4. Coding Techniques for Analog Signal

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Coding Techniques for Analog Source

- There are several analog source coding techniques
- Most of the coding techniques are applied speech and image coding

Three type of analog source encoding
- **Temporal Waveform coding**: design to represent digitally the time-domain characteristic of the signal
- **Spectral waveform coding**: signal waveform is sub divided into different frequency band and either the time waveform in each band or its spectral characteristics are encoded.
- **Model-based coding**: Based on the mathematical model of source.
Temporal Waveform Coding

Most common used methods:
- Pulse-code modulation (PCM)
- Differential pulse-code modulation (DPCM)
- Delta modulation (DM)

Pulse-code modulation (PCM)

Let’s have continuous source function $x(t)$ and each sample taken from $x(t)$ is $x_n$ at sampling rate $f_s \geq 2W$, where $W$ is the highest frequency in $x(t)$.

In PCM, each sample is quantized to one of $2^R$ amplitude levels, where $R$ is the number of binary digits used to represent each sample. The bit rate will be $Rf_s$ [bit/s].

- The quantized value will be $\tilde{x}_n$ and
  $$\tilde{x}_n = x_n - q_n,$$
  where $q_n$ is quantization error.

- Assume that a uniform quantizer is used, then PDF of quantization error is
  $$p(q) = \begin{cases} \frac{1}{\Delta} & \text{if } -\frac{\Delta}{2} \leq q \leq \frac{\Delta}{2} \\ 0 & \text{o.w.} \end{cases}$$

$\Delta$ is step size and obtained
  $$\Delta = 2^{-R}$$
Pulse-code modulation (PCM)

Mean square value of the quantization error (or noise) power is

\[
E(q^2) = \int_{-\Delta/2}^{\Delta/2} q^2 p(q) dq = \frac{1}{12} \Delta^2 = \frac{1}{12} 2^{2^R}
\]

Mean square value of the quantization error power in dB

\[
E(q^2)_{\text{dB}} = 10 \log \frac{1}{12} \Delta^2 = 10 \log \frac{1}{12} 2^{-2^R} = -10.8 - 6R [\text{dB}]
\]

Quantization noise decreases by 6dB/bit

Quantization noise for 8 bit -58.8 dB

It can be measured by signal-to-quantization noise ratio (SQNR) in dB

\[
\text{SQNR} = 10 \log \frac{\text{average power of source signal}}{E(q^2)} [\text{dB}]
\]

If source is sinusoidal

\[
\text{SQNR} = -1.76 + 6.02R [\text{dB}]
\]

The non-uniform quantizer characteristic can be obtained by passing the signal through a non-linear device that compresses the signal amplitude

For example: \(\mu\)-law compressor: A Logarithmic compressor

\[
|y| = \frac{\log(1 + \mu |x|)}{\log(1 + \mu)}
\]

\(\mu\) is a parameter that gives desired compression

\(\mu = 225\) selected for USA and Canada. \(\mu = 225\), it will drop quantization noise power about –77dB for 7 bit quantization
Example of $\mu$-law

Differential pulse-code modulation (DPCM)

- The differences between samples are expected to be smaller than the actual sampled amplitude value.
- The simple solution is to encode the differences between successive samples rather than the samples themselves.
- Fewer bits require to represent the differences.

Let $x_n$ denote the current sample from the source and let $\hat{x}_n$ denote the predicted value of $x_n$, defined as:

$$\hat{x}_n = \sum_{i=1}^{p} a_i x_{n-i}$$

- $\hat{x}_n$ is weighted linear combination of the past $p$ samples and $\{a_i\}$ are the predicted coefficient that are selected to minimize the error between $x_n$ and $\hat{x}_n$. 
Differential pulse-code modulation (DPCM)

- The mean square error between $x_n$ and $\hat{x}_n$ is given

$$\varepsilon_p = E(e_a^2) = E\left[ (x_n - \sum_{i=1}^{p} a_i x_{n-i})^2 \right]$$

$$= E(x_n^2) - 2 \sum_{i=1}^{p} a_i E(x_i x_{n-i}) + \sum_{i=1}^{p} \sum_{j=1}^{p} a_i a_j E(x_i x_{n-j})$$

- Selecting $\{a_i\}$ to minimize the MSE

Assume that source output is stationary and $\phi(m)$ shows the autocorrelation function of $x_n$

$$\varepsilon_p = \phi(0) - 2 \sum_{i=1}^{p} a_i \phi(i) + \sum_{i=1}^{p} \sum_{j=1}^{p} a_i a_j \phi(i-j)$$

- To minimize $\varepsilon_p$ set

$$\sum_{i=1}^{p} a_i \phi(i-j) = \phi(j)$$
**Differential pulse-code modulation (DPCM)**

\[ \hat{x}_n = x_n + q_n \]

The quantized sample \(\hat{x}_n\) differs from the input \(x_n\) by quantization error \(q_n\).

**Delta Modulation**

\[ \hat{\delta}_n = \hat{x}_n - \hat{x}_{n-1} \]

\[ \hat{x}_n = \hat{x}_{n-1} + \hat{\delta}_n \]

\[ q_n = \hat{e}_n - e_n = \hat{e}_n - (x_n - \hat{x}_n) \]

**Predictor**

\[ \{a_l\} \]

**To low-pass filter**

**Source Encoder**

**Source Decoder**

**Output**

**To transmitter**
Delta Modulation

Equivalent realization of Delta modulation

Source Encoder

Source Decoder

Delta Modulation

- The performance of the DM encoder is limited by two types of distortion

- Slope overload distortion
  - Step size is too small
- Granular noise
  - Step size is too large

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Delta Modulation

- Alternative solution is variable step size: Step size is increased when the waveform has steep slope and decreased when the waveform has a relatively small slope.

- One of the method is called continuous variable slope delta modulation (CVSD).

\[
\Delta_n = \alpha\Delta_{n-1} + k_1
\]

If \( \tilde{e}_n, \tilde{e}_{n-1}, \) and \( \tilde{e}_{n-2} \) has same sign.

Otherwise

\[
\Delta_n = \alpha\Delta_{n-1} = k_2
\]

where \( 0 < \alpha < 1 \)
\( k_1 \gg k_2 > 1 \)

Source: http://www.owlnet.rice.edu/~elec301/Projects99/adda/dmod.html
**Spectral Waveform Coding**

- Filter the source output signal into a number of frequency subband and separately encode the signal in each subband.
- Each subband can be encoded in time-domain waveform or each subband can be encoded in frequency-domain waveform.

**Subband Coding**

- Source signal (such as speech or image) is divided into small number of subbands and each subband is coded in time-waveform.
- More bits are used for the lower-frequency band signal and fewer bits are used for the higher-frequency band.
- Filter design is important in achieving good performance.
- Quadrature-mirror filters (QMFs) are used most often in practice.

**Subband Coding Example**

Let's assume that the speech signal bandwidth is 3200Hz.

- The first pair of QMFs divides the spectrum into two bands:
  - Low: 0-1600Hz, and High: 1600-3200Hz.
- The Low band is split into two using another pair of QMFs:
  - Low: 0-800Hz, and High: 800-1600Hz.
- The Low band is split into two again using another pair of QMFs:
  - Low: 0-400Hz, and High: 400-800Hz.
- We need 3 pairs of QMFs and we have signal in the frequency bands 0-400, 400-800, 800-1600, and 1600-3200.
## Adaptive Transform Coding (ATC)

- The source signal is sampled and subdivided into frames of $N_f$ samples.
- The data in each frame is transformed into the spectral domain for coding.
- At the decoder side, each frame of spectral samples is transformed back into the time domain and signal is synthesized from the time domain samples.
- For efficiency, more bit is assigned to more important spectral coefficients and less bit is assigned to less important coefficients.
- For transform from time to frequency domain, DFT or Discrete cosine transform (DCT) can be used.

## Model-based Source Coding

- The source is modeled as a linear system that results in the observed source output.
- Instead of transmitted samples of the source, the parameters of the linear system are transmitted with an appropriate excitation table.
- If the parameters are sufficient small, provides large compression.

### Linear predictive coding (LPC)

- Let’s have sampled sequence $x_n$, $n=0,1,...,N-1$ and assume that is generated by discrete time filter that gives transfer function

\[
H(z) = \frac{G}{1 - \sum_{k=1}^{p} a_k z^{-k}}
\]
Model-based Source Coding

- Suppose that the input sequence id denoted by $v_n$, $n=0,1,2,...$
- Then the output sequence of the digital filter satisfy the difference equation

$$x_n = \sum_{k=1}^{p} a_k z_{n-k} + Gv_n$$

If the input is a white noise sequence or an impulse, we may estimate (predict) of $x_n$ by weighted linear combination

$$\hat{x}_n = \sum_{k=1}^{p} a_k x_{n-k}, \quad n > 0$$

The difference between $x_n$ and $\hat{x}_n$

$$e_n = x_n - \hat{x}_n = x_n - \sum_{k=1}^{p} a_k x_{n-k}$$

The filter coefficients $\{a_k\}$ can be selected to minimize the mean square error.

Encoding methods for Speech signal

- Speech signal band limits 200-3200Hz.
- Sampling frequency 8000samples/s for all encoder except DM

<table>
<thead>
<tr>
<th>Encoding method</th>
<th>Quantization</th>
<th>Coder</th>
<th>Transmission rate(bits/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCM</td>
<td>Linear</td>
<td>12 bits</td>
<td>96,000</td>
</tr>
<tr>
<td>Log PCM</td>
<td>Logarithmic</td>
<td>7-8 bits</td>
<td>36,000-64,000</td>
</tr>
<tr>
<td>DPCM</td>
<td>Logarithmic</td>
<td>4-6 bits</td>
<td>32,000-48,000</td>
</tr>
<tr>
<td>ADPCM</td>
<td>Adaptive</td>
<td>3-4 bits</td>
<td>24,000-32,000</td>
</tr>
<tr>
<td>DM</td>
<td>Binary</td>
<td>1 bit</td>
<td>32,000-64,000</td>
</tr>
<tr>
<td>ADM</td>
<td>Adaptive Binary</td>
<td>1 bit</td>
<td>16,000-32,000</td>
</tr>
<tr>
<td>LPC/C ragazzoCEL</td>
<td></td>
<td></td>
<td>2400-9600</td>
</tr>
</tbody>
</table>