Consider a music database that contains information about artists, albums and individual songs. Each album has a unique numerical code, a release year, a title and features one or more artists. An album consists of one or more numbered tracks, each of which contains a song with a title and duration.

1. a) Decompose the universal relation scheme into a BCNF database scheme, showing a reasonable designated key and designated set of functional dependencies for each relation scheme.

b) Using the database scheme in part a), write a relational expression that lists the artists who have recorded a song entitled “Database Blues”.

c) Using the database scheme in part a), write a relational expression that lists the titles of albums released in 2002 that contain a song at least 15 minutes long.

2. Let \( r_1, r_2, r_3 \) be relations over \( R_1(X_1), R_2(X_2), R_3(X_2) \), respectively. Prove that
\[
    r_1 \bowtie (r_2 \cap r_3) = (r_1 \bowtie r_2) \cap (r_1 \bowtie r_3).
\]

3. Prove that every relation \( r \) over \( R(A, B, C) \) that satisfies FD \( B \rightarrow C \) has a lossless decomposition with respect to \( X_1 = \{ A, B \}, X_2 = \{ B, C \} \).

4. Show there exists a relation \( r \) over \( R(A, B, C) \) that has a lossy decomposition with respect to \( X_1 = \{ A, B \}, X_2 = \{ B, C \} \).

5. Consider the following relation instances:

\[
\begin{array}{cccc}
\text{\( r_1 \)} & \text{\( r_2 \)} & \text{\( r_3 \)} \\
\begin{array}{ccc}
A & B & \text{\( (r_1 \bowtie r_2 \bowtie r_3) \)} \\
1 & 2 & 2 & 1 & 3 \\
2 & 2 & 2 & 2 & 1 \\
3 & 3 & 2 & 2 & 1 \\
\end{array}
\end{array}
\]

a) Compute \( r_1 \bowtie r_2 \bowtie r_3 \).

b) Compute \( \rho_{E - B}(r_2) \bowtie r_3 \).

c) Is \( CD \) a superkey for \( r_2 \)?

d) Is \( DE \) a key for \( r_3 \)?