Chapter 8

Linear Programming with Matlab

The Matlab function LINPROG can be used to solve a linear programming problem with the following syntax (help LINPROG):

\[
X = \text{LINPROG}(f, A, b) \text{ solves the linear programming problem:}
\]

\[
\min f^T x \quad \text{subject to:} \quad Ax \leq b
\]

\[
X = \text{LINPROG}(f, A, b, Aeq, beq) \text{ solves the problem above while additionally satisfying the equality constraints } Aeq \cdot x = beq.
\]

\[
X = \text{LINPROG}(f, A, b, Aeq, beq, LB, UB) \text{ defines a set of lower and upper bounds on the design variables, } X, \text{ so that the solution is in the range } LB \leq X \leq UB. \text{ Use empty matrices for } LB \text{ and } UB \text{ if no bounds exist. Set } LB(i) = -\infty \text{ if } X(i) \text{ is unbounded below; set } UB(i) = \infty \text{ if } X(i) \text{ is unbounded above.}
\]

Since LINPROG minimizes an objective function, the example 3.2-1 is reformulated as

\[
\text{Minimize } z = -150x_1 - 175x_2
\]

Note: Maximizing \((150x_1 + 175x_2)\) is equivalent to minimizing \((-150x_1 - 175x_2)\)

Subject to

\[
\begin{align*}
7x_1 + 11x_2 &\leq 77 \quad \text{(material constraint)} \\
10x_1 + 8x_2 &\leq 80 \quad \text{(time constraint)} \\
x_1 &\leq 9 \quad \text{(storage constraint of regular heating gas)} \\
x_2 &\leq 6 \quad \text{(storage constraint of premium heating gas)} \\
x_1, x_2 &\geq 0 \quad \text{(positive production constraint)}
\end{align*}
\]

The coefficient vector \(f\) for the objective function is

\[
f = \begin{bmatrix} -150 \\ -175 \end{bmatrix}
\]

The matrix coefficient \(A\) for the inequality constraints is

\[
A = \begin{bmatrix} 7 & 11 \\ 10 & 8 \end{bmatrix}
\]
The right hand vector $b$ for the inequality constraints is

$$b = \begin{bmatrix} 77 \\ 80 \end{bmatrix}$$

Since there are no equality constraints in this example, $Aeq$ and $beq$ are zeros.

$$Aeq = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } beq = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The lower and upper bounds vectors are given by

$$LB = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ and } UB = \begin{bmatrix} 9 \\ 6 \end{bmatrix}$$

The following Matlab statements are used to solve this linear programming problem.

Matlab Example  

```
% Example 8.3-1
f=[-150;-175];
A=[7 11;10 8];
b=[77;80];
Aeq=[0 0;0 0];
beq=[0;0];
LB=[0;0];UB=[9;6];
x=linprog(f,A,b,Aeq,beq,LB,UB)
```

```
>> e3d2d1
Optimization terminated successfully.
x =
  4.8889
  3.8889
```

Example 8.3-2

Solve the following linear programming problem using Matlab LINPROG

Maximize $y = x_1 + 2x_2$

Subject to

$$2x_1 + x_2 \leq 10$$
$$x_1 + x_2 \leq 6$$
$$-x_1 + x_2 \leq 2$$
$$-2x_1 + x_2 \leq 1$$
$$x_1, x_2 \geq 0$$
Solution

The problem is reformulated using slack variable $S_1, S_2, S_3,$ and $S_4$

Minimize $y = -x_1 - 2x_2$

Subject to

\[
\begin{align*}
2x_1 + x_2 + S_1 &= 10 \\
x_1 + x_2 + S_2 &= 6 \\
-x_1 + x_2 + S_3 &= 2 \\
-2x_1 + x_2 + S_4 &= 1 \\
x_1, x_2, S_1, S_2, S_3, \text{ and } S_4 &\geq 0
\end{align*}
\]

The arguments for the Matlab command X=LINPROG($f, A, Aeq, beq, LB, UB$) are given as

\[
\begin{bmatrix}
-1 \\
-2 \\
0 \\
0 \\
0 \\
0
\end{bmatrix},
\begin{bmatrix}
2 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 0 & 0 \\
-1 & 1 & 0 & 0 & 1 & 0 \\
-2 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
\begin{bmatrix}
10 \\
6 \\
2 \\
2 \\
0 \\
0
\end{bmatrix},
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix},
\begin{bmatrix}
\infty \\
\infty \\
\infty \\
\infty \\
\infty \\
\infty
\end{bmatrix}
\]

The following Matlab statements are used to solve this linear programming problem.

Matlab Example

```matlab
% Example 8.3-2
f=[-1;-2;0;0;0;0];
A=zeros(6,6);
b=zeros(6,1);
Aeq=[2 1 1 0 0 0;1 1 0 1 0 0;-1 1 0 0 1 0;-2 1 0 0 0 1;0 0 0 0 0 0;0 0 0 0 0 0];
beq=[10;6;2;1;0;0];
LB=[0;0;0;0;0;0];UB=[inf;inf;inf;inf;inf;inf];
x=linprog(f,A,b,Aeq,beq,LB,UB)

>> e8d3d2
Optimization terminated successfully.
x =
2.0000
4.0000
2.0000
0.0000
0.0000
1.0000
```
The solution from the Matlab program is

\[
\begin{align*}
x_1 &= 2 \\
x_2 &= 4 \\
S_1 &= 2 \\
S_2 &= 0 \\
S_3 &= 0 \\
S_4 &= 1
\end{align*}
\]

This solution is verified with the graphical solution shown in Figure 8.3-2. The vertex D is the basic feasible solution obtained graphically.

![Figure 8.3-2 Geometric representation of the linear programming problem.](image)

The problem can also be reformulated using only one slack variable \( S_1 \)

Minimize \( y = -x_1 - 2x_2 \)

Subject to

\[
\begin{align*}
2x_1 + x_2 + S_1 &= 10 \\
x_1 + x_2 &\leq 6 \\
-x_1 + x_2 &\leq 2 \\
-2x_1 + x_2 &\leq 1 \\
x_1, x_2, \text{ and } S_1 &\geq 0
\end{align*}
\]
Consider the Matlab command \(X=\text{LINPROG}(f, A, b, Aeq, beq, LB, UB)\). The coefficient vector \(f\) for the objective function is

\[
f = \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix}
\]

The matrix coefficient \(A\) for the inequality constraints is

\[
A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ -2 & 1 & 0 \end{bmatrix}
\]

The right hand vector \(b\) for the inequality constraints is

\[
b = \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}
\]

The equality constraints \(Aeq\) and \(beq\) are given as

\[
Aeq = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad beq = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}
\]

The lower and upper bounds vectors are given by

\[
LB = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad UB = \begin{bmatrix} \infty \\ \infty \\ \infty \end{bmatrix}
\]

The following Matlab statements are used to solve this linear programming problem.

```matlab
% Example 8.3-2b
f=[-1;-2;0];
A=[1 1 0;-1 1 0;-2 1 0];
b=[6;2;1];
Aeq=[2 1 1; 0 0 0;0 0 0];
beq=[10;0;0];
LB=[0;0;0];UB=[inf;inf;inf];
x=linprog(f,A,b,Aeq,beq,UB,LB,UB)
```

>> e8d3d2b
Optimization terminated successfully.
\[ x = \begin{align*} 
2.0000 \\
4.0000 \\
2.0000 
\end{align*} \]

The solution from the Matlab program is
\[ x_1 = 2 \]
\[ x_2 = 4 \]
\[ S_1 = 2 \]