Parametric Equations

Question

Which of the following parametric curves does not trace out the unit circle?

A. $x = \cos(t), y = \sin(t), 0 \leq t \leq 2\pi$

B. $x = \sin^2(t), y = \cos^2(t), 0 \leq t \leq 2\pi$

C. $x = \sin(t^2), y = \cos(t^2), 0 \leq t \leq \sqrt{2\pi}$

D. $x = \sin(2t), y = \cos(2t), 0 \leq t \leq \pi$

Parametric Equations

Question

If a and b are positive constants, then $x = a\cos(bt), y = a\sin(bt)$ describes the motion of a particle orbiting counterclockwise about the origin. Which transformation of the motion is *not* correctly identified?

A. If a is doubled, then the radius of the orbit is doubled.

B. If b is doubled, the time to complete one orbit is doubled.

C. If the sign of a is changed, then the particle orbits clockwise.

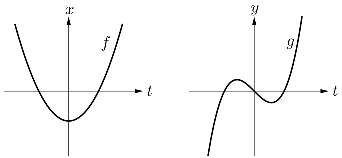
D. If the sign of b is changed, then the particle orbits clockwise.

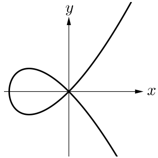
E. More than one of the above is incorrect.

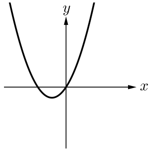
Parametric Equations

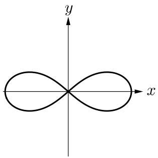
Question

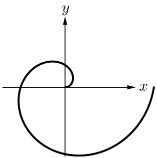
The graphs of $x = f(t)$ and $y = g(t)$ are pictured at the right. Which of the graphs below could be the graph of y versus x?



A. 

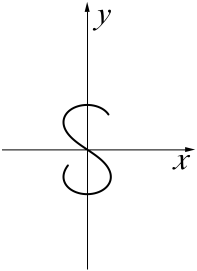
B. 

C. 

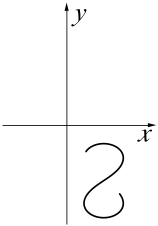
D. 

Question

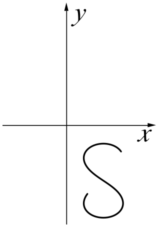
The figure shows the graph of $x = f(t), y = g(t)$. Which of the figures below could be the graph of $x = f(t) + 2$, $y = g(t) - 3$?



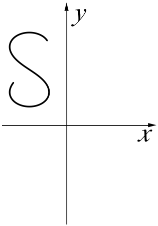
A.



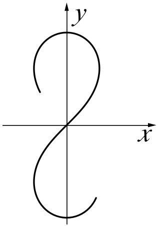
B.



C.

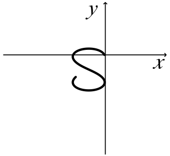


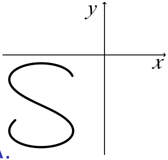
D.

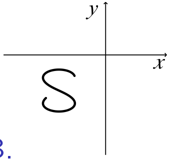


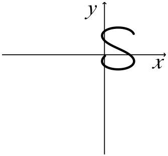
Question

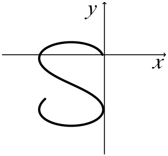
The figure shows the graph of $x = f(t), y = g(t)$. Which of the figures below could be the graph of $x = 2f(t)$, $y = 2g(t)$?



A. 

B. 

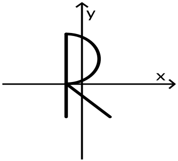
C. 

D. 

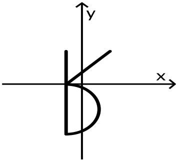
Parametric Equations

Question

The figure shows the graph of $x = f(t), y = g(t)$ on the left and a second curve on the right. Which of the following could be the equation of the second curve?



$x = f(t)$
$y = g(t)$



???

A. $x = -f(t)$, $y = g(t)$.

B. $x = f(t)$, $y = -g(t)$

C. $x = -f(t)$, $y = -g(t)$

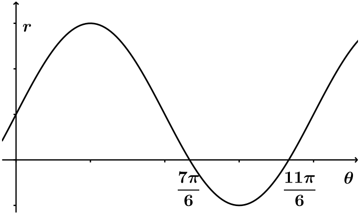
D. $x = f(-t)$, $y = g(t)$

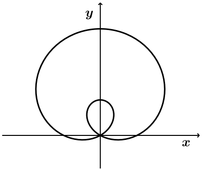
E. $x = -f(t)$, $y = g(-t)$

Polar Equations

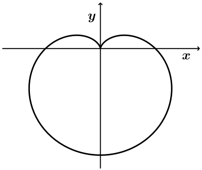
Question

The Cartesian graph of $r = f(\theta)$ is shown on the right. Which of the following is the polar graph of $r = f(\theta)$?

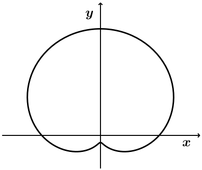




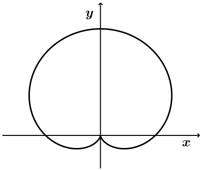
A.



B.



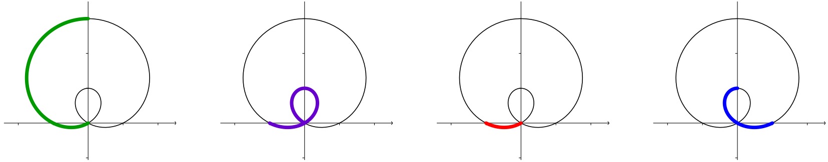
C.



D.

Question

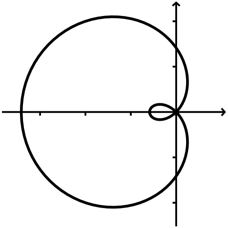
There are four copies of the graph of $r = 1 + 2\sin(\theta)$ below, each with a different arc highlighted. Which figure has the highlighted arc corresponding to $\pi \leq \theta \leq 7\pi/6$?



A. B. C. D.

Question

Consider the graph of $r = \sqrt{2} - 2\cos(\theta)$ to the right. Which of the following will *not* give us the area contained by the outer loop?



A. $2\int_{\pi/4}^{\pi} \frac{1}{2} r^2 d\theta$

B. $\int_{\pi/4}^{-\pi/4} \frac{1}{2} r^2 d\theta$

C. $\int_{\pi/4}^{7\pi/4} \frac{1}{2} r^2 d\theta$

D. $\int_{0}^{2\pi} \frac{1}{2} r^2 d\theta - \int_{-\pi/4}^{\pi/4} \frac{1}{2} r^2 d\theta$

Taylor Polynomials

Question

Suppose we want to use a Taylor polynomial to estimate $\sqrt{11}$. We could consider the function $f(x) = \sqrt{x}$ and use Taylor's formula for approximation at a. What would be the best choice for a?

A. $a = 0$
B. $a = 9$
C. $a = 10$
D. $a = 25$

L'Hospital's Rule

Question

Which of the following limits are *not* indeterminate?
(There is more than one correct answer)

A. $\lim\limits_{x \to \infty} \dfrac{\arctan(x)}{x}$

B. $\lim\limits_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^x$

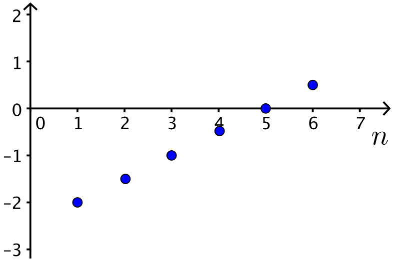
C. $\lim\limits_{x \to 0^+} x \ln(x)$

D. $\lim\limits_{x \to \infty} \dfrac{x}{e^{-x}}$

Sequences

Question

The figure shows the graph of the first six terms of a sequence $\{a_n\}_{n=1}^{\infty}$. Which of the following could be the formula for a_n?



A. $a_n = (-\frac{1}{2})^n - \frac{3}{2}$

B. $a_n = \frac{1}{2}n + 5$

C. $a_n = \frac{1}{2}n - 2$

D. $a_n = -\frac{1}{2}n + \frac{5}{2}$

E. $a_n = \frac{1}{2}n - \frac{5}{2}$

Question

Which of the following sequences is not geometric?

A. $a_n = \left(\frac{-1}{3}\right)^n + 2$

B. $a_n = 2\left(\frac{-1}{3}\right)^n$

C. $a_n = (-1)^n \frac{4}{3^n}$

D. $a_n = \left(\frac{-1}{3}\right)^n$

E. $a_n = 3^{-n}$

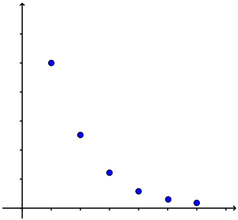
Question

Which of these sequences is not geometric?
(There may be more than one right answer.)

A. $\frac{1}{2}, \frac{3}{4}, \frac{9}{8}, \frac{27}{16}, \ldots$

B. $b_1 = \frac{1}{2},\ b_{n+1} = \frac{3}{2} b_n$

C. $a_n = \frac{2}{5^n}$

D. 

E. $a_n = \frac{1}{5} n$

Sequences

Question

Given $\{a_n\}_{n=1}^{\infty} = 3, 7, 4, 1/2, \pi, -1, \ldots$. If $b_n = a_{2n}$, which of the following is the sequence $\{b_n\}_{n=1}^{\infty}$?

A. $7, 1/2, -1, \ldots$

B. $6, 14, 8, 1, 2\pi, -2, \ldots$

C. $5, 9, 6, 5/2, \pi + 2, 1, \ldots$

D. $4, 1/2, \pi, -1, \ldots$

E. None of the above

Question

If $\{a_n\}_{n=1}^{\infty}$ is a sequence with $\lim_{n\to\infty} \frac{a_n}{n} = 0$, then which of the following must be true? (There may be more than one correct answer.)

A. $\lim_{n\to\infty} \frac{a_n}{e^n} = 0$

B. $\lim_{n\to\infty} \frac{a_n}{n!} = 0$

C. $\lim_{n\to\infty} \frac{a_n}{\sqrt{n}} = 0$

D. $\lim_{n\to\infty} \frac{a_n}{\ln(n)} = 0$

E. $\lim_{n\to\infty} a_n = 0$

Question

Given that $\lim_{n\to\infty} |a_n| = 3$, which of the following statements must be true?

A. $\lim_{n\to\infty} a_n = 3$

B. $\lim_{n\to\infty} a_n$ diverges due to oscillation.

C. $\lim_{n\to\infty} a_n$ diverges to ∞.

D. $\lim_{n\to\infty} \frac{a_n}{n}$ converges.

Question

Given that $\lim_{n\to\infty} |a_n| = 3$, which of the following statements must be false?

A. $\lim_{n\to\infty} a_n = 3$

B. $\lim_{n\to\infty} a_n$ diverges due to oscillation.

C. $\lim_{n\to\infty} a_n$ diverges to ∞.

D. $\lim_{n\to\infty} \frac{a_n}{n}$ converges.

Question

True or False?

It is always true that if $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

A. True, and I am confident.

B. True, and I am not so confident.

C. False, and I am not so confident.

D. False, and I am confident.

Series

Question

True or False?

It is always true that if $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

A. True, and I am confident.

B. True, and I am not so confident.

C. False, and I am not so confident.

D. False, and I am confident.

Question

True or False?

It is always true that if $\sum_{n=1}^{\infty} a_n = 1$, then $\lim_{n\to\infty} a_n = 1$.

A. True, and I am confident.

B. True, and I am not so confident.

C. False, and I am not so confident.

D. False, and I am confident.

Question

Two of the series below have the same value. Which two?

A. $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{n}{2^{n+1}}$

B. $\displaystyle\sum_{n=2}^{\infty}(-1)^n \frac{n-1}{2^{n+1}}$

C. $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{n+1}{2^{n+3}}$

D. $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{n+1}{2^{n+2}}$

Question

If we know that $\sum_{k=1}^{n} a_k = \dfrac{2n^2+5}{n^2+4}$, then which of the following must be true about $\sum_{n=1}^{\infty} a_n$?

A. It diverges, because $\lim_{n\to\infty} \frac{2n^2+5}{n^2+4} \neq 0$.

B. It converges to $\frac{5}{4}$.

C. It converges to 2.

D. It converges, but we can't know its exact value.

E. There is not enough information to determine whether or not it converges or diverges.

Series

Question

It is an amazing fact that $\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$. Which of the following statements is true?

I. $\sum_{k=1}^{n} \frac{1}{k^2}$ is a number close to $\frac{\pi^2}{6}$ when n is large.

II. $\sum_{k=1}^{1000} \frac{1}{k^2}$ is larger than $\frac{\pi^2}{6}$.

A. I. only

B. II. only.

C. Neither is true.

D. Both I. and II. are true.

Question

It is an amazing fact that $\sum_{k=1}^{\infty} \frac{(-1)^k}{2k+1} = \frac{\pi}{4}$. Which of the following statements is true?

I. $\sum_{k=1}^{n} \frac{(-1)^k}{2k+1}$ is a number close to $\frac{\pi}{4}$ when n is large.

II. $\sum_{k=1}^{1000} \frac{(-1)^k}{2k+1}$ is larger than $\frac{\pi}{4}$.

A. I. only

B. II. only.

C. Neither is true.

D. Both I. and II. are true.

Question

All of the following are geometric series. Which of them has first term $a = 1/3$ and common ratio $r = 3/8$?
(There may be more than one correct answer.)

A. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{3}\left(\frac{3}{8}\right)^{n-1}$

B. $\displaystyle\sum_{n=0}^{\infty} \frac{3^{n-1}}{8^n}$

C. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3}\left(\frac{3}{8}\right)^{n-1}$

D. $\displaystyle\frac{1}{3} + \frac{1}{8} + \frac{3}{64} + \frac{9}{512} + \cdots$

E. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{2^{3n}}$

Question

Which of the following inequalities is valid for all $x \geq 1$?

A. $x \leq 3x \leq x\,(2 - \sin x)$

B. $x\,(2 - \sin x) \leq x \leq 3x$

C. $x \leq x\,(2 - \sin x) \leq 3x$

D. $3x \leq x\,(2 - \sin x) \leq x$

Question

Which of the following inequalities is valid for all $x \geq 1$?

A. $\dfrac{x\,(2-\sin x)}{1+x^3} \;\leq\; \dfrac{3x}{1+x^3} \;\leq\; \dfrac{3}{x^2}$

B. $\dfrac{x\,(2-\sin x)}{1+x^3} \;\leq\; \dfrac{3}{x^2} \;\leq\; \dfrac{3x}{1+x^3}$

C. $\dfrac{3}{x^2} \;\leq\; \dfrac{x\,(2-\sin x)}{1+x^3} \;\leq\; \dfrac{3x}{1+x^3}$

D. $\dfrac{3x}{1+x^2} \;\leq\; \dfrac{x\,(2-\sin x)}{1+x^3} \;\leq\; \dfrac{3}{x^2}$

Improper Integrals

Question

True or False: $\displaystyle\int_1^{\infty} \frac{x\,(2-\sin x)}{1+x^3}\,dx$ converges.

A. True, and I am confident.

B. True, and I am not so confident.

C. False, and I am not so confident.

D. False, and I am confident.

The Integral Test

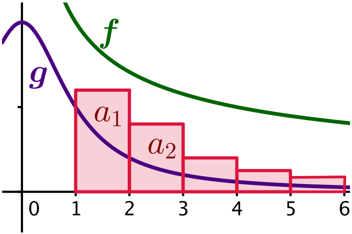
Question

The figure shows the graphs of two functions f and g along with the terms of $\sum_{k=1}^{\infty} a_k$ represented as boxes. Rank these four quantities:

$$\mathrm{Int}_f = \int_2^{\infty} f(x)\,dx \qquad \mathrm{Int}_g = \int_3^{\infty} g(x)\,dx$$

$$\mathrm{Sum}_2 = \sum_{k=2}^{\infty} a_k \qquad \mathrm{Sum}_3 = \sum_{k=3}^{\infty} a_k.$$

A. $\mathrm{Int}_g \le \mathrm{Sum}_3 \le \mathrm{Sum}_2 \le \mathrm{Int}_f$.
B. $\mathrm{Int}_g \le \mathrm{Sum}_2 \le \mathrm{Sum}_3 \le \mathrm{Int}_f$.
C. $\mathrm{Sum}_3 \le \mathrm{Int}_g \le \mathrm{Sum}_2 \le \mathrm{Int}_f$.
D. $\mathrm{Int}_f \le \mathrm{Sum}_3 \le \mathrm{Sum}_2 \le \mathrm{Int}_g$.



The Integral Test

Question

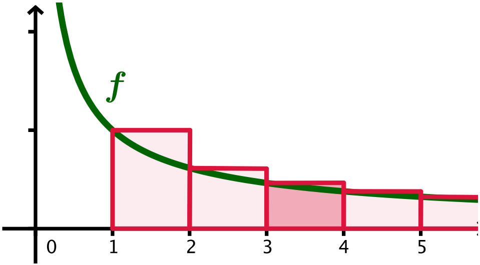
The figure shows the graph of a function f. If $a_n = f(n)$ for $n \geq 1$, then what is the area of the darker shaded region?

A. $\displaystyle\int_3^4 f(x)\,dx$

B. a_3.

C. a_4.

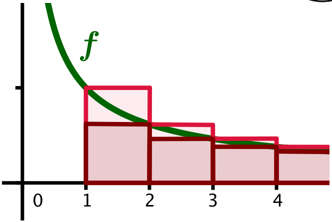
D. Not enough information is given.



The Integral Test

Question

The figure shows the graph of a function f. If $a_n = f(n)$ for $n \geq 1$, then which of the following statements must be true?



A. $$\int_1^\infty f(x)\,dx \leq \sum_{n=1}^{\infty} a_n$$

B. $$\sum_{n=2}^{\infty} a_n \leq \int_1^\infty f(x)\,dx.$$

C. If $\int_1^\infty f(x)\,dx$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

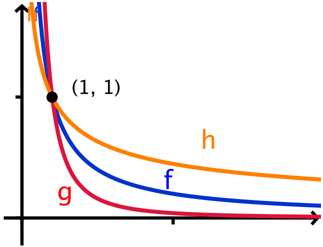
D. If $\int_1^\infty f(x)\,dx$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges.

(There could be more than one right answer.)

The Integral Test

Question

Suppose that $a_n = f(n)$ and $b_n = g(n)$. Given that $\int_1^\infty f(x)\,dx = 2$, which of the following must be true?



A. $\sum_{n=1}^{\infty} b_n$ converges.

B. $\sum_{n=1}^{\infty} a_n = 2$

C. $\int_1^\infty h(x)$ diverges.

D. $\int_1^\infty g(x)$ diverges.

Comparison Test

Question

Consider the series $\sum_{n=1}^{\infty} a_n$ where $a_n = \frac{2+(-1)^n}{1+n^3}$. Which of the following series $\sum_{n=1}^{\infty} b_n$ has $a_n \leq b_n$ for all $n \geq 1$?

A. $\sum_{n=1}^{\infty} \frac{3}{n}$

B. $\sum_{n=1}^{\infty} \frac{3}{n^3}$

C. $\sum_{n=1}^{\infty} \frac{3}{1+n^3}$

D. $\sum_{n=1}^{\infty} \frac{2}{1+n^3}$

E. More than one of the above

F. All of A through D

Comparison Test

Question

True or False: The sum $\displaystyle\sum_{n=1}^{\infty} \frac{2+(-1)^n}{1+n^3}$ converges.

A. True, and I am confident.

B. True, and I am not so confident.

C. False, and I am not so confident.

D. False, and I am confident.

Comparison Test

Question

By the p-test, $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n}}$ diverges. Which of the following statements is justified by the Comparison Test?

A. $\frac{1}{2\sqrt{n+7}} \geq \frac{1}{2\sqrt{n}}$ and therefore $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n+7}}$ diverges.

B. $\frac{1}{2\sqrt{n+7}} \leq \frac{1}{2\sqrt{n}}$ and therefore $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n+7}}$ diverges.

C. $\frac{1}{2\sqrt{n}-1} \leq \frac{1}{2\sqrt{n}}$ and therefore $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n}-1}$ diverges.

D. $\frac{1}{2\sqrt{n}-1} \geq \frac{1}{2\sqrt{n}}$ and therefore $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n}-1}$ diverges.

Comparison Test

Question

Which of the following inequalities, if true, would help you decide the convergence or divergence of $\sum_{n=1}^{\infty} a_n$?

A. $a_n \leq \dfrac{1}{\sqrt{n}}$

B. $a_n \leq \dfrac{1}{n^2}$

C. $a_n \geq \dfrac{1}{\sqrt{n^3}}$

D. $a_n \geq \left(\dfrac{1}{2}\right)^n$

Limit Comparision Test

Question

Consider the series $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^4+7n}}{n^5+\sqrt{n}}$. Which of the following simpler series would be most useful in applying the limit comparison test to this series?

A. $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{n^5}$

B. $\sum_{n=1}^{\infty} \frac{1}{n^4}$

C. $\sum_{n=1}^{\infty} \frac{1}{n^{11/3}}$

D. $\sum_{n=1}^{\infty} \frac{n^{4/3}}{n^5+\sqrt{n}}$

E. $\sum_{n=1}^{\infty} \frac{1}{n}$

Limit Comparison Test

Question

Which of the following limits would help you decide if $\sum_{n=1}^{\infty} a_n$ converges? (There may be more than one right answer.)

A. $\lim_{n\to\infty} \frac{a_n}{\frac{1}{n^2}} = 5$

B. $\lim_{n\to\infty} \frac{a_n}{\frac{1}{\sqrt{n}}} = 12$

C. $\lim_{n\to\infty} \frac{\left(\frac{1}{2}\right)^n}{a_n} = \infty$

D. $\lim_{n\to\infty} \frac{\frac{1}{n^2}}{a_n} = 0$

Alternating Series Test

Question

If a_k is positive and decreasing to 0, then the alternating series $\sum_{k=1}^{\infty}(-1)^{k-1}a_k$ converges to some value s. Rank the n^{th} partial sums s_1, s_{100}, s_{329}, the actual sum s, and the number 0, from smallest to greatest.

A. $s_{100} \leq 0 \leq s \leq s_{329} \leq s_1$

B. $0 \leq s_{100} \leq s \leq s_{329} \leq s_1$

C. $0 \leq s_{100} \leq s \leq s_1 \leq s_{329}$

D. $s_{100} \leq 0 \leq s \leq s_1 \leq s_{329}$

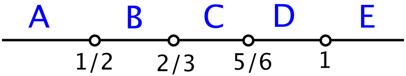
Alternating Series Test

Question

Let s be the sum of the alternating series

$$s = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$$

Which region of the number line does the number s belong to?



Strategy for Testing Series

Question

Which of the test(s) below would be the best choice for deciding the convergence of $\sum_{n=1}^{\infty} e^{1/n}$?

A. Test for Divergence
B. Geometric Series Test
C. Integral Test
D. Comparison or Limit Comparison Test
E. Alternating Series Test
F. Ratio Test

Strategy for Testing Series

Question

Which of the test(s) below would be the best choice for deciding the convergence of $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))}$?

A. Test for Divergence
B. Geometric Series Test
C. Integral Test
D. Comparison or Limit Comparison Test
E. Alternating Series Test
F. Ratio Test

Strategy for Testing Series

Question

Which of the test(s) below would be the best choice for deciding the convergence of $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{5}{n}$?

A. Test for Divergence
B. Geometric Series Test
C. Integral Test
D. Comparison or Limit Comparison Test
E. Alternating Series Test
F. Ratio Test

Strategy for Testing Series

Question

Which of the test(s) below would be the best choice for deciding the convergence of $\sum_{n=1}^{\infty} \frac{n7^n}{n!}$?

A. Test for Divergence
B. Geometric Series Test
C. Integral Test
D. Comparison or Limit Comparison Test
E. Alternating Series Test
F. Ratio Test

Strategy for Testing Series

Question

Which of the test(s) below would be the best choice for deciding the convergence of $\sum_{n=1}^{\infty} \frac{\pi^n}{3^{n+1}}$?

A. Test for Divergence

B. Geometric Series Test

C. Integral Test

D. Comparison or Limit Comparison Test

E. Alternating Series Test

F. Ratio Test

Strategy for Testing Series

Question

Which of the test(s) below would be the best choice for deciding the convergence of $\sum_{n=1}^{\infty} \frac{n^2+2n+2}{4n^4-\sqrt{n}+n}$?

A. Test for Divergence

B. Geometric Series Test

C. Integral Test

D. Comparison or Limit Comparison Test

E. Alternating Series Test

F. Ratio Test

Strategy for Testing Series

Question

Suppose $a_n = \dfrac{1}{n^4 + n + 2}$. Which of the following is a valid argument for why $\displaystyle\sum_{n=1}^{\infty} a_n$ converges?

A. $\displaystyle\lim_{n\to\infty} a_n = 0$, so $\sum a_n$ converges by the Test for Divergence.

B. $\displaystyle\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$, so $\sum a_n$ converges by the Ratio Test.

C. $\sum a_n$ is a p-series with $p = 4$, so $\sum a_n$ converges by p-test.

D. $a_n \leq \dfrac{1}{n^4}$ and $\sum \dfrac{1}{n^4}$ converges, so $\sum a_n$ converges by the Comparison Test.

Sequences vs. Series

Question

Try to think of a specific example for each of a_n, b_n, c_n and d_n. For one of them, there cannot be any example, because the situation described is impossible. Which one is impossible?

A. The sequence a_n is positive and decreasing and $\sum_{n=1}^{\infty} a_n$ diverges.

B. The sequence b_n alternates and $\sum_{n=1}^{\infty} b_n$ converges.

C. $\lim_{n\to\infty} c_n = 0$ and $\sum_{n=1}^{\infty} c_n = 1$.

D. $\lim_{n\to\infty} d_n = 1$ and $\sum_{n=1}^{\infty} d_n = 1$.

Intervals of Convergence

Question

If $\sum_{n=1}^{\infty} a_n x^n$ converges at $x = 5$, which of the following is NOT true?

A. $\sum_{n=1}^{\infty} a_n x^n$ definitely converges at $x = -5$.

B. $\sum_{n=1}^{\infty} a_n x^n$ definitely converges at $x = -3$.

C. $\sum_{n=1}^{\infty} a_n x^n$ definitely converges at $x = 0$.

D. $\sum_{n=1}^{\infty} a_n x^n$ definitely converges at $x = 3$.

Intervals of Convergence

Question

Suppose that $\sum_{n=0}^{\infty} a_n(x-2)^n$ converges when $x = 5$ and diverges when $x = -4$. Which of the following statements must be true? (There is more than one right answer)

A. $\displaystyle\sum_{n=0}^{\infty} a_n(x-2)^n$ converges when $x = 1$.

B. $\displaystyle\sum_{n=0}^{\infty} a_n(x-2)^n$ diverges when $x = 6$.

C. $\displaystyle\sum_{n=0}^{\infty} a_n(x-2)^n$ converges when $x = -1$.

D. $\displaystyle\sum_{n=0}^{\infty} a_n(x-2)^n$ diverges when $x = -5$.

Intervals of Convergence

Question

Which of the following could *not* be the interval of convergence of $\sum a_n(x+2)^n$?
(There may be more than one right answer.)

A. $\{-2\}$
B. $[-3, 3]$
C. $(-3, -1]$
D. $[-2, \infty)$
E. $(-\infty, \infty)$

Power Series

Question

If $\sum a_n x^n$ converges on the interval $[-3, 3]$, where does the series $\sum a_n(2x-1)^n$ converge?

A. $[-\frac{3}{2}, \frac{3}{2}]$
B. $[-6, 6]$
C. $[-2, 4]$
D. $[-1, 2]$

Question

If $\sum a_n x^n$ has radius of convergence 3, what is the radius of convergence of $\sum a_n x^{2n}$?

A. 3
B. $\sqrt{3}$
C. 3^2
D. Not enough information to decide

Taylor polynomials

Question

In each example below, we give an estimate for some quantity. Which one would most likely *not* be a good estimate? (Why?)

A. Estimate e by $1 + 1 + 1/2 + 1/6$.

B. Estimate $\sin(2)$ by $2 - 2^3/3! + 2^5/5!$

C. Estimate $\arctan(2)$ by $2 - 2^3/3 + 2^5/5 - 2^7/7$.

D. Estimate $\ln(1.5)$ by $0.5 - 0.5^2/2 + 0.5^3/3 - 0.5^4/4 + 0.5^5/5$.

Question

Which of the following Taylor polynomials for $\sin(x)$ would be most useful for approximating $\sin(3)$ by hand?

A. $x - \frac{1}{3!}x^3$

B. $1 - \frac{1}{2!}(x - \frac{\pi}{2})^2$

C. $-(x - \pi) + \frac{1}{3!}(x - \pi)^3$

D. $\sin(3) - \frac{1}{3!}\sin(3)(x - 3)^3$

Question

If $a_1 + a_2 + a_3 + \cdots$ and $b_1 + b_2 + b_3 + \cdots$ are convergent series of numbers and c is a real number, which of the following identities is always true?
(There may be more than one correct answer.)

A. $c(a_1 + a_2 + a_3 + \cdots) = (ca_1 + ca_2 + ca_3 + \cdots)$

B. $(a_1 + a_2 + a_3 + \cdots) + (b_1 + b_2 + b_3 + \cdots) =$
$(a_1 + b_1 + a_2 + b_2 + a_3 + b_3 + \cdots)$

C. $(a_1+a_2+a_3+\cdots)(b_1+b_2+b_3+\cdots) = (a_1b_1+a_2b_2+a_3b_3+\cdots)$

D. $(a_1 + a_2 + a_3 + \cdots)^2 = (a_1^2 + a_2^2 + a_3^2 + \cdots)$

Series

Question

If $\sum a_n$ and $\sum b_n$ are convergent series of numbers and c is a real number, which of the following identities is always true?
(There may be more than one correct answer.)

A. $c\sum a_n = \sum ca_n$

B. $\sum a_n + \sum b_n = \sum(a_n + b_n)$

C. $\left(\sum a_n\right)\left(\sum b_n\right) = \sum a_n b_n$

D. $\left(\sum a_n\right)^2 = \sum a_n^2$

Question

Which of the following calculations of the term-by-term derivatives of the power series $\sum_{n=1}^{\infty} n^2 x^n$ is valid?

A. $$\frac{d}{dx}\left(\sum_{n=1}^{\infty} n^2 x^n\right) = \sum_{n=1}^{\infty} 2nx^n$$

B. $$\frac{d}{dx}\left(\sum_{n=1}^{\infty} n^2 x^n\right) = \sum_{n=1}^{\infty} n^3 x^{n-1}$$

C. $$\frac{d}{dx}\left(\sum_{n=1}^{\infty} n^2 x^n\right) = \sum_{n=1}^{\infty} (2nx^n + n^2(nx^{n-1}))$$

D. $$\frac{d}{dx}\left(\sum_{n=1}^{\infty} n^2 x^n\right) = \sum_{n=1}^{\infty} (2n)(nx^{n-1})$$

Taylor Series

Question

Which of the following is the sum function for the power series $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{n!}$?

A. $\cos(x)$

B. x^2e^{-x}

C. e^{-x^2}

D. There is not enough information to answer.

Question

Which of the following numbers most accurately approximates the sum of $\sum_{n=0}^{\infty}(-1)^n\frac{\left(\frac{\pi}{3}\right)^{2n}}{(2n)!} = 1 - \frac{\left(\frac{\pi}{3}\right)^2}{2!} + \frac{\left(\frac{\pi}{3}\right)^4}{4!} - \cdots$?

A. 1

B. $1 - \frac{\left(\frac{\pi}{3}\right)^2}{2} \approx 0.4516$

C. $1 - \frac{\left(\frac{\pi}{3}\right)^2}{2} + \frac{\left(\frac{\pi}{3}\right)^4}{4!} \approx 0.5017$

D. 0.5

E. There is not enough information to answer.

Taylor Series

Question

Suppose the MacLaurin series on the interval $[-1, 1]$ for some function $f(x)$ is

$$f(x) = x - \frac{x^3}{9} + \frac{x^5}{25} - \cdots .$$

Which of the following statements are true? (There may be more than one correct answer.)

A. $\lim_{x \to 0} \frac{f(x)}{x} = 0$

B. $\lim_{x \to 0} \frac{f(x)}{x} = 1$

C. $f(1) > 8/9$

D. $f(0) = 1$