1. Let \( r_1, r_2 \) be relations over relation schemes \( R_1(X), R_2(X) \). Prove that
\[
r_1 - r_2 = r_1 - (r_1 \bowtie r_2).
\]

2. Let \( r_1, r_2 \) be relations over relation schemes \( R_1(X_1), R_2(X_2) \), in which \( \frac{1}{2} \) of the tuples in \( r_1 \) are dangling from \( r_2 \) and \( \frac{3}{4} \) of the tuples in \( r_2 \) are dangling from \( r_1 \). Express the minimum and maximum possible cardinality of \( r_1 \bowtie r_2 \) in terms of the individual cardinalities \( ||r_1||, ||r_2|| \).

3. Consider the following database scheme:

   FOODGROUPS (Ingredient, Group)
   INGREDIENTS (Ingredient, Recipe)
   MENUS (Date, Meal, Recipe)

Write relational expressions that retrieve the following:

a) Food groups (e.g., fruit, vegetable, meat, dairy) not served on May 4, 2004.
b) Meals (e.g., breakfast, lunch, dinner) at which chicken was served May 3–7, 2004.

4. A hotel has numbered rooms, some accommodating overnight guests and some hosting conference talks. On a given day, each conference room is dedicated to a topic and each guest room accommodates one overnight guest. A day’s time in a conference room is divided into numbered slots starting in the morning with number 1. Each conference room time slot hosts a talk having a title and a featured presenter. Conference organizers also take attendance to know who attended what conference rooms during each time slot.

   a) Decompose the universal relation scheme into a BCNF database scheme, showing an appropriate designated key and designated set of functional dependencies for each relation scheme.

Write relational expressions that retrieve the following:

b) People who presented a talk and attended a talk in the same conference room.
c) Topics of talks given in slot 1 of conferences held May 3–7, 2004.
d) Titles of talks on the topic “Effective Test-Taking”.
e) Names of attendees who stayed the same night as a guest.

5. Let \( r \) be a relation over \( R(X) \). Show that superkey is a special case of functional dependency by redefining superkey (w.r.t. \( r \)) in terms of functional dependency (w.r.t. \( r \)).
A relation $r$ over relation scheme $R(X)$ with designated key $K \subseteq X$ is physically stored on a magnetic disk in two parts—*sequential records* and *index*. Complete tuples are stored in the sequential records, wherein tuple $t$ resides in disk block $addr(t)$ at byte $offset(t)$ within the block. Key values are also stored in the index, which is an ordered tree (e.g., binary search tree or B-tree) with one node per tuple containing $(t[K], addr(t), offset(t))$, ordered by key values $t[K]$.

a) Describe the steps needed to efficiently retrieve $\pi_Y(\sigma_{K=v}(r))$, where $Y - K \neq \emptyset$.

b) Describe the steps needed to efficiently add a new tuple $t$ to $r$. 