Consider the following database scheme:

COURSES (Course#, Title, Instructor)
EXAMS (Course#, Student#, Date, Grade)
STUDENTS (Student#, LastName, FirstName, Address, Age)

Write expressions using selection, projection, and natural join that retrieve the following:

a) The exams passed by John Smith, with course titles and grades.
b) The instructors that have given at least an A to a teenage student.

Consider the following database scheme:

PERSONS (Name, Age, Sex)
FATHERHOOD (Father, Child)
MOTHERHOOD (Mother, Child)

Write an expression that retrieves the parents of all the persons for whom both parents are known to the database.

Prove that if $r_1, r_2, \ldots, r_m$ are relations over relation schemes $R_1(X_1), R_2(X_2), \ldots, R_m(X_m)$, then

$$\pi_{X_i} (\bigcup_{j=1}^m r_j) \subseteq r_1.$$

Represent as a BCNF database scheme the following information. A student may attend several courses (and a course is attended by several students). For each course, several books are adopted (and a book can be adopted by several courses). We are interested in the code, name, address, and attended courses for each student. For each course, we want to know the code, title, adopted books, and enrolled students. For each book, we want to know the code, title, and adopting courses.

Consider the relation scheme

STUDENTS (S#, C#, Grade, S-Name, Sex, Age, C-Name),

where S# and C# are the code numbers of a student and of a course, respectively. Identify appropriate functional dependencies. Define the corresponding BCNF database scheme.