Chapter 4

4.3 Applications of Energy Balance

We will discuss examples illustrating the analysis of several devices of interest in engineering, including nozzles and diffusers, turbines, compressors and pumps, heat exchangers, and throttling devices.

In a nozzle, the fluid velocity increases in the direction of flow due to the reduction in the flow area. In a diffuser, the fluid velocity decreases in the direction of flow due to the increase in the flow area.

Example 4.3-1

Steam enters a converging-diverging nozzle operating at steady state with $p_1 = 40$ bar, $T_1 = 400^\circ$C, and a velocity of 10 m/s. The steam flows through the nozzle with negligible heat transfer and no significant change in potential energy. At the exit, $p_2 = 15$ bar, and the velocity is 665 m/s. The mass flow rate is 2 kg/s. Determine the exit area of the nozzle, in m$^2$.

Solution

Apply the steady state energy balance between (1) and (2) gives

$$h_2 - h_1 + g(z_2 - z_1) + \frac{1}{2}(V_2^2 - V_1^2) = \frac{\dot{Q}}{\dot{m}} - \frac{\dot{W}_s}{\dot{m}} = 0$$

$$h_2 = h_1 + \frac{1}{2}(V_1^2 - V_2^2)$$

At $p_1 = 40$ bar (4 MPa), $T_1 = 400^\circ$C, $h_1 = 3214$ kJ/kg (Table E4.3-1)

$$h_2 = 3214 \text{ kJ/kg} + 0.5(10^2 - 665^2) \left( \frac{m^2}{s^2} \right) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 2993 \text{ kJ/kg}$$

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Table E4.3-1 Steam properties

<table>
<thead>
<tr>
<th>Temp C</th>
<th>Pressure MPa</th>
<th>Specific Volume m³/kg</th>
<th>Internal Energy kJ/kg</th>
<th>Specific Enthalpy kJ/kg</th>
<th>Specific Entropy kJ/kg/K</th>
<th>Quality</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>400</td>
<td>4</td>
<td>0.07341</td>
<td>2920</td>
<td>3214</td>
<td>6.769</td>
<td>Dense Fluid (T&gt;Tc)</td>
</tr>
<tr>
<td>2</td>
<td>280.1</td>
<td>1.5</td>
<td>0.1628</td>
<td>2749</td>
<td>2993</td>
<td>6.839</td>
<td>Superheated Vapor</td>
</tr>
</tbody>
</table>

At $p_2 = 15$ bar (1.5 MPa), $h_2 = 2993$ kJ/kg, $v_2 = 0.1628 \text{ m}^3/\text{kg}$

The exit area is then

$$A_2 = \frac{m v_2}{V_2} = \frac{(2 \text{ kg})(0.1628 \text{ m}^3/\text{kg})}{665 \text{ m/s}} = 4.9 \times 10^{-4} \text{ m}^2$$

Example 4.3-2

Steam enters a turbine operating at steady state with a mass flow rate of 4600 kg/h. The turbine develops a power output of 1000 kW. At the inlet, the pressure is 60 bar, the temperature is 400°C, and the velocity is 10 m/s. At the exit, the pressure is 0.1 bar, the quality is 0.9, and the velocity is 30 m/s. Calculate the rate of heat transfer between the turbine and surroundings, in kW.

Solution

Apply the steady state energy balance between (1) and (2) gives

$$h_2 - h_1 + g(z_2 - z_1) + \frac{1}{2}(V_2^2 - V_1^2) = \frac{\dot{Q}}{m} - \frac{\dot{W}_s}{m}$$

Solving for $\frac{\dot{Q}}{m}$ with $g(z_2 - z_1) = 0$, we obtain

$$\frac{\dot{Q}}{m} = \frac{\dot{W}_s}{m} + h_2 - h_1 + \frac{1}{2}(V_2^2 - V_1^2)$$

Properties of steam for states (1) and (2) are listed in Table E4.3-2

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### Table E4.3-2 Steam properties

<table>
<thead>
<tr>
<th>Temp C</th>
<th>Pressure MPa</th>
<th>Specific Volume m³/kg</th>
<th>Internal Energy kJ/kg</th>
<th>Specific Enthalpy kJ/kg</th>
<th>Specific Entropy kJ/kg/K</th>
<th>Quality</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>400</td>
<td>6</td>
<td>0.04739</td>
<td>2893</td>
<td>3177</td>
<td>6.541</td>
<td>Dense Fluid (T&gt;TC)</td>
</tr>
<tr>
<td>2</td>
<td>45.81</td>
<td>0.01</td>
<td>13.21</td>
<td>2213</td>
<td>2345</td>
<td>7.4</td>
<td>0.9 Liquid Vapor Mixture</td>
</tr>
</tbody>
</table>

The change in kinetic energy is evaluated:

\[
\frac{1}{2} (V_2^2 - V_1^2) = 0.5(30^2 - 10^2) \left( \frac{m^2}{s^2} \right) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right) = 0.4 \text{ kJ/kg}
\]

The change in enthalpy is

\[
h_2 - h_1 = 2345 \text{ kJ/kg} - 3177 \text{ kJ/kg} = -832 \text{ kJ/kg}
\]

\[
\dot{Q} = \dot{W}_s + h_2 - h_1 + \frac{1}{2} (V_2^2 - V_1^2) = \dot{W}_s - 832 \text{ kJ/kg} + 0.4 \text{ kJ/kg}
\]

\[
\dot{Q} = \frac{\dot{W}_s}{m} - 831.6 \text{ kJ/kg}
\]

\[
\dot{Q} = 1000 \text{ kW} - 4600 \left( \frac{\text{kg}}{\text{h}} \right) (831.6 \text{ kJ/kg}) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \left( \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right) = -62.6 \text{ kW}
\]

Heat transfer from the turbine to the surroundings.

**Example 4.3-3**

Air enters a compressor operating at steady state at a pressure of 1 bar, a temperature of 290 K, and a velocity of 6 m/s through an inlet area of 0.1 m². At the exit, the pressure is 7 bar, the temperature is 450 K, and the velocity is 2 m/s. Heat transfer from the compressor to its surroundings occurs at a rate of 180 kJ/min. Employing the ideal gas model, calculate the power input to the compressor, in kW.

**Solution**

\[
\frac{p_1 = 1 \text{ bar}}{T_1 = 290 \text{ K}} \quad \frac{V_1 = 6 \text{ m/s}}{A_1 = 0.1 \text{ m}^2} \quad \frac{p_2 = 7 \text{ bar}}{T_2 = 450 \text{ K}} \quad \frac{V_2 = 2 \text{ m/s}}{Q_{cc} = -180 \text{ kJ/min}}
\]

---

Apply the steady state energy balance between (1) and (2) gives

\[ h_2 - h_1 + g(z_2 - z_1) + \frac{1}{2}(V_2^2 - V_1^2) = \frac{\dot{Q}}{m} - \frac{\dot{W}_s}{m} \]

Solving for \( \dot{W}_s \) with \( g(z_2 - z_1) = 0 \), we obtain

\[ \dot{W}_s = \dot{Q} + m \left[ (h_i - h_f) + \left( \frac{V_1^2 - V_2^2}{2} \right) \right] \]

The mass flow rate is given by

\[ \dot{m} = \frac{AV_i}{v_i} \]

The specific velocity can be determined from the ideal gas law

\[ v_1 = \frac{(R/M)T_i}{p_1} = \frac{8314 \text{ N} \cdot \text{m}}{28.97 \text{ kg} \cdot \text{K}} \frac{(290 \text{ K})}{10^5 \text{ N/m}^2} = 0.8324 \text{ kg/m}^3 \]

The mass flow rate is then

\[ \dot{m} = \frac{AV_i}{v_i} = \frac{(0.1 \text{ m}^2)(6 \text{ m/s})}{0.8324 \text{ kg/m}^3} = 0.7209 \text{ kg/s} \]

The change in air enthalpy can be obtained from the CATT2 program

\[ h_1 - h_2 = 290.6 \text{ kJ/kg} - 452.3 \text{ kJ/kg} = -161.7 \text{ kJ/kg} \]

**Table E4.3-3** Air properties from CATT2

<table>
<thead>
<tr>
<th>Temp</th>
<th>Pressure</th>
<th>Specific Enthalpy (Mass)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>MPa</td>
<td>kJ/kg</td>
</tr>
<tr>
<td>290</td>
<td>0.1</td>
<td>290.6</td>
</tr>
<tr>
<td>450</td>
<td>0.7</td>
<td>452.3</td>
</tr>
</tbody>
</table>

The change in kinetic energy is evaluated:

\[ \frac{1}{2}(V_1^2 - V_2^2) = 0.5(6^2 - 2^2) \left( \frac{\text{m}^2}{\text{s}^2} \right) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right) = 0.02 \text{ kJ/kg} \]

The power input to the compressor is then
\[ \dot{W}_j = \dot{Q} + \dot{m} \left( (h_1 - h_2) + \left( \frac{V_1^2 - V_2^2}{2} \right) \right) \]

\[ \dot{W}_j = -\frac{180}{60} \text{ kJ/s} + (0.7209 \text{ kg/s})(-161.7 + 0.02) \text{ kJ/kg} = -119.6 \text{ kW} \]

### 4.4 Heat Exchanger

Heat exchangers are devices for transferring heat between two fluid streams. Heat exchangers can be classified as indirect contact type and direct contact type. Indirect contact type heat exchangers have no mixing between the hot and cold streams, only energy transfer is allowed as shown in Figure 4.4-1.

![Indirect contact type heat exchanger](image)

**Figure 4.4-1** Indirect contact type heat exchanger.

Direct contact type heat exchangers have no wall to separate the cold from the hot streams as shown in Figure 4.4-2.

![Direct contact type heat exchanger](image)

**Figure 4.4-2** Direct contact type heat exchanger.

We will briefly discuss two types of tubular heat exchangers: *concentric tube* and *shell-and-tube* heat exchangers. A *concentric tube* or *double pipe* heat exchanger is the simplest heat exchanger for which the hot and cold fluids move in the same or opposite directions as shown in Figure 4.4-3.
Shell-and-tube heat exchanger is the most common configuration. There are many different forms of shell-and-tube heat exchangers according to the number of shell-and-tube passes. A common form with one shell pass and two tube passes is shown in Figure 4.4-4. Baffles are usually installed to increase the heat transfer coefficient of the fluid by introducing turbulence and cross-flow in the shell side.
Example 4.4-1. Saturated steam at 99.63°C condenses on the outside of a 5-m long, 4-cm-diameter thin horizontal copper tube by cooling liquid water that enters the tube at 25°C at an average velocity of 3 m/s and leaves at 45°C. Liquid water density is 997 kg/m³, \( c_p \) of liquid water is 4.18 kJ/kg°C. (a) Determine the rate of heat transfer to water. (b) If the rate of heat transfer to water is 200 kW, determine the rate of condensation of steam.

Solution

(a) The rate of heat transfer to water is given by

\[
\dot{Q} = \dot{m} c_p (T_e - T_i)
\]

In this equation, the mass flow rate of water is given by

\[
\dot{m} = \rho V_{vel} A_{tube}
\]

\[
\dot{m} = (997 \text{ kg/m}^3)(3 \text{ m/s})(\pi \times 0.02 \text{ m}^2) = 3.7586 \text{ kg/s}
\]

The heat transfer rate is then

\[
\dot{Q} = (3.7586 \text{ kg/s})(4.18 \text{ kJ/kg°C})(45 - 25) \text{ °C} = 314.2 \text{ kW}
\]

(b) If the rate of heat transfer to water is 200 kW, determine the rate of condensation of steam.

We need the enthalpy for saturated liquid and saturated vapor

<table>
<thead>
<tr>
<th>Temp C</th>
<th>Pressure MPa</th>
<th>Specific Enthalpy kJ/kg</th>
<th>Quality</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>99.63</td>
<td>0.1</td>
<td>2675</td>
<td>1</td>
<td>Saturated Vapor</td>
</tr>
<tr>
<td>99.63</td>
<td>0.1</td>
<td>417.5</td>
<td>0</td>
<td>Saturated Liquid</td>
</tr>
</tbody>
</table>

The rate of heat transfer to water can also be determined from

\[
\dot{Q} = \dot{m}_{steam} (h_g - h_l) \Rightarrow \dot{m}_{steam} = \frac{\dot{Q}}{h_g - h_f}
\]

\[
\dot{m}_{steam} = \frac{200 \text{ kJ/s}}{(2675 - 417.5) \text{ kJ/kg}} = 0.0886 \text{ kg/s}
\]
Example 4.4-2. 

A light oil is to be preheated before being fed to a distillation tower. The preheating is to be accomplished in a heat exchanger in which the heating medium is hot oil. The following data are available:

<table>
<thead>
<tr>
<th></th>
<th>Light oil</th>
<th>Hot oil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass flow rate, kg/hr</td>
<td>160,000</td>
<td>120,000</td>
</tr>
<tr>
<td>Heat capacity, kJ/kg·K</td>
<td>1.7</td>
<td>1.8</td>
</tr>
<tr>
<td>Entry temperature, K</td>
<td>300</td>
<td>500</td>
</tr>
</tbody>
</table>

1) If the light oil leaves the heat exchanger at 360 K, determine the temperature of the exit hot oil.

2) If the two fluids flow co-currently through the heat exchanger, determine the maximum attainable temperature of the light oil.

3) If the two fluids flow counter-currently through the heat exchanger, determine the maximum attainable temperature of the light oil.

**Solution**

1) Determine the temperature of the exit hot oil.

\[
\dot{Q} = \dot{m}_c c_{pc} (T_{ce} - T_{ci}) = \dot{m}_h c_{ph} (T_{hi} - T_{he}) \Rightarrow T_{he} = T_{hi} - \frac{\dot{m}_c c_{pc}}{\dot{m}_h c_{ph}} (T_{ce} - T_{ci})
\]

\[
T_{he} = 500 \text{ K} - \frac{(160,000)(1.7)}{(120,000)(1.8)} (360 - 300) \text{ K} = 424.4 \text{ K}
\]

2) If the two fluids flow co-currently through the heat exchanger, determine the maximum attainable temperature of the light oil.

For cocurrent flow, the maximum attainable temperature of the light oil is \( T_{ce} = T_{he} \)

\[
\dot{m}_c c_{pc} (T_{ce} - T_{ci}) = \dot{m}_h c_{ph} (T_{hi} - T_{ce}) \Rightarrow T_{ce} = \frac{\dot{m}_c c_{pc} T_{ci} + \dot{m}_h c_{ph} T_{hi}}{\dot{m}_c c_{pc} + \dot{m}_h c_{ph}}
\]

\[
T_{ce} = \frac{(16)(1.7)(300 \text{ K}) + (12)(1.8)(500 \text{ K})}{(16)(1.7) + (12)(1.8)} = 388.5 \text{ K}
\]

3) If the two fluids flow counter-currently through the heat exchanger, determine the maximum attainable temperature of the light oil.

Since \( \dot{m}_c c_{pc} = (160,000)(1.7) = 272,000 \text{ kJ/hr·s} > \dot{m}_h c_{ph} = (120,000)(1.8) = 216,000 \text{ kJ/hr·s} \)

\[
T_{he} = T_{ci} \Rightarrow \dot{m}_c c_{pc} (T_{ce} - T_{ci}) = \dot{m}_h c_{ph} (T_{hi} - T_{ci}) \Rightarrow T_{ce} = T_{ci} + \frac{\dot{m}_h c_{ph}}{\dot{m}_c c_{pc}} (T_{hi} - T_{ci})
\]

\[
T_{ce} = 300 \text{ K} + \frac{(120,000)(1.8)}{(160,000)(1.7)} (500 - 300) \text{ K} = 458.8 \text{ K}
\]