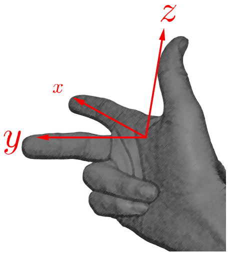
Three Dimensional Coordinates Systems

Question

Holding up your right hand as a coordinate system, use your left hand to point to the following locations relative to your coordinates.

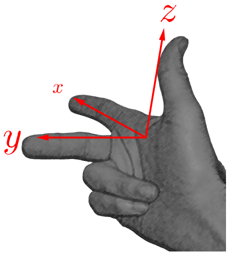


- $(0, 0, 0)$
- $(1, 0, 0)$
- $(1, 1, 0)$
- $(0, 1, 1)$
- $(-1, 0, 1)$

Three Dimensional Coordinates Systems

Question

Work with a partner. Partner 1 holds up their hand as a right handed coordinate system pointed at Partner 2, while Partner 2 locates the following points relative to the coordinates of Partner 1.



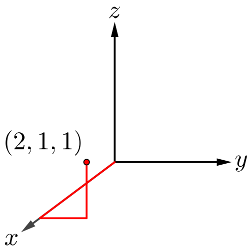
- $(0, 1, 0)$
- $(0, 0, -1)$
- $(0, 0, 1)$
- $(0, -1, 1)$

Three Dimensional Coordinate Systems

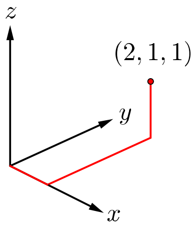
Question

Which of the following pictures does NOT correctly plot the point $(2, 1, 1)$ with respect to the coordinates indicated?

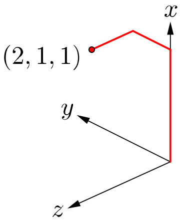
A.



B.



C.

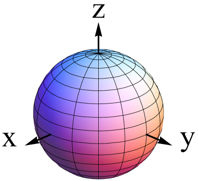


Three Dimensional Coordinate Systems

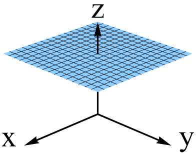
Question

Which of the following depicts the set of all points which are one unit away from the xy-plane?

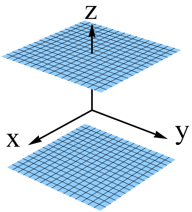
A.



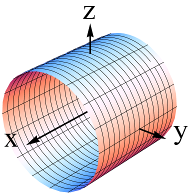
B.



C.



D.



Three Dimensional Coordinate Systems

Question

Which of the following statements is(are) true about the set of points (x, y, z) that satisfy the equation $y = x^2$?

I. The set is a surface in space.
II. The set lies in the xy-plane.
III. Any number can occur as the z-coordinate of a point in the set.

A. I only.
B. II only.
C. I and III only.
D. II and III only.
E. None of these.

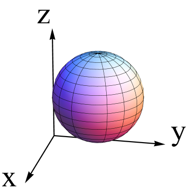
Three Dimensional Coordinate Systems

Question

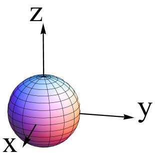
Which of the following correctly depicts the sphere given by the equation

$$x^2 + (y-2)^2 + (z-2)^2 = 4?$$

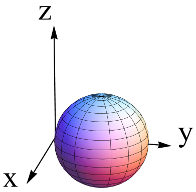
A.



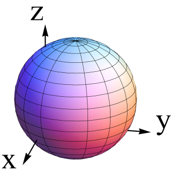
B.



C.



D.



Vectors

Question

True or False? The displacement vector from (a, b) to (c, d) is the same as the displacement vector from (c, d) to (a, b).

A. True, and I am confident.
B. True, but I am not so confident.
C. False, but I am not so confident.
D. False, and I am confident.

Vectors

Question

True or False? The vectors $2\vec{i} - \vec{j} + \vec{k}$ and $\vec{i} - 2\vec{j} + \vec{k}$ are parallel.

A. True, and I am confident.

B. True, but I am not so confident.

C. False, but I am not so confident.

D. False, and I am confident.

Vectors

Question

True or False? The vector $\vec{v} + \vec{w}$ is always of greater magnitude than either $\vec{v}$ or $\vec{w}$.

A. True, and I am confident.

B. True, but I am not so confident.

C. False, but I am not so confident.

D. False, and I am confident.

Vectors

Question

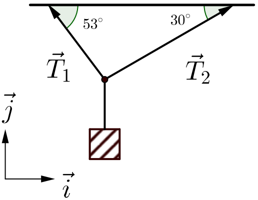
True or False? The vectors $\vec{i} + \vec{j}$ and $\vec{i} - \vec{j}$ are perpendicular.

A. True, and I am confident.
B. True, but I am not so confident.
C. False, but I am not so confident.
D. False, and I am confident.

Vectors

Question

A weight hangs from two wires as in the figure, creating two tension forces $\vec{T}_1$ and $\vec{T}_2$. If $|\vec{T}_1| = 4$, then which of the following equations for $\vec{T}_1$ is correct?



A. $\vec{T}_1 = -4\cos(53°)\vec{i} + 4\sin(53°)\vec{j}$

B. $\vec{T}_1 = -4\sin(53°)\vec{i} + 4\cos(53°)\vec{j}$

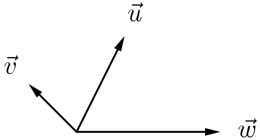
C. $\vec{T}_1 = -4\cos(37°)\vec{i} + 4\sin(37°)\vec{j}$

D. $\vec{T}_1 = -4\cos(30°)\vec{i} - 4\sin(30°)\vec{j}$

E. $\vec{T}_1 = 4\cos(53°)\vec{i} + 4\sin(53°)\vec{j}$

Question

The figure shows two vectors $\vec{v}$ and $\vec{w}$. Which of these could be an expression for the vector $\vec{u}$?



A. $2\vec{v} + \vec{w}$

B. $\vec{v} + \vec{w}$

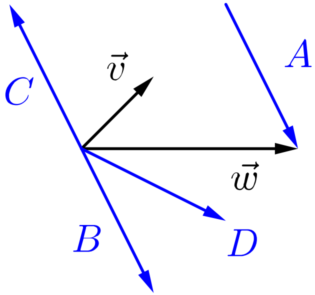
C. $\vec{v} + 2\vec{w}$

D. $2\vec{v} + 2\vec{w}$

Vectors

Question

The figure shows two vectors $\vec{v}$ and $\vec{w}$ and a number of other vectors built from $\vec{v}$ and $\vec{w}$. Which of these vectors corresponds to $\vec{w} - 2\vec{v}$?



The Dot Product

Question

Which of the following formulas involving vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ with $\vec{a} \neq \vec{0}$ are meaningless?

A. $\left(\dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}\right)\vec{a}$

B. $\dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$

C. $(\vec{a} \cdot \vec{b})\vec{c} - (\vec{c} + \vec{b}) \cdot \vec{a}$

D. $(\vec{b} \cdot \vec{a})\vec{c} - (\vec{c} \cdot \vec{a})\vec{b}$

Question

The figure shows a pair of vectors in the plane. The value of $\vec{v} \cdot \vec{w}$ is:

A. Positive

B. Zero

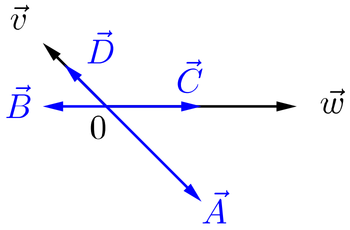
C. Negative

D. There is not enough information to determine.

The Dot Product

Question

The figure shows vectors $\vec{v}$ and $\vec{w}$ in the plane and a number of other vectors built from $\vec{v}$ and $\vec{w}$. Which of the vectors shown could be $\mathrm{proj}_{\vec{v}}(\vec{w})$?



E. None of these.

The Dot Product

Question

Suppose that a force of 30 N is applied in the $\vec{i}$-direction while an object is displaced 1 m from the origin into the 2nd quadrant. Which of the following could be the work done by the force on the object?

A. -15 N-m

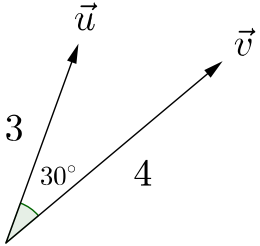
B. 20 N-m

C. -50 N-m

D. -30 N

E. 20 N

The Cross Product



Question

The figure shows two vectors with $|\vec{u}| = 3$ and $|\vec{v}| = 4$. Which of the following numbers is $|\vec{u} \times \vec{v}|$?

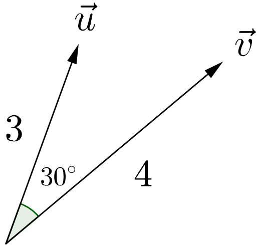
A. 6

B. 12

C. $6\sqrt{3}$

D. 30°

E. There is not enough information to determine.



Question

The figure shows two vectors in the xy-plane with $|\vec{u}| = 3$ and $|\vec{v}| = 4$. Which of the following could be $\vec{u} \times \vec{v}$?

A. $-6\vec{k}$

B. $6\vec{k}$

C. $6(\vec{i} + \vec{j})$

D. $6\vec{j}$

The Cross Product

Question

Which of the following formulas involving the vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ is meaningless?

A. $(\vec{a}\cdot\vec{b})\vec{c}-(\vec{a}\times\vec{b})\times\vec{c}$

B. $\vec{a}\cdot(\vec{b}\times\vec{c})$

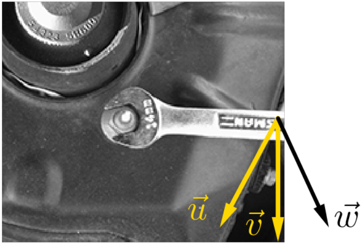
C. $|\vec{a}\times\vec{b}|\vec{c}+\vec{b}\times\vec{a}$.

D. $(\vec{a}\times\vec{b})\cdot\vec{c}-(\vec{a}\cdot\vec{b})\vec{c}$

The Cross Product

Question

In applying a force of a given magnitude to the end of the wrench, which of the directions $\vec{u}$, $\vec{v}$, or $\vec{w}$ will generate the maximum torque on the nut?



A. $\vec{u}$

B. $\vec{v}$

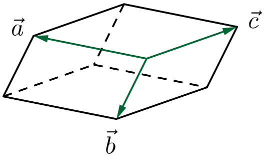
C. $\vec{w}$

D. All three options produce a torque of the same magnitude.

The Cross Product

Question

The figure shows a parallelepiped spanned by vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$. Which of the following formulas computes its volume?



A. $|\vec{a} \cdot (\vec{b} \times \vec{c})|$
B. $|\vec{b} \cdot (\vec{c} \times \vec{a})|$
C. $|\vec{c} \cdot (\vec{b} \times \vec{a})|$
D. All of the above.
E. More than one but not all of the above.

True or False?

If two lines in space are perpendicular to the same plane, then the lines are parallel.

A. True, and I am confident.

B. True, but I am not confident.

C. False, but I am not confident.

D. False, and I am confident.

True or False?

If two lines in space are parallel to the same plane, then the lines are parallel.

A. True, and I am confident.

B. True, but I am not confident.

C. False, but I am not confident.

D. False, and I am confident.

True or False?

If two lines in space are skew, then they cannot be perpendicular to the same plane.

A. True, and I am confident.
B. True, but I am not confident.
C. False, but I am not confident.
D. False, and I am confident.

Lines and Planes

Question

Which completes the sentence: Two different planes in space can ______________.

A. fail to intersect.

B. intersect in a point.

C. intersect in a line.

D. More than one but not all of the above.

E. All of the above.

Lines and Planes

Question

Which completes the sentence: A line and a plane in space can ______________.

A. fail to intersect.

B. intersect in a point.

C. intersect in a line.

D. More than one but not all of the above.

E. All of the above.

Lines and Planes

Question

Consider the line with parametric equations $x = 1 + 3t$, $y = 2 - 2t$, $z = 1 + t$. Which of the following vectors is parallel to the line?

A. $\langle 1, 2, 1 \rangle$

B. $\langle 3, -2, 1 \rangle$

C. $\langle 4, 0, 2 \rangle$

D. None of these.

Question

Consider the line with parametric equations $x = 1 + 3t$, $y = 2 - 2t$, $z = 1 + t$. Which of the following points lies on the line?

A. $(3, -2, 1)$
B. $(1, 2, 1)$
C. $(4, 0, 2)$
D. More than one of these.

Question

Which of the following is NOT the equation of a line?

A. $\dfrac{x-2}{3} = \dfrac{y+1}{9} = \dfrac{z}{2}$

B. $x = 1 + 3t,\ y = 2 - 2t,\ z = 1 + t$

C. $3x + 4y - 5z = 2$

D. $\vec{r}(t) = \langle 4, 2, 7 \rangle + t\langle 3, -2, 1 \rangle$

Lines and Planes

Question

Which of the following is NOT the equation of a plane with normal vector $\langle 3, -1, 1 \rangle$ passing through the point $(0, 0, 2)$?

A. $\langle 3, -1, 1 \rangle \cdot \langle x, y, z - 2 \rangle = 0$

B. $3x - y + z = 2$

C. $3x - y + z = 0$

D. $12x - 4y + 4z = 8$

Question

A plane through the origin contains the vector $\langle 1, 1, 1 \rangle$. Which of the following equations could NOT be the equation of the plane?

A. $3x - 2y + 2z = 0$

B. $3x - 2y - z = 0$

C. $y - z = 0$

D. $-4x + 9y - 5z = 0$

Lines and Planes

Question

For the plane $y = x$, rank the following quantities in order from smallest to largest.

I. The distance from $(1, 0, 0)$ to the plane.

II. The distance from $(-1, 1, 0)$ to the plane.

III. The distance from $(1, 1, 2)$ to the plane.

A. I $<$ II $<$ III

B. I $=$ II $<$ III

C. III $<$ I $<$ II

D. III $<$ I $=$ II

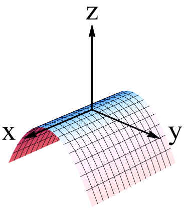
E. None of these.

Cylinders and Quadric Surfaces

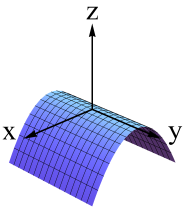
Question

Which of the following depicts the graph of $z = -y^2$?

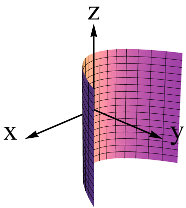
A.



B.



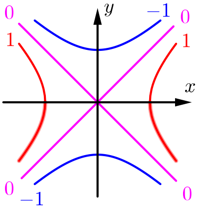
C.



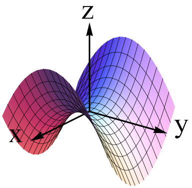
Cylinders and Quadric Surfaces

Question

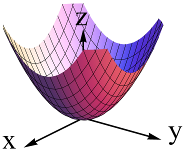
The figure shows the traces of a surface in the planes $z = -1$, $z = 0$, and $z = 1$. Which of the following could be a picture of this surface?



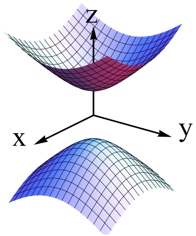
A.



B.

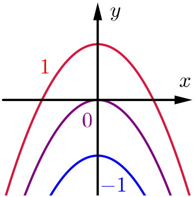


C.

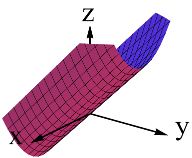


Question

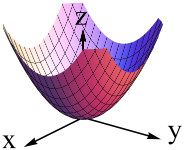
The figure shows the traces of a surface in the planes $z = -1$, $z = 0$, and $z = 1$. Which of the following could be a picture of this surface?



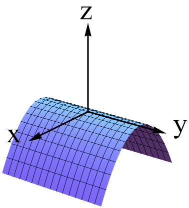
A.



B.



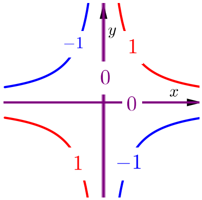
C.



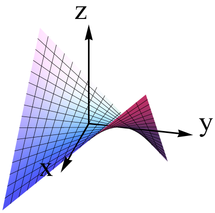
Cylinders and Quadric Surfaces

Question

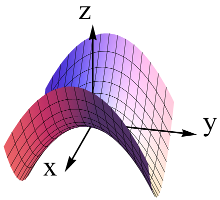
The figure shows the traces of a surface in the planes $z = -1$, $z = 0$, and $z = -1$. Which of the following could be a picture of this surface?



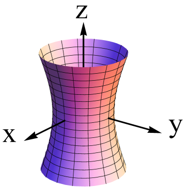
A.



B.



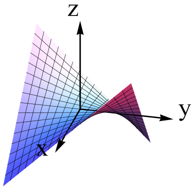
C.



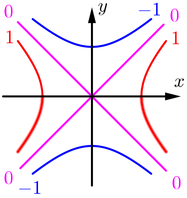
Cylinders and Quadric Surfaces

Question

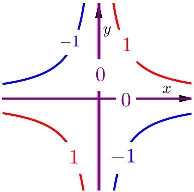
Consider the surface shown at right. Which of the following figures could be the traces of this surface in the planes $z = -1$, $z = 0$, and $z = 1$?



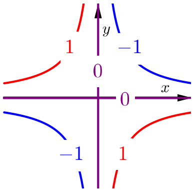
A.



B.



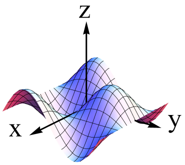
C.



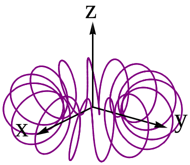
Discussion

Which of the following do you think can NOT be a picture of the range of continuous function $\vec{r}(t) = f(t)\vec{i} + g(t)\vec{j} + h(t)\vec{k}$.

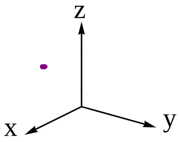
A. Surface



B. Curve



C. Point



Vector Functions and Space Curves

Question

Which of these vector functions does NOT have range which is contained in a line?

A. $\vec{r}(t) = \langle 3, 1, 2 \rangle + t^2 \langle 1, -1, 0 \rangle$

B. $\vec{r}(t) = \langle 3, 1, 2 \rangle + t \langle 1, -1, 0 \rangle$

C. $\vec{r}(t) = \langle 2t - 1, t, -3t + 5 \rangle$

D. $\vec{r}(t) = t\vec{i} + t\vec{j} + t^2\vec{k}$

E. $\vec{r}(t) = \langle 1, 2, t \rangle$

Vector Functions and Space Curves

Question

The vector function $\vec{r}(t) = \cos(t)\vec{j} + \sin(t)\vec{k}$ parameterizes the intersection of two surfaces in space. Which of the following surfaces could form the pair?

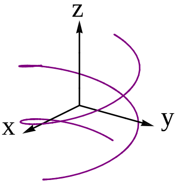
I. the plane $x = 0$
II. the circular cylinder $y^2 + z^2 = 1$
III. the sphere $x^2 + y^2 + z^2 = 1$
IV. the hyperbolic cylinder $y^2 - z^2 = 1$

A. The pairs (I,II) and (I,III) only.
B. The pair (I,II) only.
C. The pairs (I,II) and (I,III) and (I,IV) only.
D. The pairs (I,II) and (I,III) and (II,III) only.
E. There is not enough information to decide.

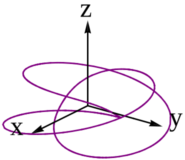
Question

If f, g, and h are continuous functions on $[0,1]$, which of the following figures could NOT be a picture of the range of $\vec{r}(t) = f(t)\vec{i} + g(t)\vec{j} + h(t)\vec{k}$?

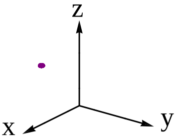
A.



B.



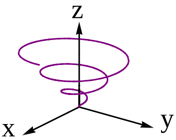
C.



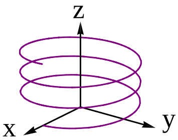
Question

Which of the following space curves could be parameterized by $\vec{r}(t) = \langle t\cos(t), t\sin(t), t\rangle$ for $t \geq 0$?

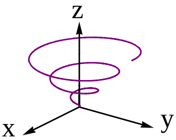
A.



B.



C.



Question

Consider the space curve parametrized by $x = \cos(t), y = \sin(t)$, and $z = t$. Which of the following is an equation of the tangent line to the curve at the point where $t = \pi/4$?

A. $\vec{r}(t) = \left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 1 \right\rangle + t \langle -\sin(t), \cos(t), 1 \rangle$

B. $\vec{r}(t) = \left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 1 \right\rangle + t \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{\pi}{4} \right\rangle$

C. $\vec{r}(t) = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{\pi}{4} \right\rangle + t \left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 1 \right\rangle$

D. $\vec{r}(t) = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{\pi}{4} \right\rangle + t \left\langle -1, 1, \frac{1}{\sqrt{2}} \right\rangle$

E. More than one of the above.

Question

Suppose that a plane curve is parametrized by $\vec{r}(t) = f(t)\vec{i} + g(t)\vec{j}$ and $\vec{r}\,'(3) = \vec{0}$. What can you conclude about the curve near $\vec{r}(3)$?

A. The tangent line to the curve is horizontal at $\vec{r}(3)$.

B. The tangent line to the curve is vertical at $\vec{r}(3)$.

C. There is a corner in the curve at $\vec{r}(3)$.

D. There is not enough information to decide.

Derivatives and Integrals of Vector Functions

Question

If $\vec{u}$ and $\vec{v}$ are differentiable vector functions and f is a differentiable scalar function, which of the following formulas is meaningless?

A. $f'(t)\vec{u}(t) + f(t)\vec{u}\,'(t)$

B. $\vec{u}\,'(t) \times \vec{v}(t) + \vec{u}(t) \times \vec{v}\,'(t)$

C. $\vec{u}(t) \cdot \int \vec{v}(t)dt$

D. $f(t) + \int (\vec{u}(t) \times \vec{v}(t))dt$

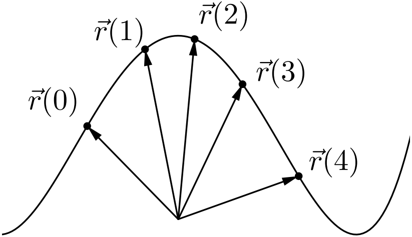
E. $f(t) + \int (\vec{u}(t) \cdot \vec{v}(t))dt$

Derivatives and Integrals of Vector Functions

Question

The figure shows the position vector $\vec{r}(t)$ along a curve in the plane. Which of the following tangent vectors do you you think should have the smallest magnitude?

A. $\vec{r}\,'(0)$
B. $\vec{r}\,'(1)$
C. $\vec{r}\,'(2)$
D. $\vec{r}\,'(3)$



Arc length and Curvature

Question

Suppose you move with constant velocity $\langle -3, 4, 0 \rangle$ m/s for 2 seconds. How far did you travel?

A. -10 m

B. -6 m

C. 5 m

D. 8 m

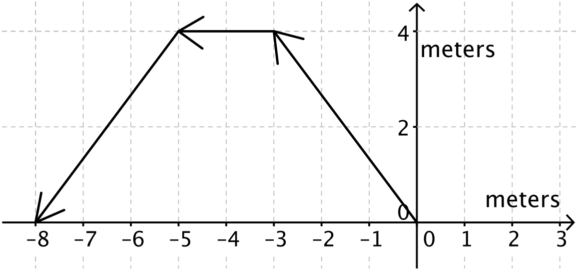
E. 10 m

Arc length and Curvature

Question

Suppose an object is displaced from the origin by a sequence of 3 steps as indicated in the figure. How far did the object travel?

A. -12 m
B. 12 m
C. -8 m
D. 8 m
E. 2 m

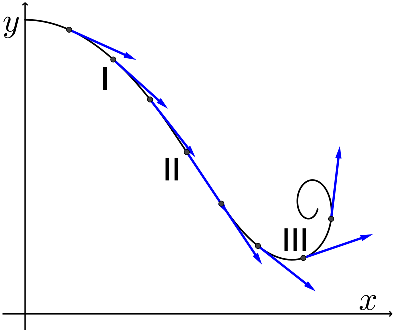


Arc length and Curvature

Question

The figure shows a curve along with the unit tangent vector $\vec{T}$ at a number of equally spaced points.

Rank the values of $\left|\dfrac{d\vec{T}}{ds}\right|$ at the points I, II, and III.



A. smallest at I, largest at II

B. smallest at III, largest at II

C. smallest at II, largest at III

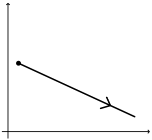
D. the same at I and II, largest at III

Arc length and Curvature

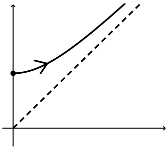
Question

Suppose that $\left|\frac{d\vec{T}}{ds}\right| = s$ for a curve in the plane. Which of the following could be a picture of the curve?

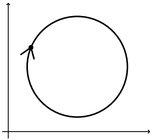
A.



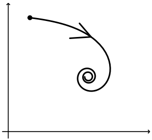
B.



C.



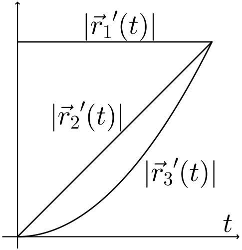
D.



Question

Suppose we have 3 curves with parametrizations $\vec{r_1}(t)$, $\vec{r_2}(t)$, and $\vec{r_3}(t)$ with speed functions whose graphs are shown in the figure. Which curve has the greatest length?

A. Curve 1
B. Curve 2
C. Curve 3
D. Not enough information to decide



Question

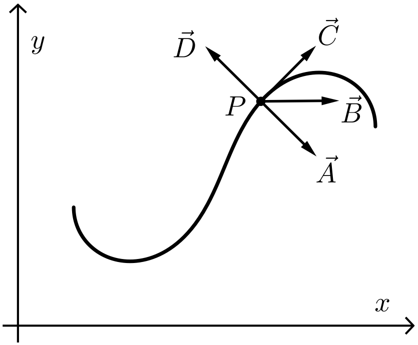
If the binormal vector $\vec{B}(t)$ is constant in t, what can you conclude about the curve?

A. The curve is a straight line in space.
B. The curve is a circle in space.
C. The curve is a helix.
D. The curve is planar.

Arc length and Curvature

Question

Suppose $\vec{T}(t)$ is the unit tangent vector function of a curve. Which of the vectors shown at P could represent $\vec{N} = \frac{\vec{T}\,'}{|\vec{T}\,'|}$ at P?



Motion in Space: Velocity and Acceleration

True or False?

Recall that in general the acceleration $\vec{a} = v'\vec{T} + \kappa v^2\vec{N}$. Suppose $\vec{T}$ and $\vec{N}$ are nonzero and $\vec{a} = 2\vec{T} + 3\vec{N}$ throughout a particular motion. Which of the following could be a true statement about the motion?

A. The speed could be constant.

B. The motion could be along a straight line.

C. The motion could be around a circle in space.

D. None of the above.

Question

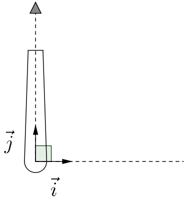
A projectile is fired straight up with initial speed v_0. Assuming that air resistance is negligible and the external force is due to gravity, which the following statements about the position function is NOT true?

A. $|\vec{r}\,'(0)| = v_0$.

B. $\vec{r}\,''(t) = -g\vec{j}$.

C. The $\vec{j}$-component of $\vec{r}(t)$ is zero.

D. The $\vec{i}$-component of $\vec{r}(t)$ is zero.



Motion in Space: Velocity and Acceleration

Question

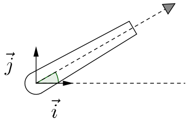
The cannon from the previous question is tilted to have angle of elevation 30°. Which of the following statements about the position function is NOT true?

A. $\vec{r}\,''(t) = -g\vec{j}$.

B. The $\vec{j}$-component of $\vec{r}\,'(0)$ is smaller than the $\vec{i}$-component of $\vec{r}\,'(0)$.

C. $|\vec{r}\,'(0)| = v_0$.

D. The $\vec{i}$-component of $\vec{r}(t)$ is zero.



Motion in Space: Velocity and Acceleration

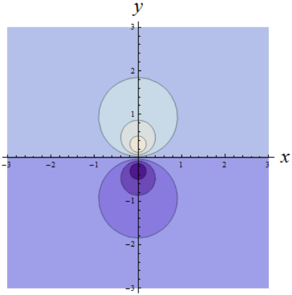
Question

The motion of a particle is given by the function $\vec{r}(t)$. If $\vec{r}(t) \times \vec{r}\,'(t)$ is parallel to $\vec{j}$ for all t, what can be said about the motion?

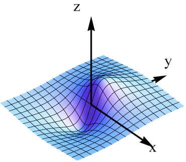
A. The acceleration is constant.
B. The motion is parallel to the y-axis.
C. The motion occurs in the xz-plane.
D. The motion is along a helix.

Question

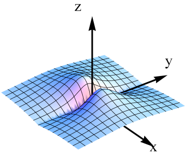
The figure shows the level curves of a function of two variables $f(x, y)$. Which of the figures below could be the graph of f?



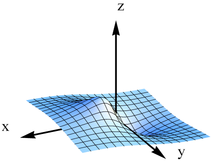
A.



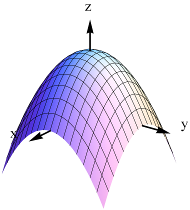
B.



C.



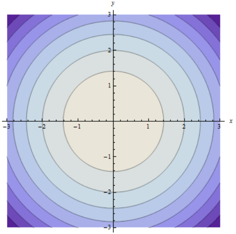
Functions of Several Variables



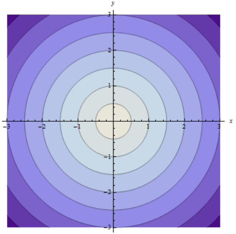
Question

The figure shows the graph of a function of two variables $f(x, y)$. Which of the figures below could be the level curves of f?

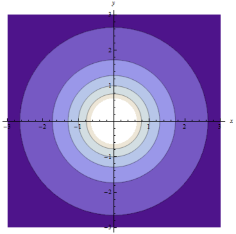
A.



B.



C.



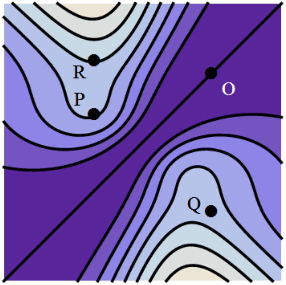
Question

The figure shows a contour map with four labeled points. Which of the following statements about those points are true?

I. From Q, one looks down at P.

II. From P, one looks down at R.

III. From R, one looks down at O.



A. I and III only.

B. I and II only.

C. III only.

D. I only.

Question

True or False? For a function $f(x, y)$, it is possible for the level 3 curve and the level 5 curve to intersect.

A. True, and I am confident.

B. True, but I am not so confident.

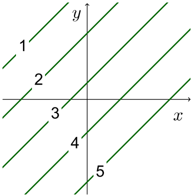
C. False, but I am not so confident.

D. False, and I am confident.

Question

Which of the following could NOT represent a linear function of x and y?

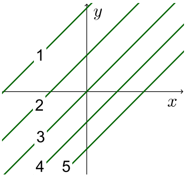
A.



B.

$x\backslash y$	-1	0	1	2
-4	-2	0	2	4
-3	-1	1	3	5
-2	0	2	4	6
-1	1	3	5	7

C.

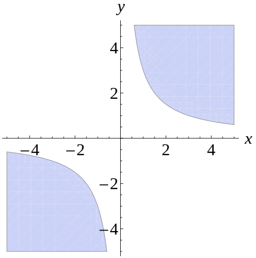


Functions of Several Variables

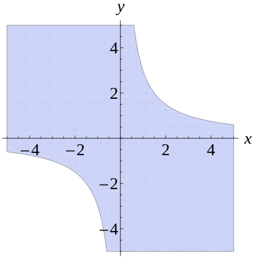
Question

Which of the shaded regions depicts the domain of $f(x, y) = xy\sqrt{xy - 3}$?

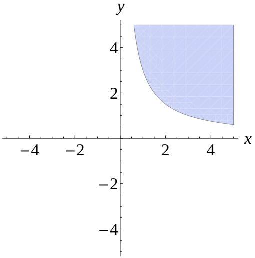
A.



B.



C.



Functions of Several Variables

Question

Which is the best description of all of the level *surfaces* of $f(x, y, z) = x^2 + y^2 - z^2$?

A. All level surfaces are spheres centered at the origin.

B. All level surfaces are paraboloids centered on the z-axis.

C. All level surfaces are cones.

D. All level surfaces are hyperboloids of 2 sheets.

E. All level surfaces are hyperbolic paraboloids.

F. None of these.

Question

True or false? If the values of $f(x, y)$ tend to 1 as $(x, y) \to (0, 0)$ along both the x-axis and the y-axis, then

$$\lim_{(x,y)\to(0,0)} f(x, y) = 1.$$

A. True, and I am confident.

B. True, but I am not so confident.

C. False, but I am not so confident.

D. False, and I am confident.

Question

If the values of $f(x,y)$ tend to 1 as $(x,y) \to (0,0)$ along the line $y=x$, but the values of $f(x,y)$ tend to -1 as $(x,y) \to (0,0)$ along the line $y=-x$, then

$$\lim_{(x,y)\to(0,0)} f(x,y) \text{ does not exist.}$$

A. True, and I am confident.

B. True, but I am not so confident.

C. False, but I am not so confident.

D. False, and I am confident.

Question

For the function $f(x, y) = \dfrac{xy}{x^2 + y^2}$, which of the following expressions would help investigate the trend in the values of f as (x, y) approaches $(0, 0)$ along the line $y = mx$?

A. $\dfrac{m^2x^2}{(2m^2)x^2}$

B. $\dfrac{mx^2}{(1+m)x^2}$

C. $\dfrac{mxy}{m^2x^2 + y^2}$

D. $\dfrac{mx^2}{(1+m^2)x^2}$

Question

If

$$f(x,mx)=\frac{mx}{x^3+m^3},\qquad g(x,mx)=\frac{m(x+1)}{1+mx},$$

and

$$h(x,mx)=\frac{x^2+2x}{2x^2+x},$$

then which limit can you conclude must NOT exist?

A. $\lim\limits_{(x,y)\to(0,0)} f(x,y)$

B. $\lim\limits_{(x,y)\to(0,0)} g(x,y)$

C. $\lim\limits_{(x,y)\to(0,0)} h(x,y)$

D. There is not enough information to decide.

Limits and Continuity

Question

If

$$f(x, mx) = \frac{mx}{x^3 + m^3}, \qquad g(x, mx) = \frac{m(x+1)}{1+mx},$$

and

$$h(x, mx) = \frac{x^2 + 2x}{2x^2 + x},$$

then which limit can you conclude must exist?

A. $\lim\limits_{(x,y)\to(0,0)} f(x, y)$

B. $\lim\limits_{(x,y)\to(0,0)} g(x, y)$

C. $\lim\limits_{(x,y)\to(0,0)} h(x, y)$

D. There is not enough information to decide.

Question

Where is the function $\arctan\left(\frac{y}{x}\right)$ continuous?

A. Everywhere except at the origin.

B. Everywhere except along the x-axis.

C. Everywhere except along the y-axis.

D. Everywhere except along the line $y = x$.

Question

The table shows the values of a function a function $f(x, y)$. Do you think that the limit of $f(x, y)$ as $(x, y) \to (0, 0)$ exists?

$x \backslash y$	-1.0	-0.5	-0.2	0	0.2	0.5	1.0
-1.0	0.00	0.60	0.92	1.00	0.92	0.60	0.00
-0.5	-0.60	0.00	0.72	1.00	0.72	0.00	-0.6
-0.2	-0.92	-0.72	0.00	1.00	0.00	-0.72	-0.92
0	-1.00	-1.00	-1.00		-1.00	-1.00	-1.00
0.2	-0.92	-0.72	0.00	1.00	0.00	-0.72	-0.92
0.5	-0.60	0.00	0.72	1.00	0.72	0.00	-0.6
1.0	0.00	0.60	0.92	1.00	0.92	0.60	0.00

A. I think the limit exists, and I am confident.

B. I think the limit exists, but I am not confident.

C. I think the limit does not exist, but I am not confident.

D. I think the limit does not exist, and I am confident.

Partial Derivatives

Question

In economics, the production $P(L, K)$ of a sector is modelled by a function of labor L and capital K. Suppose that at a labor level of 10 thousand hours of work and a capital investment of \$5 million, $P(10, 5)$ is \$20 million of goods and $P_L(10, 5) = 0.7$. Which of these is NOT true?

A. If Labor is increased from 10 thousand to 11 thousand hours of work while capital investment remains the same, then we expect production to increase by about \$0.7 million in goods.

B. If Labor is increased from 10 thousand to 11 thousand hours of work while capital investment remains the same, then we expect production to be about \$0.7 million in goods.

C. $P(11, 5) \approx 20.7$

Question

The table shows values of a function $f(x, y)$. Which of the following equations are most accurate?

A. $f_x(1, 2) \approx -1$.

B. $f_y(1, 2) \approx 2$.

C. $f_x(3, 2) \approx 1$.

D. $f_y(3, 2) \approx 4$.

E. More than one of the above.

$x \backslash y$	0	1	2	3
0	3	5	7	9
1	2	4	6	8
2	1	3	5	7
3	0	2	4	6

Partial Derivatives

Question

Rank the values of $f_y(3,0)$, $f_x(0,0)$, and $f_y(2,1)$ in ascending order.

A. $f_x(0,0) \leq f_y(2,1) \leq f_y(3,0)$

B. $f_y(3,0) \leq f_y(2,1) \leq f_x(0,0)$

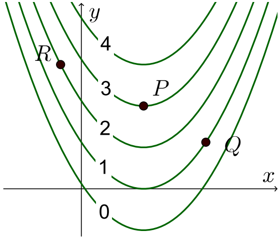
C. $f_x(0,0) \leq f_y(3,0) \leq f_y(2,1)$

D. $f_y(3,0) \leq f_x(0,0) \leq f_y(2,1)$

$x \backslash y$	0	1	2	3
0	5	4	3	1
1	3	2	2	4
2	1	1	1	1
3	-1	0	0	-1

Question

The figure shows the level curves of a function $f(x, y)$. Rank the values of f_x at the points P, Q, R.



A. $f_x(Q) < f_x(P) < f_x(R)$

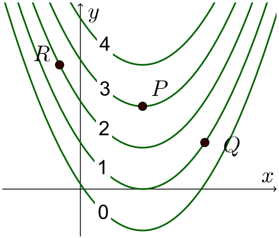
B. $f_x(R) < f_x(P) < f_x(Q)$

C. $f_x(Q) < f_x(R) < f_x(P)$

D. $f_x(P) < f_x(Q) < f_x(R)$

Question

The figure shows the level curves of a function $f(x, y)$. At which of the points P, Q, or R does f_y have the smallest value?



A. P

B. Q

C. R

D. There is not enough information to decide.

Partial Derivatives

Question

For the function $f(x, y) = e^x \sin(xy^2)$ which formula is correct?

A. $f_y = e^x \cos(xy^2)$

B. $f_y = 2ye^x \cos(xy^2)$

C. $f_y = 2xye^x \cos(xy^2)$

D. $f_y = e^x \sin(xy^2) + 2ye^x \cos(xy^2)$

E. None of the above.

Question

For the function $f(x, y) = e^x \sin(xy^2)$ which formula is correct?

A. $f_x = e^x \cos(xy^2)$

B. $f_x = y^2 e^x \cos(xy^2)$

C. $f_x = e^x \sin(xy^2) + e^x \cos(xy^2)$

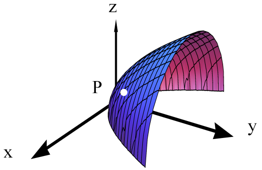
D. $f_x = e^x \sin(xy^2) + y^2 e^x \cos(xy^2)$

E. None of the above.

Partial Derivatives

Question

The graph of a function $f(x, y)$ together with a point $P = (a, b, f(a, b))$ is shown at right. What are the signs of the partial derivatives at the point point (a, b)?



A. $f_x(a, b) < 0,\ f_y(a, b) < 0$

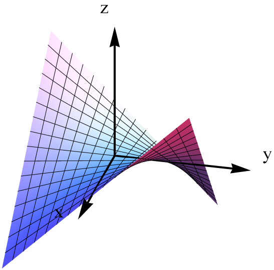
B. $f_x(a, b) > 0,\ f_y(a, b) < 0$

C. $f_x(a, b) < 0,\ f_y(a, b) > 0$

D. $f_x(a, b) > 0,\ f_y(a, b) > 0$

Question

Consider the graph of $f(x, y)$ shown at right. True or False? $f_{xy}(0,0)$ is positive.



A. True, and I am confident.

B. True, but I am not confident.

C. False, but I am not confident.

D. False, I am confident.

Partial Derivatives

Question

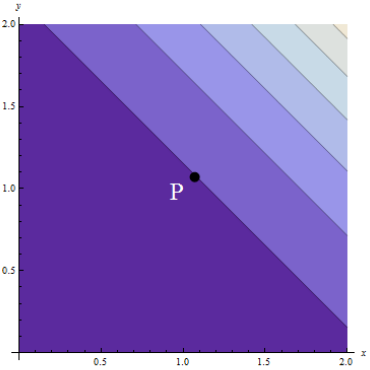
The level curves of a function $f(x, y)$ are shown at right. Determine the signs of the partial derivatives at the point P.

A. $f_x(P) < 0$, $f_y(P) < 0$

B. $f_x(P) > 0$, $f_y(P) < 0$

C. $f_x(P) < 0$, $f_y(P) > 0$

D. $f_x(P) > 0$, $f_y(P) > 0$



Partial Derivatives

Question

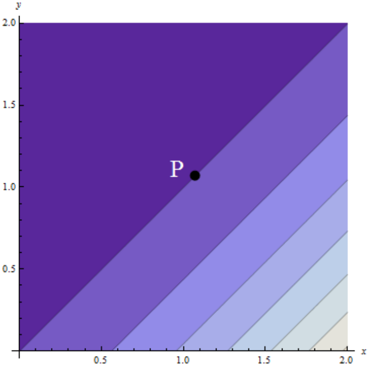
The level curves of a function $f(x, y)$ are shown at right. Determine the signs of the pure second partial derivatives at the point P.

A. $f_{xx}(P) < 0$, $f_{yy}(P) < 0$

B. $f_{xx}(P) > 0$, $f_{yy}(P) < 0$

C. $f_{xx}(P) < 0$, $f_{yy}(P) > 0$

D. $f_{xx}(P) > 0$, $f_{yy}(P) > 0$



Tangent Planes and Linear Approximation

Question

For a particular function $f(x, y)$, the equation of the tangent plane at the point $(1, 2)$ has the form

$$z - 7 = f_x(1, 2)(x - 1) + f_y(1, 2)(y - 2).$$

Which of the following vectors is normal to the plane?

A. $\langle 1, f_x(1, 2), f_y(1, 2)\rangle$

B. $\langle f_x(1, 2), f_y(1, 2), -1\rangle$

C. $\langle f_x(1, 2), f_y(1, 2), 1\rangle$

D. $\langle -f_x(1, 2), -f_y(1, 2), 1\rangle$

E. More than one of the above.

Tangent Planes and Linear Approximation

Question

For the function $f(x, y) = xy$, find the equation of the tangent plane to the graph of f at the point $(2, 3)$.

A. $z - 6 = x(x - 2) + y(y - 3)$

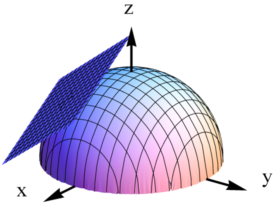
B. $z - 6 = y(x - 2) + x(y - 3)$

C. $z - 6 = 2(x - 2) + 3(y - 3)$

D. $z - 6 = 3(x - 2) + 2(y - 3)$

Question

The figure shows the graph of $f(x, y)$ together with its tangent plane at $(1, -1, \sqrt{2})$. What are the signs of the partial derivatives of f at $(1, -1)$?



A. $f_x(1, -1) < 0$, $f_y(1, -1) < 0$

B. $f_x(1, -1) > 0$, $f_y(1, -1) < 0$

C. $f_x(1, -1) < 0$, $f_y(1, -1) > 0$

D. $f_x(1, -1) > 0$, $f_y(1, -1) > 0$

Tangent Planes and Linear Approximation

Question

For a particular function, $f(2,3) = -1$, $f_x(2,3) = 5$, and $f_y(2,3) = -7$. Which of the following approximations of f is valid near $(2,3)$?

A. $f(x,y) \approx -1 + 5(x-2) - 7(y-3)$

B. $f(x,y) \approx 5(x-2) - 7(y-3)$

C. $f(x,y) \approx -1 + 5x - 7y$

D. $f(x,y) \approx 5x - 7y$

Tangent Planes and Linear Approximation

Question

Suppose $L(x, y) = -1 + 5(x-2) - 7(y-3)$ is the linearization of $f(x, y)$ at $(2, 3)$. The $y = 3$ trace of the graph of $z = f(x, y)$ is a curve. What is the slope of the tangent line to that curve at the point $(2, 3, -1)$?

A. -7

B. 5

C. -1

D. There is not enough information to decide.

Tangent Planes and Linear Approximation

Question

A cylinder of height h cm and radius r cm is heated and expands by 0.1 cm in height and 0.3 cm in radius. Which formula estimates the expansion in volume in cubic cm?

A. $dV = 0.6\pi rh + 0.1\pi r^2$
B. $dV = \pi r^2(h + 0.1)$
C. $dV = \pi(r + 0.3)^2 h$
D. $dV = \pi(r + 0.3)^2(h + 0.1)$

The Chain Rule

Question

Select the appropriate chain rule for computing $\dfrac{\partial w}{\partial u}$ where $w = w(x, y), x = x(u)$, and $y = y(u, v)$.

A. $\dfrac{\partial w}{\partial u} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial u} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial u}$

B. $\dfrac{\partial w}{\partial u} = \dfrac{\partial w}{\partial x}\dfrac{dx}{du} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial u}$

C. $\dfrac{\partial w}{\partial u} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial u} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial u} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial v}$

D. $\dfrac{\partial w}{\partial u} = \dfrac{\partial w}{\partial x}\dfrac{dx}{du} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial u} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial v}$

The Chain Rule

Question

If $w = xy$ where $x = r\cos(\theta)$ and $y = r\sin(\theta)$, which expression computes $\frac{\partial w}{\partial \theta}$?

A. $\frac{\partial w}{\partial \theta} = 0$

B. $\frac{\partial w}{\partial \theta} = r^2\cos(2\theta)$

C. $\frac{\partial w}{\partial \theta} = -r^3\sin^2(\theta)\cos(\theta) + r^3\sin(\theta)\cos^2(\theta)$

D. None of the above

The Chain Rule

Question

If $w = f(x, y)$, $x = t^2$, and $y = 5t + 3$, what is $\left.\dfrac{dw}{dt}\right|_{t=1}$?

A. $\left.\dfrac{dw}{dt}\right|_{t=1} = f_x(x, y) \cdot 2 + f_y(x, y) \cdot 5$

B. $\left.\dfrac{dw}{dt}\right|_{t=1} = f_x(x, y) \cdot 2t + f_y(x, y) \cdot 5$

C. $\left.\dfrac{dw}{dt}\right|_{t=1} = f_x(1, 8) \cdot 2 + f_y(1, 8) \cdot 5$

D. $\left.\dfrac{dw}{dt}\right|_{t=1} = f_x(1, 8) \cdot 2t + f_y(1, 8) \cdot 5$

The Chain Rule

Question

The intensity I of sunlight varies with position and time. A solar car is traveling along the ground. Which chain rule would help you compute $\frac{df}{dt}$, where f is the intensity of sunlight on the panel of the car and t is time?

A. $\frac{df}{dt} = \frac{\partial I}{\partial t}$

B. $\frac{df}{dt} = \frac{\partial I}{\partial x} + \frac{\partial I}{\partial y} + \frac{\partial I}{\partial t}$

C. $\frac{df}{dt} = \frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial y}\frac{dy}{dt}$

D. $\frac{df}{dt} = \frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial y}\frac{dy}{dt} + \frac{\partial I}{\partial t}$

Question

Using the table of values of $f(x,y)$ shown at right, estimate $D_{\vec{u}}f(1,2)$ where $\vec{u} = \frac{1}{\sqrt{2}}(\vec{i}+\vec{j})$.

$x \backslash y$	0	1	2	3
0	-1	1	0	2
1	0	5	3	2
2	1	6	6	5
3	3	7	9	7

A. 2

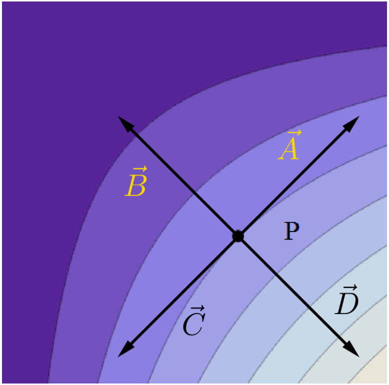
B. $-1/\sqrt{2}$

C. -3

D. $2/\sqrt{2}$

Question

On the contour plot of f shown, there are four directions at P indicated by unit vectors. For which direction $\vec{u}$ is $D_{\vec{u}}f$ at P the largest?



Question

If ∇f at P is $\vec{v} = \langle 3, 4, 12 \rangle$, then what is $D_{\vec{u}} f$ at P when the angle between $\vec{u}$ and $\vec{v}$ is 60°?

A. 13

B. $\dfrac{13}{2}$

C. $\dfrac{13\sqrt{3}}{2}$

D. Not enough information to answer.

Question

If $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$ is a curve in space and $x(t)^2 + y(t)^2 + z(t)^2 = 1$, what can you conclude about $\vec{r}\,'(0)$?

A. $\vec{r}\,'(0)$ is normal to a sphere of radius 1 centered at the origin.

B. $\vec{r}\,'(0) = \vec{0}$ because $|\vec{r}(t)|$ is constant.

C. $\vec{r}\,'(0)$ is tangent to a sphere of radius 1 centered at the origin.

D. $\vec{r}\,'(0) = 2x(0)x'(0)\vec{i} + 2y(0)y'(0)\vec{j} + 2z(0)z'(0)\vec{k}$.

Question

If $x(t)^2 + y(t)^2 + z(t)^2 = 1$ for all t and we differentiate both sides with respect to t, what do we get?

A. $2x(t)x'(t) + 2y(t)y'(t) + 2z(t)z'(t) = 1$

B. $2x(t) + 2y(t) + 2z(t) = 0$

C. $2x(t) + 2y(t) + 2z(t) = 1$

D. $2x(t)x'(t) + 2y(t)y'(t) + 2z(t)z'(t) = 0$

Question

Find the equation of the tangent plane to the surface $3x^2 + y^2 - z^2 = 1$ at the point $(0, 1, 0)$.

A. $y = 1$

B. $y = 0$

C. $6x(x - 0) + 2y(y - 1) - 2z(z - 0) = 0$

D. $6x + 2y - 2z = 0$

Directional Derivatives and the Gradient

Question

What is the equation of the tangent plane to the surface $z = f(x, y)$ when $x = 2$ and $y = 3$?

A. $-f_x(x, y)(x - 2) - f_y(x, y)(y - 3) + (z - f(x, y)) = 0$

B. $-f_x(2, 3)(x - 2) - f_y(2, 3)(y - 3) + (z - f(2, 3)) = 0$

C. $-f_x(2, 3)(x - 2) - f_y(2, 3)(y - 3) = 0$

D. $f_x(2, 3)(x - 2) + f_y(2, 3)(y - 3) + (z - f(2, 3)) = 0$

Directional Derivatives and the Gradient

Question

If f is not a constant function, what angle θ should $\vec{u}$ make with ∇f at P in order that $D_{\vec{u}}f(P)$ is as large as possible?

A. 0°

B. 90°

C. 180°

D. Not enough information

Maximum and Minimum Values

Question

The figure shows four points on the contour map of a function $f(x, y)$. Which of the four marked points could be critical points of the function?

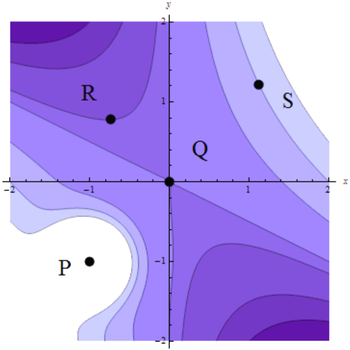
A. P only

B. R and S only

C. R only

D. P and Q only

E. All four



Maximum and Minimum Values

Question

Suppose f has a critical point at $(3, 2)$, $f_{xx}(3, 2) < 0$ and $D = f_{xx}(3, 2) f_{yy}(3, 2) - (f_{xy}(3, 2))^2 > 0$. What can you conclude?

A. f has a local minimum at $(3, 2)$.
B. f has a local maximum at $(3, 2)$.
C. $f(3, 2)$ is not a local maximum nor a local minimum of f.
D. There is not enough information to decide.

Maximum and Minimum Values

Question

If f has a critical point at $(3, 2)$, $f_{yy}(3, 2) < 0$ and $D = f_{xx}(3, 2)f_{yy}(3, 2) - (f_{xy}(3, 2))^2 > 0$, can you conclude that f has a local maximum $(3, 2)$?

A. Yes, because $f_{xx}(3, 2)$ must also be negative.

B. No, the second derivative test refers only to f_{xx}.

C. Yes, whenever $D > 0$, f has a local maximum.

D. No, because $f_{xx}(3, 2)$ could be positive.

Maximum and Minimum Values

Question

For the functions $f(x, y) = x^2y^2$ and $g(x, y) = x^3y^3$ we get $D_f = -12x^2y^2$ and $D_g = -45x^4y^4$ which both equal zero at $(0, 0)$. What is true about f and g?

A. Both f and g have a local minimum at $(0, 0)$.

B. f has a local minimum at $(0, 0)$ and g has a local maximum at $(0, 0)$.

C. f has a local minimum at $(0, 0)$, g has a saddle point at $(0, 0)$.

D. Both f and g have saddle points at $(0, 0)$.

Maximum and Minimum Values

Question

Suppose you wanted to find the point (x, y, z) on the surface $z = x^2 + y^2$ which is closest to the point $(1, 2, 3)$. Which of the following functions of x and y would you minimize?

A. $\sqrt{(x-1)^2 + (y-2)^2 + (x^2 + y^2 - 3)^2}$

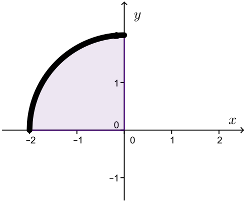
B. $(x-1)^2 + (y-2)^2 + (x^2 + y^2 - 3)^2$

C. $\sqrt{(x-1)^2 + (y-2)^2}$

D. More than one of the above.

Question

Suppose you wanted to investigate the values of a function $f(x, y)$ along the arc of the quarter circle shown. Which of these substitutions would you plug into f?



A. $x = -\sqrt{4 - y^2}$ with $0 \leq y \leq 2$

B. $y = \sqrt{4 - x^2}$ with $0 \leq x \leq 2$

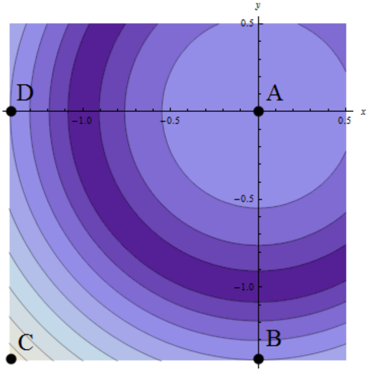
C. $x = \sqrt{4 - y^2}$ with $0 \leq y \leq 2$

D. $y = -\sqrt{4 - x^2}$ with $-2 \leq x \leq 0$

Maximum and Minimum Values

Question

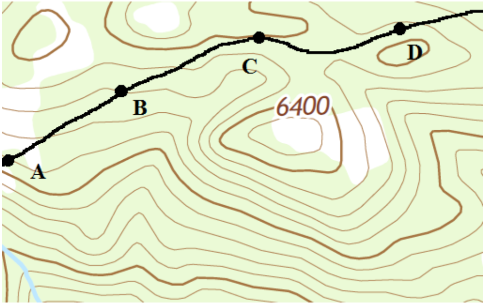
The figure shows the contours of a function $f(x, y)$ over a closed region R bounded by a square and four marked points in that region. Which point is the location of the absolute maximum of f over R?



Lagrange Multipliers

Question

The map shows a hiking path on a topographic map and four marked points. Which point is the highest along the path?



Lagrange Multipliers

Question

A farmer has 1000 ft of fencing and wants to make an animal pen next to river, where fencing would be needed on only three sides. To find the dimensions of the pen with maximum area by the method of Lagrange Multipliers, what would you use for f and g?

A. $f(x,y) = xy$ and $g(x,y) = 2x + y$

B. $f(x,y) = 2x + 2y$ and $g(x,y) = xy$

C. $f(x,y) = xy$ and $g(x,y) = x + y$

D. $f(x,y) = x + y$ and $g(x,y) = xy$

Lagrange Multipliers

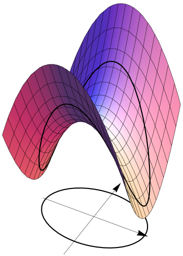
Question

Match the Lagrange Multiplier Problem

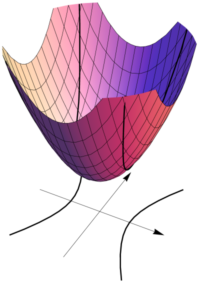
$$-2x = 2x\lambda,\ 2y = 4y\lambda,\ x^2 + 2y^2 = 1$$

with the graph of the function being optimized and the constraint.

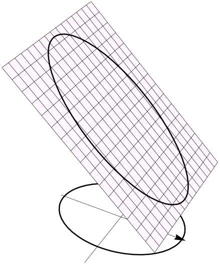
A.



B.



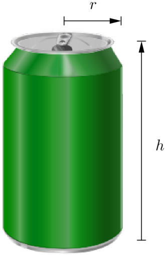
C.



Lagrange Multipliers

Question

If you want to design a soda can to hold 350 cm^3 of soda using the least amount of metal to make the can, which optimization problem should you solve?



A. $\min\ 2\pi r h$ subject to $\pi r^2 h = 350$

B. $\min\ \pi r^2 h$ subject to $2\pi r h + 2\pi r^2 = 350$

C. $\min\ 2\pi r h + 2\pi r^2$ subject to $\pi r^2 h = 350$

D. $\min\ \pi r^2 h$ subject to $2\pi r h = 350$

Lagrange Multipliers

Question

Which of these word problems could the equations

$$\nabla(x+y+z) = \lambda\nabla(x^2+y^2+z^2-1) \text{ and } x^2+y^2+z^2-1=0$$

be used to solve?

A. Find the maximum of the temperature function $f(x,y,z) = x+y+z$ over the unit sphere centered at the origin.

B. Find the maximum of the temperature function $f(x,y,z) = x^2+y^2+z^2-1$ over the plane $x+y+z=0$ in space.

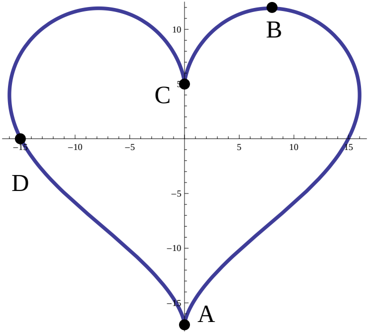
C. Find the minimum of the temperature function $f(x,y,z) = x+y+z$ over the surface given by the equation $x^2+y^2+z^2=1$.

D. More than one of the above.

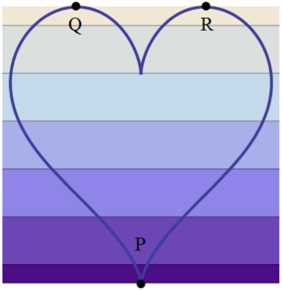
Lagrange Multipliers

Question

Which of the points shown is the location of the minimum of $f(x, y) = y$ along the curve?



Lagrange Multipliers



Question

In minimizing $f(x, y) = y$ along the curve $g(x, y) = 0$ shown, the method of Lagrange Multipliers will fail. Why?

A. The minimum occurs at P but $\nabla g = \vec{0}$ at P.

B. g is not differentiable at P.

C. The method of Lagrange Multipliers will only detect the points Q and R.

D. More than one but not all of the above.