{(2.3461)(8.314)(296.4)}{1.1334 \times 10^{-2}} = 510,000 \text{ Pa} \]

The following Matlab program determines the final gas temperature and pressure for three cases: (1) Piston rises, (2) Piston moves horizontally, and (3) Piston moves down. The program can be saved as ex2d4d2.m

```matlab
mp=226;                   % kg
g=9.81;                     % m/s2
dx=.305                     % m
R=8.314; Cv=20.93 % J/mol×K
Vi=5.6669e-3          % m^3
Vf=2*Vi; Tgi=294.3 % K
Pg=10.13e5       % Pa
Ng=Pg*Vi/(R*Tgi)  % Moles
Pa=1.013e5;           % Pa
% Case 3: Piston rises
disp('Piston rises')
W=Pa*(Vf-Vi)+mp*g*dx;
Tgf=Tgi-W/(Ng*Cv);
Pgf=Ng*R*Tgf/Vf;
fprintf('Work on the surroundings (J) = %g
',W)
fprintf('Final gas temperature (K)       = %g
',Tgf)
fprintf('Final gas pressure (Pa)       = %g
',Pgf)
% Case 2: Piston moves horizontally
disp(' ')
disp('Piston moves horizontally')
W=Pa*(Vf-Vi);
Tgf=Tgi-W/(Ng*Cv);
Pgf=Ng*R*Tgf/Vf;
fprintf('Work on the surroundings (J) = %g
',W)
fprintf('Final gas temperature (K)       = %g
',Tgf)
fprintf('Final gas pressure (Pa)       = %g
',Pgf)
% Case 3: Piston moves down
disp(' ')
disp('Piston moves down')
W=Pa*(Vf-Vi)-mp*g*dx;
Tgf=Tgi-W/(Ng*Cv);
Pgf=Ng*R*Tgf/Vf;
fprintf('Work on the surroundings (J) = %g
',W)
fprintf('Final gas temperature (K)       = %g
',Tgf)
fprintf('Final gas pressure (Pa)       = %g
',Pgf)
>> ex2d4d2
```
Piston rises
Work on the surroundings (J) = 1250.26
Final gas temperature (K) = 268.839
Final gas pressure (Pa) = 462681

Piston moves horizontally
Work on the surroundings (J) = 574.057
Final gas temperature (K) = 282.61
Final gas pressure (Pa) = 486380

Piston moves down
Work on the surroundings (J) = -102.146
Final gas temperature (K) = 296.38
Final gas pressure (Pa) = 510080

Example 2.4-3
The rate of heat transfer between a certain electric motor and its surroundings varies with time as

\[ \dot{Q} = -0.2\left[1 - e^{(-0.05t)}\right] \]

In this equation, \( t \) is in seconds and \( \dot{Q} \) is in kW. The shaft of the motor rotates at a constant speed of \( \omega = 100 \text{ rad/s} \) and applies a constant torque of \( \mathcal{S} = 18 \text{ N\cdotm} \) to an external load. The motor draws a constant electric power input equal to 2.0 kW. For the motor, plot \( \dot{Q} \) and \( \dot{W} \), each in kW, and the change in energy, \( \Delta E \), in kJ, as function of time from \( t = 0 \) to \( t = 100 \text{ s} \).

Solution
Let the system be the motor, the instantaneous time rate form of the energy balance is

\[ \frac{dE}{dt} = \dot{Q} + \dot{W} \]

\( \dot{W} \) is the net power on the system, therefore \( \dot{W} = \dot{W}_{\text{elec}} - \dot{W}_{\text{shaft}} \)

\( \dot{W}_{\text{elec}} = 2.0 \text{ kW} \)

\( \dot{W}_{\text{shaft}} = \mathcal{S}\omega = (18 \text{ N\cdotm})(100 \text{ rad/s}) = 1,800 \text{ W} = 1.8 \text{ kW} \)

\( \dot{W} = \dot{W}_{\text{elec}} - \dot{W}_{\text{shaft}} = 2.0 - 1.8 = 0.2 \text{ kW} \)

The accumulation of energy is

\[ \frac{dE}{dt} = -0.2\left[1 - e^{(-0.05t)}\right] + 0.2 = 0.2e^{(-0.05t)} \]

The change in energy at any time is \( \Delta E = \int_0^t \frac{dE}{dt} \, dt = \int_0^t 0.2e^{(-0.05t)} \, dt \)

\[ \Delta E = E_2 - E_1 = \left. \frac{0.2}{(-0.05)} e^{(-0.05t)} \right|_0^t = 4[1 - e^{(-0.05t)}] \]

The following Matlab program plots \( \dot{Q} \) and \( \dot{W} \), and the change in energy, \( \Delta E \), as a function of time.

```matlab
% Example 2.4-3
tw=[0 100]; Power=[0.2 0.2];
t=0:100;
Heat_rate=-0.2*(1-exp(-0.05*t));
Del_E=4*(1-exp(-0.05*t));
% Plot Del_E
subplot(2,1,1);plot(t,Del_E)
xlabel('Time, s');ylabel('E_2-E_1, kJ')
% Plot Power and Heat_rate
subplot(2,1,2),plot(tw,Power,t,Heat_rate,:)
axis([0 100 -0.2 0.3])
xlabel('Time, s');ylabel('Power, Heat_rate, kW')
legend('Power','Heat_rate')
```