PROBLEMS

14.1 You are asked to design a digital lowpass FIR filter with –3dB at $f_1 = 200$ Hz, –33dB at $f_2 = 300$ Hz, and sampling rate $f_s = 800$ Hz.

(a) Choose appropriate window.
(b) Select appropriate value for $N$, the length of the impulse response $h(k)$.
(c) Select the cutoff frequency $\theta_c$.
(d) Find the impulse response $h(k)$ of the desired FIR filter.
(e) If Kaiser window is employed, what would you choose for $N$ and $\beta$?

14.2 You are asked to design a digital FIR bandstop filter that eliminates 60 Hz signal coupled from the power line. The specification of this filter is: –3dB at $f_1 = 50$ Hz, –65dB at $f_2 = 55$ Hz, –65dB at $f_3 = 65$ Hz. –3dB at $f_4 = 70$ Hz, and the sampling rate is $f_s = 150$ Hz.

14.3 Design an FIR linear phase digital filter approximating the ideal frequency response

$$H_d(\theta) = \begin{cases} 
1, & \text{for } |\theta| \leq \frac{\pi}{6} \\
0, & \text{for } \frac{\pi}{6} < |\theta| \leq \pi 
\end{cases}$$

(a) Determine the coefficients $h(k)$ of a 25-tap ($N = 25$) filter based on the window method with a rectangular window.
(b) Repeat part (a) using Hamming window.
(c) Repeat part (a) using Bartlett window.

14.4 Use the window method with a Hamming window to design a 21-tap differentiator characterized by

$$H_d(\theta) = |\theta| \text{ for } -\pi \leq \theta \leq \pi$$

14.5 You are designing a 5-point ($N = 5$) digital lowpass filter with cutoff angle $\theta_c = \pi/4$ using rectangular window.

(a) Plot the window.
(b) Calculate and plot the shifted desired response $h_{ds}(k) = h_d(k - 2)$ for $k = 0, 1, 2, 3, 4$.
(c) Calculate and plot the impulse response $h(k) = h_{ds}(k) w(k)$ of the FIR filter.
(d) Find the DTFT $H(\theta)$ of $h(k)$ and plot $|H(\theta)|$.
(e) Determine the maximum sidelobe level and express it in dB.

14.6 Repeat Problem 14.5 when Bartlett window is employed.
14.7 Repeat Problem 14.5 when Hamming window is employed.

14.8 Design a digital FIR lowpass filter with length 15 (\(N = 15\)) and cutoff frequency 0.3 (or \(\theta_c = 0.3 \times 2\pi = 0.6\pi\)) using Hamming window.

(a) Find and plot the desired shifted impulse response.
(b) Find and plot the weighting function.
(c) Find and plot the impulse response.
(d) Plot the magnitude response \(\lvert H(\theta) \rvert\) for \(-\pi \leq \theta \leq \pi\).
(e) Plot the phase response \(\angle H(\theta)\) for \(-\pi \leq \theta \leq \pi\).

14.9 Design a digital FIR highpass filter with length 15 (\(N = 15\)) and cutoff frequency 0.2 (or \(\theta_c = 0.2 \times 2\pi = 0.4\pi\)) using Hanning window.

(a) Find and plot the desired shifted impulse response.
(b) Find and plot the weighting function.
(c) Find and plot the impulse response.
(d) Plot the magnitude response \(\lvert H(\theta) \rvert\) for \(-\pi \leq \theta \leq \pi\).
(e) Plot the phase response \(\angle H(\theta)\) for \(-\pi \leq \theta \leq \pi\).

14.10 Design a digital FIR bandpass filter with length 15 (\(N = 15\)), low cutoff frequency 0.2 (or \(\theta_l = 0.2 \times 2\pi = 0.4\pi\)), and high cutoff frequency 0.3 (or \(\theta_h = 0.3 \times 2\pi = 0.6\pi\)), using Blackman window.

(a) Find and plot the desired shifted impulse response.
(b) Find and plot the weighting function.
(c) Find and plot the impulse response.
(d) Plot the magnitude response \(\lvert H(\theta) \rvert\) for \(-\pi \leq \theta \leq \pi\).
(e) Plot the phase response \(\angle H(\theta)\) for \(-\pi \leq \theta \leq \pi\).

14.11 Design a digital FIR bandstop filter with length 15 (\(N = 15\)), low cutoff frequency 0.2 (or \(\theta_l = 0.2 \times 2\pi = 0.4\pi\)), and high cutoff frequency 0.3 (or \(\theta_h = 0.3 \times 2\pi = 0.6\pi\)), using Hamming window.

(a) Find and plot the desired shifted impulse response.
(b) Find and plot the weighting function.
(c) Find and plot the impulse response.
(d) Plot the magnitude response \(\lvert H(\theta) \rvert\) for \(-\pi \leq \theta \leq \pi\).
(e) Plot the phase response \(\angle H(\theta)\) for \(-\pi \leq \theta \leq \pi\).

14.12 Design a digital FIR lowpass filter with \(ap = 2\) dB, \(aa = 35\) dB, \(fplpf = 1800\) Hz, \(fslpf = 2700\) Hz, sampling rate \(fs = 10000\) Hz using Kaiser window.
(a) $\delta_1, \delta_2, \delta, \alpha, \beta, D, f_{clpf}, \theta_{clpf}, bt, N$.
(b) Find and plot the desired shifted impulse response.
(c) Find and plot the weighting function.
(d) Find and plot the impulse response.
(e) Plot the magnitude response $|H(\theta)|$ for $-\pi \leq \theta \leq \pi$.
(f) Plot the phase response $\angle H(\theta)$ for $-\pi \leq \theta \leq \pi$.

14.13 Design a digital FIR highpass filter with $ap = 3$ dB, $aa = 32$ dB, $f_{plpf} = 2900$ Hz, $f_{slpf} = 1600$ Hz, sampling rate $f_s = 10000$ Hz using Kaiser window.

(a) $\delta_1, \delta_2, \delta, \alpha, \beta, D, f_{chpf}, \theta_{chpf}, bt, N$.
(b) Find and plot the desired shifted impulse response.
(c) Find and plot the weighting function.
(d) Find and plot the impulse response.
(e) Plot the magnitude response $|H(\theta)|$ for $-\pi \leq \theta \leq \pi$.
(f) Plot the phase response $\angle H(\theta)$ for $-\pi \leq \theta \leq \pi$.

14.14 Design a digital FIR bandpass filter with $ap = 2.5$ dB, $aa = 38$ dB, $f_{1bpf} = 2100$ Hz, $f_{2bpf} = 2900$ Hz, $f_{3bpf} = 1200$ Hz, $f_{4bpf} = 3600$ Hz, sampling rate $f_s = 10000$ Hz using Kaiser window.

(a) $\delta_1, \delta_2, \delta, \alpha, \beta, D, f_{1bpf}, f_{2bpf}, \theta_{1bpf}, \theta_{2bpf}, bt, N$.
(b) Find and plot the desired shifted impulse response.
(c) Find and plot the weighting function.
(d) Find and plot the impulse response.
(e) Plot the magnitude response $|H(\theta)|$ for $-\pi \leq \theta \leq \pi$.
(f) Plot the phase response $\angle H(\theta)$ for $-\pi \leq \theta \leq \pi$.

14.15 Design a digital FIR bandstop filter with $ap = 2.3$ dB, $aa = 43$ dB, $f_{1bsf} = 1200$ Hz, $f_{2bsf} = 3900$ Hz, $f_{3bsf} = 1900$ Hz, $f_{4bsf} = 2900$ Hz, sampling rate $f_s = 10000$ Hz using Kaiser window.

(a) $\delta_1, \delta_2, \delta, \alpha, \beta, D, f_{1bsf}, f_{2bsf}, \theta_{1bsf}, \theta_{2bsf}, bt, N$.
(b) Find and plot the desired shifted impulse response.
(c) Find and plot the weighting function.
(d) Find and plot the impulse response.
(e) Plot the magnitude response $|H(\theta)|$ for $-\pi \leq \theta \leq \pi$.
(f) Plot the phase response $\angle H(\theta)$ for $-\pi \leq \theta \leq \pi$.

14.16 Design a type 1 FIR highpass filter with length 15 ($N = 15$) and
\( A_0 = 0, A_1 = 0, A_2 = 0, A_3 = 0, A_4 = 0, A_5 = 1, A_6 = 1, A_7 = 1, A_8 = 1, A_9 = 1, A_{10} = 1, A_{11} = 0, A_{12} = 0, A_{13} = 0, A_{14} = 0. \)

(a) Find and plot the impulse response of the filter.
(b) Plot the magnitude response \(|H(\theta)|\) for \(-\pi \leq \theta \leq \pi\).
(c) Plot the phase response \(\angle H(\theta)\) for \(-\pi \leq \theta \leq \pi\).

14.17 Design a type 2 FIR lowpass filter with length 16 (N = 16) and
\( A_0 = 1, A_1 = 1, A_2 = 1, A_3 = 1, A_4 = 1, A_5 = 0, A_6 = 0, A_7 = 0, A_8 = 0, A_9 = 0, A_{10} = 0, A_{11} = 0, A_{12} = 1, A_{13} = 1, A_{14} = 1, A_{15} = 1. \)

(a) Find and plot the impulse response of the filter.
(b) Plot the magnitude response \(|H(\theta)|\) for \(-\pi \leq \theta \leq \pi\).
(c) Plot the phase response \(\angle H(\theta)\) for \(-\pi \leq \theta \leq \pi\).

14.18 Design a type 3 FIR bandstop filter with length 16 (N = 16) and
\( A_0 = 1, A_1 = 1, A_2 = 1, A_3 = 1, A_4 = 0, A_5 = 0, A_6 = 0, A_7 = 1, A_8 = 1, A_9 = 1, A_{10} = 0, A_{11} = 0, A_{12} = 0, A_{13} = 1, A_{14} = 1, A_{15} = 1. \)

(a) Find and plot the impulse response of the filter.
(b) Plot the magnitude response \(|H(\theta)|\) for \(-\pi \leq \theta \leq \pi\).
(c) Plot the phase response \(\angle H(\theta)\) for \(-\pi \leq \theta \leq \pi\).

14.19 Design a type 4 FIR bandpass filter with length 15 (N = 15) and
\( A_0 = 0, A_1 = 0, A_2 = 0, A_3 = 1, A_4 = 1, A_5 = 1, A_6 = 0, A_7 = 0, A_8 = 0, A_9 = 0, A_{10} = 1, A_{11} = 1, A_{12} = 1, A_{13} = 0, A_{14} = 0. \)

(a) Find and plot the impulse response of the filter.
(b) Plot the magnitude response \(|H(\theta)|\) for \(-\pi \leq \theta \leq \pi\).
(c) Plot the phase response \(\angle H(\theta)\) for \(-\pi \leq \theta \leq \pi\).

14.20 Given \( \theta_0 = 0, \theta_1 = 2\pi/6, \theta_2 = 3\pi/6, \theta_3 = 5\pi/6, \theta_4 = \pi, \delta_1 = 0.15, \delta_2 = 0.25, D_0 = 1, D_1 = 1, D_2 = 1, D_3 = 0, D_4 = 0, \)

(a) find \( c_0, c_1, c_2, c_3, c_4 \)
(b) plot \(|H(\theta)|\) for \(-\pi \leq \theta \leq \pi\).
(c) plot \(\angle H(\theta)\) for \(-\pi \leq \theta \leq \pi\).