1. The following table gives the probability distribution of a discrete random variable X.

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(x)</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Find the following probabilities:

(a) \( P(1 \leq x < 4) = 0.4 + 0.3 + 0.2 = 0.9 \)

(b) \( P(x \geq 3) = 0.2 + 0.1 = 0.3 \)

(c) \( E(X) = 1(0.4) + 2(0.3) + 3(0.2) + 4(0.1) \)
   \( = 0.4 + 0.6 + 0.6 + 0.4 = 2 \)

(d) \( E(5 - 2x) = 5 - 2E(x) \)
   \( = 5 - 2(2) = 5 - 4 = 1 \)

2. A particular type of tennis racket comes in a midsize version and an oversize version. 60% of all customers at a certain store want the oversize version. Among ten randomly selected customers who want this type of racket, Using the Binomial distribution find the probability that at most six want the oversize version?

\( P(x \leq 6) = 0.6177 \)

3. Consider writing onto a computer disk and then sending it through a certifier that counts the number of missing pulses. Suppose this number \( X \) has a Poisson distribution with parameter \( \lambda = 0.3 \). Using the Poisson distribution, a) What is the probability that a disk has exactly one missing pulse? b) What is the probability that a disk has at least two missing pulses?

(a) \( P(x = 1) = P(x \leq 1) - P(x \leq 0) \)
   \( = 0.963 - 0.741 = 0.222 \)

(b) \( P(x \geq 2) = 1 - P(x \leq 1) \)
   \( = 1 - 0.963 = 0.037 \)
4. An article suggests that yield strength (ksi = kilo pound per square inch) for A36 grade steel is normally distributed with $\mu = 43$ and $\sigma = 4.5$. [Note: A36 steel is a standard steel alloy which is a common structural steel used in the United States.]

a. What is the probability that yield strength between 40 ksi and 45 ksi? (Accurate to 4 decimal places)

\[
\begin{align*}
p(40 < X < 45) &= \Phi(-1.67) - \Phi(-2.54) \\
&= 0.1416
\end{align*}
\]

\[
2 = \frac{45 - 43}{4.5} = -0.44 \\
2 = \frac{40 - 43}{4.5} = -1.67
\]

b. What yield strength value (in ksi) separates the strongest 75% from the others? [Hint: The yield strength separating the bottom 25% from the top 75%]

\[
\begin{align*}
2 &= \frac{x - \mu}{\sigma} \\
-0.167 &= \frac{x - 43}{4.5} \\
x &= 39.1915 \approx 40 \text{ ksi}
\end{align*}
\]

5. Suppose only 75% of all drivers in a certain state regularly wear a seat belt. A random sample of 500 drivers is selected. Using the normal approximation (with the continuity correction) find the probability that fewer than 400 of those in the sample regularly wear a seat belt.

\[
\begin{align*}
p(X < 400) &= p(X < 399.5) = p\left(2.53 < \frac{399.5 - 325}{4.682958}\right) \\
p = n \cdot p = 500 \cdot 0.75 = 375 \\
\sigma = \sqrt{375 \cdot 0.25} = 9.68241 \\
p(2.53) &= 0.9943
\end{align*}
\]
6. The weekly gravel sales is a continuous random variable \( X \) with pdf

\[
f(x) = \begin{cases} 
\frac{3}{2}(1-x^2) & \text{if } 0 \leq x \leq 1 \\
0 & \text{elsewhere}
\end{cases}
\]

\[ \int_{-\infty}^{\infty} f(x) \, dx = 1 \]

a. Find \( P(X \leq 0.3) \) (Accurate to 4 decimal places)

\[ P(X \leq 0.3) = \int_{-\infty}^{0.3} f(x) \, dx = \int_{0}^{0.3} \frac{3}{2}(1-x^2) \, dx \]

\[ = \frac{3}{2} \left[ x - \frac{x^3}{3} \right]_{0}^{0.3} = 0.4365 \]

b. Find \( E(X) \)

\[ E(X) = \int_{-\infty}^{\infty} x \cdot f(x) \, dx = \int_{0}^{1} \frac{3}{2}(1-x^2) \cdot x \, dx + \int_{0}^{1} \frac{3}{2}x \, dx = 0.375 \]

7. A random sample of 50 helmets used by motorcycle riders was subjected to an impact test, and on 18 of these helmets some damage was observed.

\[ \hat{p} = \frac{18}{50} = 0.36 \]

a. Compute a 95% confidence interval for \( p \), where \( p \) = the true population proportion of helmets of this type that would show damage from this test. (Accurate to 4 decimal places)

\[ \sigma = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.36(1-0.36)}{50}} = 0.05 \]

\[ \hat{p} \pm z_{0.025} \cdot \sigma = 0.36 \pm 1.96 \cdot 0.05 = 0.36 - 0.13 < p < 0.36 + 0.13 \]

b. Interpret your answer in the context of the problem.

we are 95% confident that ...

\[ 0.22 < p < 0.49 \]

c. Find the sample size \( n \) to estimate the true proportion \( p \) with the maximum error 0.14 and with 95% confidence.

\[ n = \frac{(0.36)(0.64)(1.96)^2}{0.14^2} \]

\[ n \approx 46 \]
8. A random sample of 110 lightning flashes in a certain region resulted in a sample average radar echo duration of 0.81 sec and a sample standard deviation of 0.34 sec.

a. Find a 99% confidence interval for $\mu$. (Accurate to 2 decimal places)

\[
\begin{align*}
E &= 2t_{0.01} \frac{s}{\sqrt{n}} = 2.575 \frac{0.34}{\sqrt{110}} \\
&= 0.813475 \\
\end{align*}
\]

\[
\begin{align*}
1.03 &< \mu < 1.83 \\
1.727 &< \mu < 1.893 \\
\end{align*}
\]

b. Interpret your answer in the context of the problem.

we are 99% confident that...

9. The following data represent the diameter (in mm) of a random sample of 9 metal rods used in an automobile suspension system. (Assume $\bar{x} = 8.2322$ and $s = 0.0268$).

8.24, 8.23, 8.20, 8.21, 8.20, 8.28, 8.23, 8.26, 8.24

a. Assuming that the diameter follows a normal distribution, compute a 99% confidence interval for the true average diameter $\mu$. (Accurate to 4 decimal places)

\[
\begin{align*}
E &= t_{0.005} \frac{s}{\sqrt{n}} = 2.34 \frac{0.0268}{\sqrt{9}} \\
&= 0.02587 \\
\end{align*}
\]

\[
\begin{align*}
8.2322 - 0.02587 &< \mu < 8.2322 + 0.02587 \\
8.2063 &< \mu < 8.2580 \\
\end{align*}
\]

b. Interpret your answer in the context of the problem.

we are 98% confident that...