Frequency Response of a Circuit

Band-Reject Filter

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A Serial RLC Circuit

\[ V(s) = \frac{sL + \frac{1}{sC}}{sL + R + \frac{1}{sC}} \]

To find frequency response, substitute \( s = j\omega \) in equation

\[ H(j\omega) = \frac{-\omega^2 + \frac{1}{LC}}{-\omega^2 + \frac{R}{L}j\omega + \frac{1}{LC}} \]

Magnitude Response

\[ |H(j\omega)| = \frac{1}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{R}{L}\right)^2}} \]

Phase Response

\[ \theta(j\omega) = -\tan^{-1}\left(\frac{-\omega}{\frac{R}{L} - \omega^2}\right) \]
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At resonance frequency, the transfer function will be real. Or system total impedance will be real

\[ j\omega_0 L + \frac{1}{j\omega_0 C} = 0 \]

\[ \omega_0 = \sqrt{\frac{1}{LC}} \]

\[ \omega_0 = \frac{1}{2\pi\sqrt{LC}} \]

\( H_{\text{min}} \) will be at \(|H(j\omega_0)|\)

\[ |H(j\omega_0)| = \frac{1}{\sqrt{\left(\frac{1}{LC} - \omega_0^2\right)^2 + \left(\frac{R}{L} \omega_0\right)^2}} \]

substitute \( \omega_0 = \sqrt{1/LC} \)

Result

\[ |H(j\omega_0)| = \frac{1}{\sqrt{\left(\frac{1}{LC} - \omega_0^2\right)^2 + \left(\frac{R}{L} \omega_0\right)^2}} = 0 \]

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Set \((1/\sqrt{2})H_{\text{max}}\) to find cutoff frequencies

\[ \frac{1}{\sqrt{2}} = \sqrt{\left(\frac{1}{LC} - \omega_0^2\right)^2 + \left(\frac{R}{L} \omega_0\right)^2} \]

\[ \frac{1}{2} \left[ \left(\frac{1}{LC} - \omega_0^2\right)^2 + \left(\frac{R}{L} \omega_0\right)^2 \right] = \left(\frac{1}{LC} - \omega_0^2\right)^2 \]

\[ \left(\frac{1}{LC} - \omega_0^2\right)^2 = \left(\frac{R}{L} \omega_0\right)^2 \]

Result

Confirm

\[ \omega_0^2 + R \omega_0 - \frac{1}{LC} = 0 \]

\[ \omega_{01} = \frac{-R + \sqrt{R^2 + \frac{4}{LC}}}{2L} \]

\[ \omega_0 = \sqrt{\frac{R \omega_0}{L}} \]

\[ \omega_{02} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \]

\( \frac{1}{LC} \)

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The Bandwidth $\beta$ is

$$\beta = \omega_{c2} - \omega_{c1} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)^2} - \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)^2}$$

$$\beta = \frac{R}{L}$$

The Quality factor $Q$ is

$$Q = \frac{\omega_c}{\beta} = \frac{\sqrt{\frac{1}{LC}}}{\frac{R}{L}}$$

$$Q = \frac{L}{\sqrt{CR^2}}$$

The cutoff frequencies in terms of $\beta$ is $\omega_0$

$$\omega_{c1} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + (\omega_c)^2}$$

$$\omega_{c2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + (\omega_c)^2}$$

The cutoff frequencies in terms of $\beta$ is $\omega_0$

$$\omega_{c1} = \omega_0 - \frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + \left(\frac{1}{2Q}\right)^2}$$

$$\omega_{c1} = \omega_0 + \frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + \left(\frac{1}{2Q}\right)^2}$$
Example Using serial RLC circuit, design band reject filter that bandwidth 250 Hz and a center frequency of 750 Hz. Use a 100nF capacitor, find $R, L, \omega_1, \omega_2,$ and $Q$

Let’s find $L$ first.

$$L = \frac{1}{\omega_0^2 C} = \frac{1}{(2\pi 750)^2 100 \times 10^{-9}} = 450 \text{ mH}$$

Calculate $R$

$$R = \beta L = 2\pi 750 (450) 10^{-3} = 707 \Omega$$

The quality factor is

$$Q = \frac{\omega_0}{\beta} = 3$$

The cutoff frequencies

$$\omega_1 = -\frac{\beta}{2} \sqrt{\left(\frac{\beta}{2}\right)^2 + \left(\omega_0\right)^2} = 3992 \text{ rad/s}$$

$$\omega_2 = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \left(\omega_0\right)^2} = 5562.2 \text{ rad/s}$$

**MatLab**

```matlab
>> R=707;
>> L=0.450;
>> C=0.0000001;
>> f=100:10:3000;
>> w=2*pi*f;
>> h=abs((-w.^2+1./(L*C))./(-w.^2+(R/L)*(j*w)+(1./(L*C))));
>> subplot (2,1,1)
>> semilogx(w,h)
>> grid on
>> title('|H(j\omega)|')
>> xlabel('\omega')
>> ylabel('|H(j\omega)|')
>> subplot (2,1,2)
>> theta=angle ((-w.^2+1./(L*C))./(-w.^2+(R/L)*j*w)+(1./(L*C)));
>> degree=theta*180/pi;
>> semilogx(w,degree)
>> grid on
>> title('\theta(j\omega)')
>> xlabel('\omega')
>> ylabel('\theta(j\omega)')
```

$R=707 \Omega, L=450 \text{ mH}, C=100 \text{ nF},$ Plot $F=0 - 3 \text{ KHz.}$
Frequency Response of a Circuit

R=707 Ω, L=450 mH, C=100 nF, Plot F=0 – 3 KHz.

\[ H(s) = \frac{s^2 + \frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} \]

MatLab

\[
\begin{align*}
R &= 707; \\
L &= 0.450; \\
C &= 0.000001; \\
\omega &= L/(L*C); \\
\beta &= R/L; \\
n &= [1 \ 0 \ a]; \\
dn &= [1 \ b \ a]; \\
\text{freqs}(n, dn)
\end{align*}
\]

Frequency Response of a Circuit

Example

OrCad Capture

Edit Simulation Profile
Frequency Response of a Circuit

PSpice Result

Frequency

V(L1:1) V(V1:+)
0V 0.5V 1.0V
(887.300,711.065m)
(635.877,708.111m)
(751.142,3.3697m)

Frequency

100Hz 300Hz 1.000Hz 3.000Hz

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