Discrete-Time Signals

- A discrete-time signal $x(n)$ is a function of an independent integer variable $n$. The signal $x(n)$ is not defined for non-integer values of $n$.

We can represent a discrete-time signal different ways;

1. **Graphical representation**

   Such as
Discrete-Time Signals

2. Functional representation

Such as

\[ x(n) = \begin{cases} 
1, & \text{for } n = 1, 3 \\
4, & \text{for } n = 2 \\
0, & \text{elsewhere} 
\end{cases} \]

3. Tabular representation

Such as

| \( n \)   | ...-2 | -1  | 0   | 1    | 2  | 3  | 4  | 5  | ...
|----------|-------|-----|-----|------|----|----|----|----|-----|
| \( x(n) \) | 0    | 0   | 0   | 1    | 4  | 1  | 0  | 0  | ...

4. Sequential representation

Such as

\[ x(n) = \{ \ldots 0,0,1,4,1,0,0,\ldots \} \]

The time origin (\( n=0 \)) is indicated by the symbol \( \uparrow \).

Some Elementary Discrete-Time Signals

1. The unit sample sequence:

\[ \delta(n) = \begin{cases} 
1, & \text{for } n = 0 \\
0, & \text{for } n \neq 0 
\end{cases} \]

The unit sample sequence is often referred to as a discrete-time impulse or an impulse.

2. The unit step signal:

\[ u(n) = \begin{cases} 
1, & \text{for } n \geq 0 \\
0, & \text{for } n < 0 
\end{cases} \]
Some Elementary Discrete-Time Signals

3. The unit ramp signal:

\[ u_r(n) = \begin{cases} 
  n, & \text{for } n \geq 0, \\
  0, & \text{for } n < 0 
\end{cases} \]

4. The exponential signal:

\[ x(n) = a^n \quad \text{for all } n \]

If \( a \) is real, then \( x(n) \) is a real signal.

Some Elementary Discrete-Time Signals

If \( a \) is complex, \( x(n) \) can be expressed as

\[ x(n) = a^n = \left( r e^{j\theta} \right)^n = r^n e^{jn\theta} \]

The real part is

\[ x_r(n) = r^n \cos \theta n \]

The imaginary part

\[ x_i(n) = r^n \sin \theta n \]
Some Elementary Discrete-Time Signals

Or we can represent the amplitude function

\[ |x(n)| = r^n \]

and the phase function

\[ \angle x(n) = \phi(n) = \theta n \]

Classification of Discrete-Time Signal

**Energy signals and Power signals**

The energy \( E \) of a signal \( x(n) \) is defined as

\[ E = \sum_{n=-\infty}^{\infty} |x(n)|^2 \]

If \( E \) is finite ( \( 0 < E < \infty \) ) then \( x(n) \) is called an energy signal

The average power of \( x(n) \) is defined as

\[ P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 \]
Classification of Discrete-Time Signal

Energy signals and Power signals (cont)

The energy of $x(n)$ is be found over the finite interval $-N < n < N$ as

$$E_s = \sum_{n=0}^{N} |x(n)|$$

The signal energy $E$ is

$$E = \lim_{N \to \infty} E_N$$

The average power of $x(n)$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} E_s$$

Classification of Discrete-Time Signal

Periodic Signals

A signal $x(n)$ is periodic with period $N(N>0)$ if and only if

$$x(n + N) = x(n) \quad \text{for all } n.$$ 

A sinusoidal signal

$$x(n) = A \sin 2\pi f_0 n$$

is periodic when $f_0$ is a rational number, that can be expressed as

$$f_0 = \frac{k}{N}$$

where $k$ and $N$ are integer

The average power in periodic $x(n)$ is given by

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$$
Classification of Discrete-Time Signal

Symmetric (even) and anti-symmetric (odd) signals

A real-valued signal $x(n)$ is called symmetric (even) if

$$x(-n) = x(n)$$

A signal $x(n)$ is called anti-symmetric (odd) if

$$x(-n) = -x(n)$$

Simple Manipulation of $x(n)$

Transformation of the time

A signal may be shifted in time replacing the time variable $n$ by $n-k$, where $k$ is integer.

- If $k$ is a positive integer, the results in a delay of the signal by $k$ unit of time
- If $k$ is a negative integer, the results in a advance of the signal by $k$ unit of time

If the time base is to replace the independent variable $n$ by $-n$, it is called folding or a reflection of the signal the time origin $n=0$
Let’s denote the Time-delay operation by $TD$ and the folding operation by $FD$:

\[
TD[x(n)] = x(n-k), \quad k > 0
\]

\[
FD[x(n)] = x(-n)
\]

\[
TD\{FD[x(n)]\} = TD[x(-n)] = x(-n+k)
\]

\[
FD\{TD[x(n)]\} = FD[x(n-k)] = x(-n-k)
\]

The analog signal $x(t)$ is almost linear between quantization levels. The quantization error output of these two systems are not the same.
Simple Manipulation of \( x(n) \)

**Example:**

\[
x(n) = [0, 0, 0, -3, -2, -1, 0, 1, 2, 3, 4, 4, 4, 4, 4, 4, 0, 0, 0]
\]

\( y(n) = x(2n) \)  \[ \text{Find } y(n). \]

\[
y(0) = x(0), \ y(-1) = x(-2), \ y(1) = x(2), \ y(-2) = x(-4), \ y(2) = x(4)
\]