Scheduling Algorithms

- Scheduling is concerned with the allocation of scarce resources to activities so as to optimize one or more performance measures.

- Depending on the situation, resources and activities can take many different forms. Resources may be machines in a workshop, runways at an airport, crews at a construction site, CPU and I/O devices in a computing system, and so on.

- Activities may be operations in a production process, landings and take offs at airport, stages in a construction project, executions of computer programs, and so on.

- Performance measures can also take many forms. One performance measure may be the minimization of late jobs, while another performance measure may be the average completion time of the jobs.
Deterministic models & the alpha|beta|gamma notation

\[ \alpha | \beta | \gamma \]

- \( \alpha \) describes the machine environment.
- \( \beta \) describes the processing characteristics and constraints of the jobs.
- \( \gamma \) describes the performance measures to optimize.

The number of jobs is denoted by \( n \)
The number of machines is denoted by \( m \)
The subscript \( j \) refers to a job
The subscript \( i \) refers to a machine
\( P_j \) denotes the processing time at job \( j \).

1.
- \( \alpha \) - machine environment
  - 1 – single machine
  - \( P_m \) – identical machines in parallel
    - Ex. \( P_2 \) – 2 identical machines in parallel
  - \( P \) – arbitrary no. of identical machines
  - \( Q_m \) – uniform machines in parallel
    - Each machine has its own speed. The speed of machine \( i \) is denoted by \( V_i \)
    - Job \( j \) will take \( P_j/V_i \) units of real time to finish
- \( Q \) – arbitrary no. of uniform machines
• R_m – unrelated machines in parallel.
  
  Each machine can process different jobs at different speeds
  
  Machine i can process job j at speed V_{ij}
  
  Job j will take P_j/V_{ij} units of real time to finish

• R – arbitrary no. of related machines.

2.

• β - job characteristics, processing times and restrictions
  
  • Release dates (r_j) – If this symbol is in the β field, job j may not start its
    processing before r_j. If this symbol is not in β field, job j may start at any time.
  
  • Deadline (d_j) – Job j must be completed by its deadline (d_j). This is **hard**
    deadline.

  • Preemptions (prmp) – Preemption means that a job can be removed from the
    machine before it is completed. It will resume processing later on, possibly on
    a different machine. If this symbol is not in the β field, non-preemptive
    scheduling is implied. Preemptive will not cause loss of time.
• Precedence constraints (prec) – If this symbol is not in the $\beta$ field, it means that the jobs are independent to each other. Otherwise, they have precedence relationships among each other.

Most general prec is an arbitrary directed acyclic graph.

DAG - Directed Acyclic Graph

in-tree
• γ - performance measures

The performance measures we want to optimize are always a function of the completion time of the job, which of course depends on the schedule.

\( C_j \) – completion time of the job \( j \).

• Makespan or Schedule Length (\( C_{\text{max}} \))

\[ C_{\text{max}} = \max \{C_1, C_2, \ldots, C_n\} \]

• Total Completion Time (\( \sum C_j \)) (flow time)

• Total Weighted Completion Time (\( \sum W_j C_j \))

Each job has a weight associated with it. Job \( j \) has weight \( W_j \)

• Maximum Lateness (\( L_{\text{max}} \))

Each job has a due date (soft deadline)

Due date of job \( j \) is \( d_j \)

\[ L_j = C_j - d_j \] (can be positive, zero, or negative)

\[ L_{\text{max}} = \max \{L_1, L_2, \ldots, L_n\} \]

For this and later performance measures, each job has a due date.
• Number of Late Jobs ($\Sigma U_j$)

$$U_j = \begin{cases} 1 & \text{if } C_j > d_j \\ 0 & \text{otherwise} \end{cases}$$

• Weighted Number of Late Jobs ($\Sigma W_j U_j$)

• Total Tardiness ($\Sigma T_j$)

$$T_j = \max \{0, C_j - d_j\}$$

• Total Weighted Tardiness ($\Sigma W_j T_j$)

Examples

1. $1 \mid \text{chains} \mid \Sigma W_j C_j$

2. $P_2 \mid \text{prec}, P_j = 1 \mid C_{\text{max}}$

3. $P \mid \text{pemp}, dj \mid \Sigma C_j$

4. $1 \mid \Sigma T_j$

5. $P \mid \text{prmp} \mid L_{\text{max}}$

6. $1 \mid r_j, dj \mid \Sigma C_j$