Double Integrals over Rectangles

Question

The chart shows the depths of several locations in a 10 ft $\times$ 12 ft rectangular pool. Which of these is the best estimate for the volume of water in the pool?

	0	5	10
0	3	3	2
6	4	4	2
12	6	5	3

A. 240 ft^3

B. 330 ft^3

C. 450 ft^3

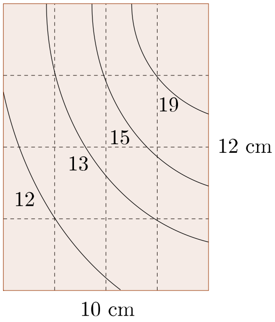
D. 570 ft^3

E. 720 ft^3

Double Integrals over Rectangles

Question

The contours of temperature (in °C) for a $12\text{cm} \times 10\text{cm}$ rectangular plate are shown at right. Which of these is the best estimate for the average temperature across the plate?



A. $19\ ^\circ\text{C}$

B. $15.5\ ^\circ\text{C}$

C. $14.2\ ^\circ\text{C}$

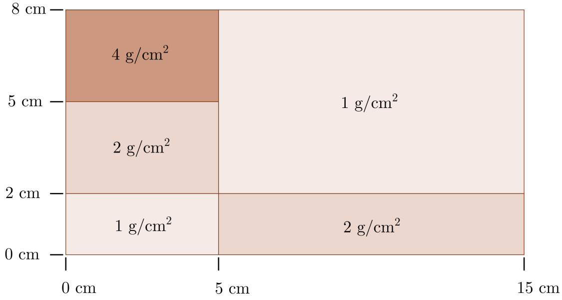
D. $12\ ^\circ\text{C}$

Double Integrals over Rectangles

Question

A rectangular part is being made of rectangular components of different sizes and densities as shown. Which of these is the mass of the part in grams?

A. 200 g
B. 100 g
C. 150 g
D. 300 g



Double Integrals over Rectangles

Question

Suppose we estimate the volume V of the solid lying below the graph of $f(x, y) = 4 - x^2 - y^2$ and above the square $\mathcal{R}$ given by $0 \leq x \leq 1$ and $0 \leq y \leq 1$, using a division into 4 equal squares. If L and U are the Riemann sums using lower left and upper right corners, respectively, how do V, L, and U compare?

A. $L < V < U$

B. $U < V < L$

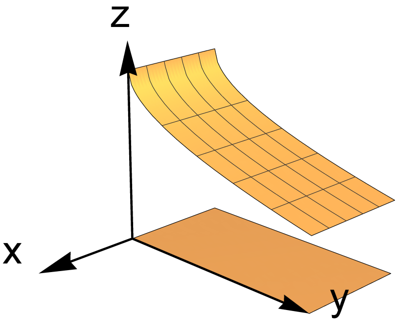
C. $L < U < V$

D. $V < L < U$

Double Integrals over Rectangles

Question

The figure shows the graph of $z = f(x, y) = 3 - \sqrt{y}$ over the region $\mathcal{R} = [-1, 0] \times [0, 3]$. Which is the best overestimate for the value of $\iint_{\mathcal{R}} f(x, y)dA$?



A. 9

B. -9

C. 3

D. $3(3 - \sqrt{3})$

Iterated Integrals

Question

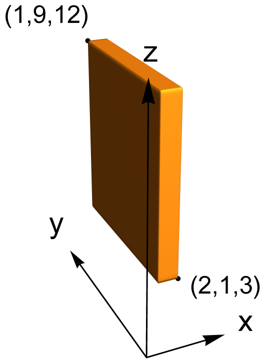
A rectangular solid S with sides parallel to the coordinate planes is shown in the figure. Which of the following iterated integrals definitely does NOT compute the volume of S?

A. $\int_1^9 \int_1^2 9\, dx dy$

B. $\int_3^{12} \int_1^9 1\, dy dz$

C. $\int_1^9 \int_1^2 9\, dy dx$

D. $\int_1^9 \int_3^{12} 1\, dz dy$



Iterated Integrals

Question

To compute $\iint_{\mathcal{R}} xe^{xy}\, dA$ over the rectangle $\mathcal{R}$ given by $2 \leq x \leq 4$, $-1 \leq y \leq 5$, which iterated integral would be easiest to compute?

A. $\int_{-1}^{5} \int_{2}^{4} xe^{xy}\, dxdy$

B. $\int_{2}^{4} \int_{-1}^{5} xe^{xy}\, dydx$

C. It doesn't matter.

D. There is not enough information to decide.

Iterated Integrals

Question

Which of these is equal to $\int_0^{1/2} \int_0^{\pi} y \cos(xy)\, dxdy$?

A. $\int_0^{1/2} \sin(\pi y)\, dy$

B. $\int_0^{1/2} y \sin(\pi y)\, dy$

C. $\int_0^{1/2} \sin(xy)\, dy$

D. $\int_0^{1/2} -y^2 \sin(xy)\, dy$

Question

Which of the following integral formulas is true?

A. $\displaystyle\int_a^b \int_c^d e^{x+y}\, dydx = \int_a^b e^x\, dx \int_c^d e^y\, dy$

B. $\displaystyle\int_a^b \int_c^d \cos(xy)\, dydx = \int_a^b \cos(x)\, dx \int_c^d \cos(y)\, dy$

C. $\displaystyle\int_a^b \int_c^d \sin(x+y)\, dydx = \int_a^b \sin(x)\, dx \int_c^d \sin(y)\, dy$

D. $\displaystyle\int_a^b \int_c^d \ln(x+y)\, dydx = \int_a^b \ln(x)\, dx \int_c^d \ln(y)\, dy$

Question

True or False: If $\mathcal{R} = [0,1] \times [0,1]$, then

$$\iint_{\mathcal{R}} e^{-x^2-y^2}\, dA = \left(\int_0^1 e^{-t^2}\, dt \right)^2.$$

A. True, and I am confident

B. True, but I am not confident.

C. False, but I am not confident.

D. False, and I am confident.

Double Integrals over General Regions

Question

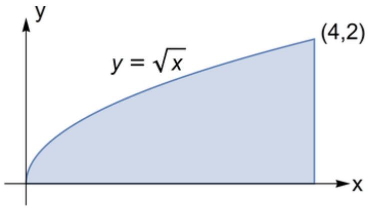
Which of these iterated integrals computes the area of the region shown?

A. $\int_0^4 \int_0^{\sqrt{x}} 1 \, dydx$

B. $\int_0^4 \int_0^2 \sqrt{x} \, dydx$

C. $\int_0^2 \int_{\sqrt{x}}^4 1 \, dxdy$

D. $\int_0^2 \int_0^{y^2} 1 \, dxdy$



Question

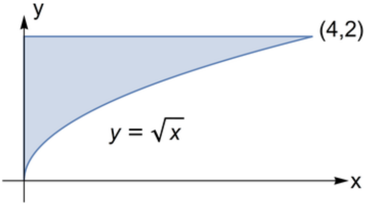
Which of these iterated integrals computes the area of the region shown?

A. $\int_0^4 \int_0^{\sqrt{x}} 1 \, dydx$

B. $\int_0^4 \int_0^2 (1 - \sqrt{x}) \, dydx$

C. $\int_0^2 \int_0^{\sqrt{x}} 1 \, dxdy$

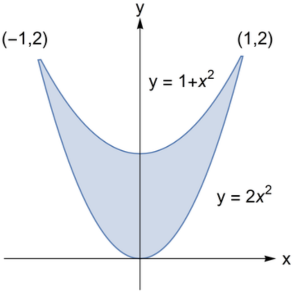
D. $\int_0^2 \int_0^{y^2} 1 \, dxdy$



Double Integrals over General Regions

Question

Which of the following formulas does NOT compute the area of the region shown?



A. $\displaystyle\int_{-1}^{1}\int_{2x^2}^{1+x^2} 1\,dydx$

B. $\displaystyle 2\int_{0}^{1}\int_{2x^2}^{1+x^2} 1\,dydx$

C. $\displaystyle\int_{0}^{1}\int_{-\sqrt{y}/2}^{\sqrt{y}/2} 1\,dxdy+\int_{1}^{2}\int_{-\sqrt{y}/2}^{-\sqrt{y-1}} dxdy+\int_{1}^{2}\int_{\sqrt{y-1}}^{\sqrt{y}/2} 1\,dxdy$

D. $\displaystyle 2\int_{0}^{2}\int_{\sqrt{y-1}}^{\sqrt{y}/2} 1\,dxdy$

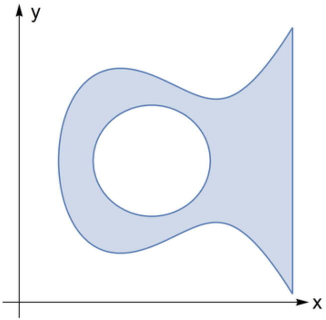
Double Integrals over General Regions

Question

To compute $\iint_{\mathcal{R}} 1 \, dA$ using iterated integrals of the form

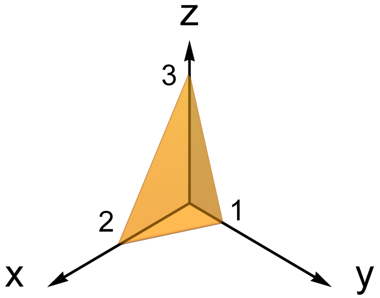
$$\int_a^b \int_{f(x)}^{g(x)} 1 \, dy dx,$$

how many sub-regions must the region $\mathcal{R}$ shown be broken into?



A. 2 B. 3 C. 4 D. 5 or more

Double Integrals over General Regions



Question

For the tetrahedron shown, bound by the coordinate planes and the plane $3x + 6y + 2z = 6$, which formula would we integrate with respect to z and then y to compute the volume?

A. $3 - \frac{3}{2}x - 3y$

B. $2 - 2y - \frac{2}{3}z$

C. $1 - \frac{1}{2}x - \frac{1}{3}z$

D. $3x + 6y + 2z$

Double Integrals over General Regions

Question

Determine which of the following integrals is equal to

$$\int_0^{\pi} \int_x^{\pi} \frac{\sin(y)}{y} \, dydx.$$

A. $\int_0^{\pi} \int_0^{x} \frac{\sin(y)}{y} \, dxdy$

B. $\int_0^{\pi} \int_0^{y} \frac{\sin(y)}{y} \, dxdy$

C. $\int_0^{\pi} \int_y^{\pi} \frac{\sin(y)}{y} \, dxdy$

D. $\int_x^{\pi} \int_0^{\pi} \frac{\sin(y)}{y} \, dxdy$

Question

Determine which of the following integrals is equal to

$$\int_0^{\ln(10)} \int_1^{e^x} ye^x \, dydx.$$

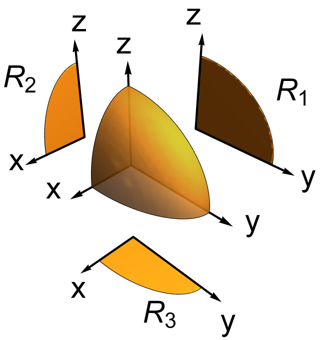
A. $\int_1^{10} \int_0^{\ln(y)} ye^x \, dxdy$

B. $\int_1^{10} \int_{\ln(y)}^{\ln(10)} ye^x \, dxdy$

C. $\left(\int_0^{\ln(y)} x \, dx\right) \left(\int_1^{e^x} y \, dy\right)$

D. $\left(\int_0^{\ln(10)} x \, dx\right) \left(\int_1^{10} y \, dy\right)$

Double Integrals over General Regions



Question

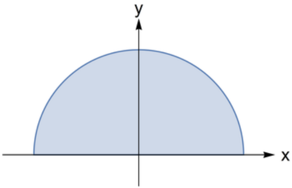
The integral $\iint_{\mathcal{R}_2} \sqrt{4 - 4x^2 - \frac{4}{3}z^2}\, dA$ computes the volume of the solid shown in the figure along with its shadows in the coordinate planes. Which of the following integrals also computes the volume?

A. $\displaystyle\iint_{\mathcal{R}_3} \sqrt{12 - 12x^2 - 3y^2}\, dA$

B. $\displaystyle\iint_{\mathcal{R}_3} \sqrt{4 - 4x^2 - \frac{4}{3}z^2}\, dA$

C. $\displaystyle\iint_{\mathcal{R}_1} \sqrt{1 - \frac{y^2}{4} - \frac{z^2}{3}}\, dA$

D. $\displaystyle\iint_{\mathcal{R}_1} \sqrt{12 - 3x^2 - 4z^2}\, dA$



Question

True or False? If $\mathcal{R}$ is the semi-circular region shown then the integral $\iint_{\mathcal{R}} (x-y)\,dA$ is negative.

A. True, and I am confident

B. True, but I am not confident.

C. False, but I am not confident.

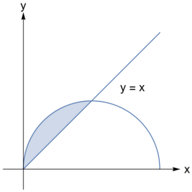
D. False, and I am confident.

Double Integrals in Polar Coordinates

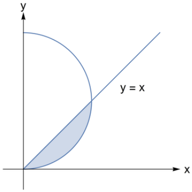
Question

The integral $\int_0^{\pi/4} \int_0^{2\sin\theta} r \, dr \, d\theta$ is a double integral over which of the regions below?

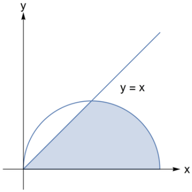
A.



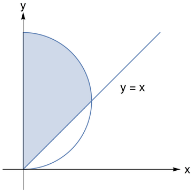
B.



C.



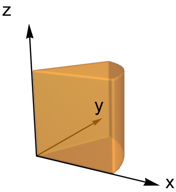
D.



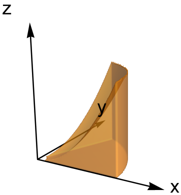
Question

The integral $\int_0^{\pi/4} \int_0^1 r^2 \, dr \, d\theta$ computes the volume of which of the solids shown?

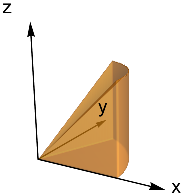
A.



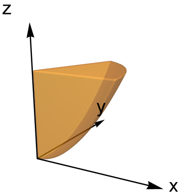
B.



C.



D.

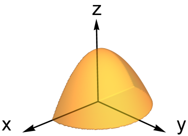


Double Integrals in Polar Coordinates

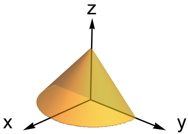
Question

The integral $\int_{-\pi/2}^{\pi/2} \int_0^1 (1-r)r\,dr\,d\theta$ computes the volume of which of the solids shown?

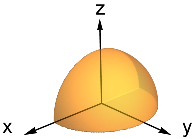
A.



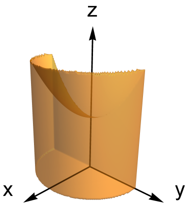
B.



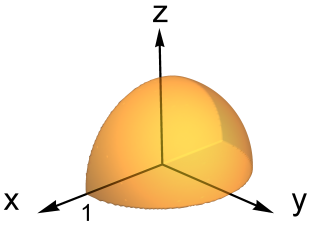
C.



D.



Double Integrals in Polar Coordinates



Question

Which of the iterated integrals in polar coordinates computes the volume of the quarter hemisphere shown in the figure?

A. $\int_0^{\pi}\int_0^1 (r - r^3)\, dr\, d\theta$

B. $\int_0^{\pi}\int_0^1 (r - r^2)\, dr\, d\theta$

C. $\int_0^{\pi}\int_0^1 (\sqrt{1 - r^2})\, r\, dr\, d\theta$

D. $\int_0^{\pi}\int_0^1 (r + r^3)\, dr\, d\theta$

Double Integrals in Polar Coordinates

Question

Which of the integrals below computes the volume of the solid below the hemisphere $z = \sqrt{4 - x^2 - y^2}$ and above the cone $z = \sqrt{x^2 + y^2}$?

A. $\displaystyle\int_0^{2\pi}\int_0^{1} (\sqrt{4 - r^2} - r) r \, dr \, d\theta$

B. $\displaystyle\int_0^{2\pi}\int_0^{\sqrt{2}} (\sqrt{4 - r^2} - r) \, r \, dr \, d\theta$

C. $\displaystyle\int_0^{2\pi}\int_0^{\sqrt{3}} (\sqrt{4 - r^2} - r) \, r \, dr \, d\theta$

D. $\displaystyle\int_0^{2\pi}\int_0^{2} (\sqrt{4 - r^2} - r) \, r \, dr \, d\theta$

Double Integrals in Polar Coordinates

Question

Consider the double integral formula in Cartesian coordinates

$$\int_0^1 \int_{\sqrt{1-x^2}}^{\sqrt{9-x^2}} \frac{1}{\sqrt{x^2+y^2}}\,dy\,dx + \int_1^3 \int_0^{\sqrt{9-x^2}} \frac{1}{\sqrt{x^2+y^2}}\,dy\,dx$$

Which of the following integral formulas in polar coordinates is this equivalent to?

A. $\int_0^{\pi/2} \int_0^1 dr\,d\theta + \int_0^{\pi} \int_0^3 dr\,d\theta$

B. $\int_0^{\pi/2} \int_1^3 dr\,d\theta$

C. $\int_0^{\pi} \int_0^1 r\,dr\,d\theta + \int_0^{\pi} \int_0^3 r\,dr\,d\theta$

D. $\int_0^{\pi/2} \int_1^3 r\,dr\,d\theta$

Double Integrals in Polar Coordinates

Question

Consider the double integral formula in Polar coordinates

$$\int_0^{\pi/3} \int_0^{1/\cos(\theta)} r^3 \, dr \, d\theta.$$

Which of the following integral formulas in Cartesian coordinates is this equivalent to?

A. $\displaystyle\int_0^1 \int_0^{x/\sqrt{3}} (x^2+y^2)^{3/2} \, dy \, dx$

B. $\displaystyle\int_0^1 \int_0^{\sqrt{3}\,x} (x^2+y^2)^{3/2} \, dy \, dx$

C. $\displaystyle\int_0^1 \int_0^{x/\sqrt{3}} (x^2+y^2) \, dy \, dx$

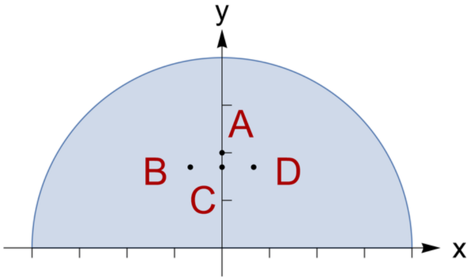
D. $\displaystyle\int_0^1 \int_0^{\sqrt{3}\,x} (x^2+y^2) \, dy \, dx$

Applications of Double Integrals

Question

The figure shows a steel semi-circular plate described by a region $\mathcal{R}$, along with four marked points. Which of the points could be $(\bar{x}, \bar{y})$ where

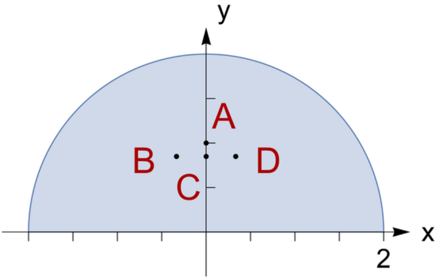
$$\bar{x} = \frac{1}{m} \iint_{\mathcal{R}} x \, dA, \; \bar{y} = \frac{1}{m} \iint_{\mathcal{R}} y \, dA, \; m = \iint_{\mathcal{R}} dA?$$



Applications of Double Integrals

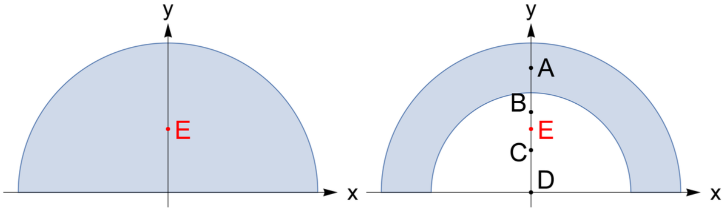
Question

The figure shows a semi-circular plate made with a material whose density varies as $\rho(x, y) = 3 - x$. Which of the four marked points could be the center of mass of the plate?



Question

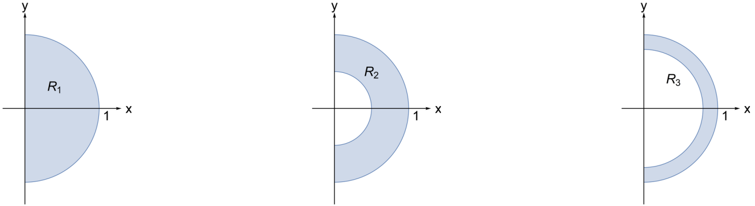
The left figure shows a steel half-disk with its center of mass E. The right figure shows the half-ring obtained by removing material from the half-disk, along with four additional points. Which point could be the location of the center of mass of the half-ring?



Applications of Double Integrals

Question

The centroid of each of the regions R_i shown has the form $(\bar{x}_i, 0)$, for $i = 1, 2, 3$. How do the numbers $\bar{x}_i$ compare?



A. $\bar{x}_1 < \bar{x}_2 < \bar{x}_3$

B. $\bar{x}_1 = \bar{x}_2 = \bar{x}_3$

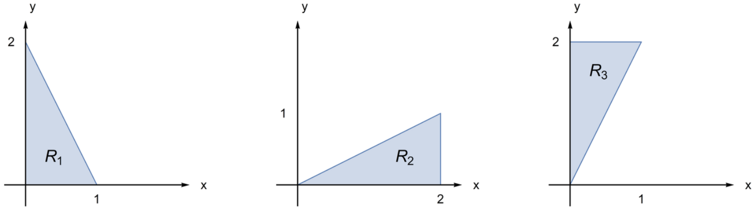
C. $\bar{x}_1 > \bar{x}_2 > \bar{x}_3$

D. $\bar{x}_2 < \bar{x}_1 < \bar{x}_3$

Applications of Double Integrals

Question

Rank the moments of inertia I_j about the x-axis for the three orientations R_j of a triangular steel plate shown.



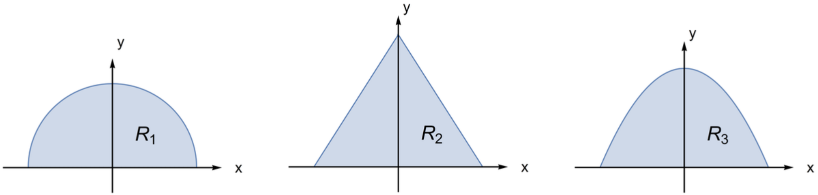
A. $I_1 < I_2 < I_3$ B. $I_2 < I_1 < I_3$

C. $I_2 < I_1 = I_3$ D. $I_3 < I_1 < I_2$

Applications of Double Integrals

Question

The regions R_j shown all have the same area. Rank the moments I_j of inertia about the x-axis.



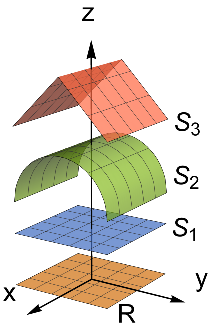
A. $I_2 < I_1 < I_3$

B. $I_2 > I_1 > I_3$

C. $I_1 = I_2 = I_3$

D. $I_1 < I_2 < I_3$

Surface Area



Question

The figure shows the graphs of 3 functions over the same domain R in the xy-plane. Rank the surface areas from smallest to largest.

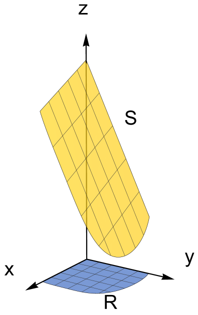
A. $\text{Area}(S_1) < \text{Area}(S_2) < \text{Area}(S_3)$

B. $\text{Area}(S_1) < \text{Area}(S_3) < \text{Area}(S_2)$

C. $\text{Area}(S_2) < \text{Area}(S_3) < \text{Area}(S_1)$

D. $\text{Area}(S_3) < \text{Area}(S_2) < \text{Area}(S_1)$

Surface Area



Question

The figure shows a surface S lying in the plane $x - 4y + 2z = 8$ with shadow R in the xy-plane. Which formula is correct?

A. $\text{Area}(S) = \frac{\sqrt{21}}{2}\text{Area}(R)$

B. $\text{Area}(S) = 3\sqrt{2}\text{Area}(R)$

C. $\text{Area}(S) = \sqrt{21}\text{Area}(R)$

D. $\text{Area}(S) = \text{Area}(R)$

Surface Area

Question

The vector $\vec{v}$ is tangent to the graph of $z = 4 - x^2 - y^2$ at the point $(x, y, 4 - x^2 - y^2)$ and has the form $\vec{v} = \Delta x\,\vec{i} + T\,\vec{k}$ for some Δx and formula T depending on x, y, and Δx. What is the formula for T?

A. 0

B. $-2x$

C. $-2x\Delta x$

D. $-4y\Delta y$

E. Δx

Surface Area

Question

Which of the following integrals computes the area of the surface $z = x^2 - y^2$ lying inside the cylinder $x^2 + y^2 = 9$?

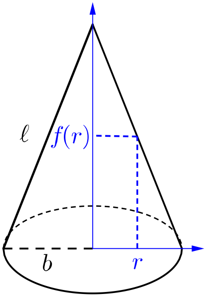
A. $\displaystyle\int_0^{2\pi}\int_0^3 r\sqrt{1+4r^2}\,dr\,d\theta$

B. $\displaystyle\int_0^{2\pi}\int_0^3 \sqrt{1+4r^2}\,dr\,d\theta$

C. $\displaystyle\int_0^{2\pi}\int_0^9 r\sqrt{1+4r^2}\,dr\,d\theta$

D. $\displaystyle\int_0^{2\pi}\int_0^9 \sqrt{1+4r^2}\,dr\,d\theta$

Surface Area



Question

If the graph of a function of (x, y) is a surface of revolution about the z-axis, then $z = f(r)$ and the surface area over the disk D can be computed by

$$\text{Area} = \iint_D \sqrt{1 + (f'(r))^2}\, r\, dr\, d\theta.$$

Which integral computes the surface area of the cone shown?

A. $\displaystyle\int_0^{2\pi}\int_0^b r\sqrt{1 + \frac{\ell^2 - b^2}{b^2}}\, dr\, d\theta$

B. $\displaystyle\int_0^{2\pi}\int_0^b \sqrt{1 + \frac{\ell^2 - b^2}{b^2}}\, dr\, d\theta$

C. $\displaystyle\int_0^{2\pi}\int_0^b r\sqrt{1 + \left(\frac{\ell^2 - b^2}{b^2}\right) r^2}\, dr\, d\theta$

D. $\displaystyle\int_0^{2\pi}\int_0^b r\sqrt{1 + \left(\frac{\ell^2 - b^2}{b^2}\right)(b - r)^2}\, dr\, d\theta$

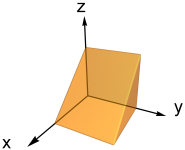
Triple Integrals

Question

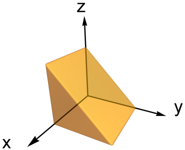
Identify the region of integration of the iterated triple integral

$$\int_0^1 \int_x^1 \int_0^1 f(x, y, z)\, dz\, dy\, dx.$$

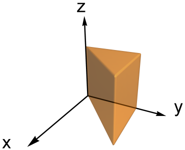
A.



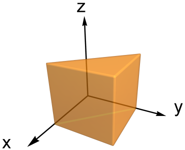
B.



C.



D.



Question

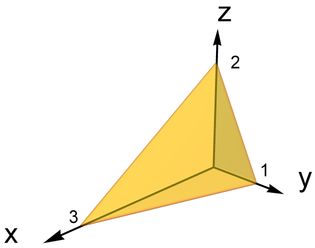
Consider the tetrahedron T in the 1st octant bound by the coordinate planes and the plane $2x + 6y + 3z = 6$. Which of the following triple integrals does NOT represent the volume of T?

A. $\displaystyle\int_0^1 \int_0^{3-3y} \int_0^{2-\frac{2}{3}x-2y} dz\, dx\, dy$

B. $\displaystyle\int_0^1 \int_0^{2-2y} \int_0^{3-3y-\frac{3}{2}z} dx\, dz\, dy$

C. $\displaystyle\int_0^2 \int_0^{3-\frac{3}{2}z} \int_0^{1-\frac{1}{3}x-\frac{1}{2}z} dy\, dx\, dz$

D. $\displaystyle\int_0^2 \int_0^{2-2y} \int_0^{3-3y-\frac{3}{2}z} dx\, dy\, dz$



Triple Integrals

Question

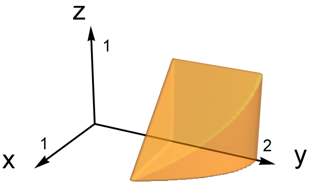
The figure shows a solid bound by the planes $x + z = 1$, $z = 0$, $x = 0$, and $y = 1$, along with the surface $y = 2 - x^2$. Which of the these iterated triple integrals computes its volume?

A. $\int_0^1 \int_0^{2-x^2} \int_0^{1-x} dz\, dy\, dx$

B. $\int_0^1 \int_1^{1-x} \int_0^{2-x^2} dz\, dy\, dx$

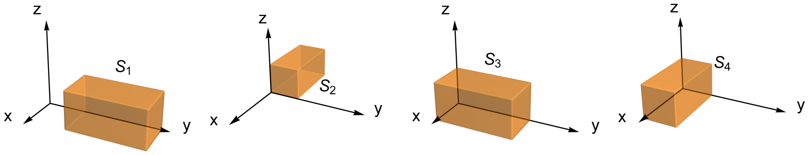
C. $\int_0^1 \int_1^{2-x^2} \int_0^{1-x} dz\, dy\, dx$

D. $\int_0^1 \int_0^{1-x} \int_0^{2-x^2} dz\, dy\, dx$



Question

The solids S_k $(k = 1, 2, 3, 4)$ shown are all congruent. Rank the triple integrals $I_k = \iiint_{S_k} x\, dV$ from smallest to largest.



A. $I_1 < I_2 < I_3 < I_4$

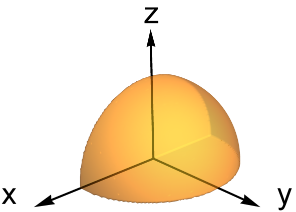
B. $I_2 < I_1 < I_4 < I_3$

C. $I_2 < I_1 = I_3 < I_4$

D. $I_1 = I_2 = I_3 = I_4$

E. $I_2 < I_1 < I_3 < I_4$

Triple Integrals



Question

True or False? If S is the quarter sphere shown, then the integral $\iiint_S (x-y)\,dA$ is negative.

A. True, and I am confident

B. True, but I am not confident.

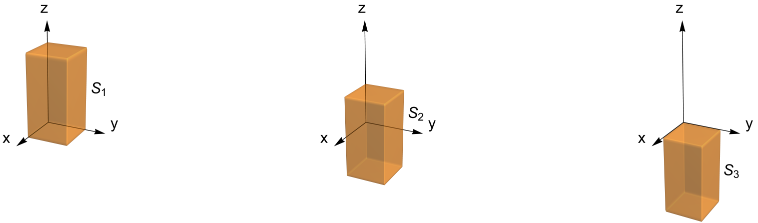
C. False, but I am not confident.

D. False, and I am confident.

Triple Integrals

Question

The solids S_k $(k = 1, 2, 3)$ shown are congruent. Rank the triple integrals $I_k = \iiint_{S_k} (y - x)\, dV$ from smallest to largest.



A. $I_1 < I_2 < I_3$ B. $I_3 < I_2 < I_1$

C. $I_1 = I_2 = I_3$ D. None of the these

Triple Integrals

Question

Which of the following is NOT a valid interpretation of this triple integral?

$$\int_0^1 \int_0^1 \int_0^1 (y^2 + z^2)\, dx\, dy\, dz$$

A. Volume of a solid between the surface $x = y^2 + z^2$ and the coordinate planes.

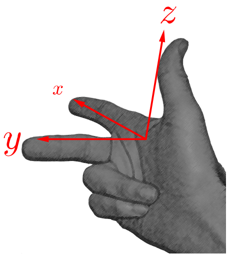
B. Average value of $f(x, y, z) = y^2 + z^2$ over a particular cube.

C. The mass of a particular cube with density $\rho(x, y, z) = y^2 + z^2$.

Cylindrical Coordinates

Question

Holding up your right hand as a Cartesian coordinate system, use your left hand to point to the following locations expressed in cylindrical coordinates.



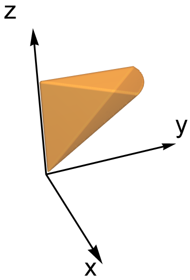
- $(r, \theta, z) = (1, 0, 0)$
- $(r, \theta, z) = (\sqrt{2}, \frac{\pi}{4}, 0)$
- $(r, \theta, z) = (1, \frac{\pi}{2}, 1)$
- $(r, \theta, z) = (1, \pi, 1)$
- $(r, \theta, z) = (\sqrt{2}, \frac{\pi}{4}, 1)$

Cylindrical Coordinates

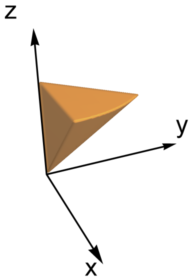
Question

The regions shown are bound by portions of planes and a cone. Which of them could be described by the inequalities $0 \leq r \leq z$, $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$, and $0 \leq z \leq 1$?

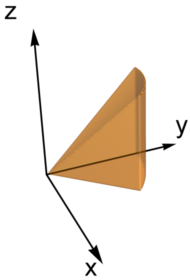
A.



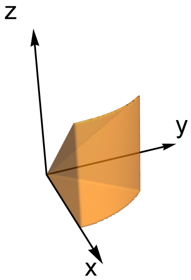
B.



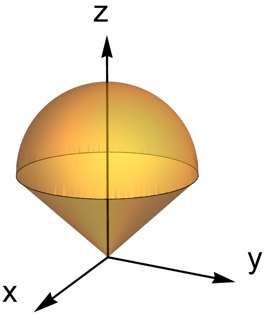
C.



D.



Cylindrical Coordinates



Question

The region depicted is above the standard cone and below the unit sphere centered at $(0,0,1)$. Select the correct description in cylindrical coordinates.

A. $\begin{cases} r \leq z \leq \sqrt{1-r^2}+1 \\ 0 \leq r \leq 1 \\ 0 \leq \theta \leq 2\pi \end{cases}$

B. $\begin{cases} r \leq z \leq \sqrt{1-r^2} \\ 0 \leq r \leq 1 \\ 0 \leq \theta \leq 2\pi \end{cases}$

C. $\begin{cases} r \leq z \leq \sqrt{4-z^2} \\ 0 \leq r \leq 1 \\ 0 \leq \theta \leq 2\pi \end{cases}$

Question

What does the following integral compute?

$$\int_0^{2\pi} \int_0^{\sqrt{3}} \int_0^{3-r^2} r\,dz\,dr\,d\theta$$

A. The volume under the paraboloid $z = 3 - x^2 - y^2$ above the xy-plane.

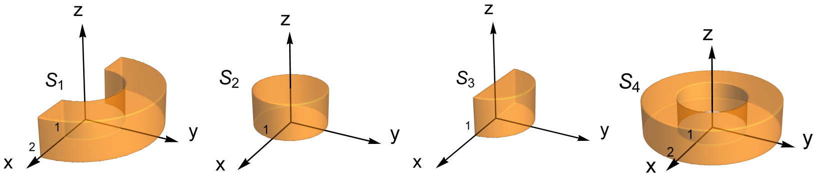
B. The volume enclosed by the upper half-ball $x^2 + y^2 + z^2 \leq 3$ and $z \geq 0$.

C. The mass of the solid described by $0 \leq z \leq 3 - r^2$, $0 \leq r \leq 3$, with density $f(r, \theta, z) = r$.

D. More than one of the above.

Question

For the solids S_k $(k = 1, 2, 3, 4)$ shown, rank the triple integrals $I_k = \iiint_{S_k} (1 - r)\, dV$ from smallest to largest.



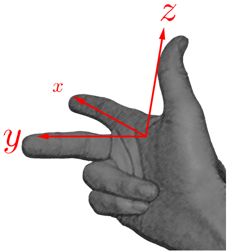
A. $I_4 < I_1 < I_3 < I_2$ B. $I_3 < I_2 < I_1 < I_4$

C. $I_3 < I_1 < I_2 < I_4$ D. $I_1 < I_3 < I_4 < I_2$

Spherical Coordinates

Question

Holding up your right hand as a Cartesian coordinate system, use your left hand to point to the following locations expressed in spherical coordinates.



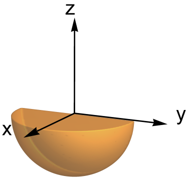
- $(\rho, \theta, \phi) = (1, 0, 0)$
- $(\rho, \theta, \phi) = (\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{2})$
- $(\rho, \theta, \phi) = (\sqrt{2}, \frac{\pi}{2}, \frac{\pi}{4})$
- $(\rho, \theta, \phi) = (\sqrt{2}, \pi, \frac{\pi}{4})$
- $(\rho, \theta, \phi) = (\sqrt{3}, \frac{\pi}{4}, \arcsin(\sqrt{\frac{2}{3}}))$

Spherical Coordinates

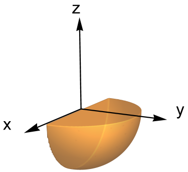
Question

Which of the following quarter spheres is described by the inequalities $0 \leq \rho \leq 1$, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, and $\frac{\pi}{2} \leq \phi \leq \pi$ in spherical coordinates?

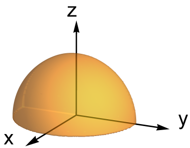
A.



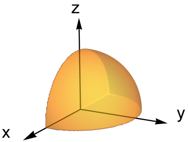
B.



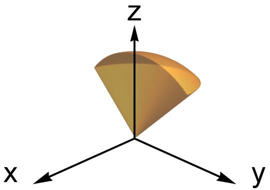
C.



D.



Spherical Coordinates



Question

The region depicted is below the unit sphere but above the standard cone and on one side of the plane $x = 0$. Select the correct description in spherical coordinates.

A. $\begin{cases} 0 \leq \rho \leq 1 \\ 0 \leq \phi \leq \pi/4 \\ \pi/2 \leq \theta \leq 3\pi/2 \end{cases}$

B. $\begin{cases} 0 \leq \rho \leq 1 \\ 0 \leq \phi \leq \pi/4 \\ \pi \leq \theta \leq 2\pi \end{cases}$

C. $\begin{cases} 0 \leq \rho \leq 1 \\ \pi/4 \leq \phi \leq \pi/2 \\ \pi/2 \leq \theta \leq 3\pi/2 \end{cases}$

D. $\begin{cases} 0 \leq \rho \leq 1 \\ \pi/4 \leq \phi \leq \pi/2 \\ \pi \leq \theta \leq 2\pi \end{cases}$

Spherical Coordinates

Question

What does the following integral compute?

$$\int_{\pi/2}^{\pi} \int_{0}^{2\pi} \int_{0}^{9} \rho^2 \sin(\phi)\, d\rho\, d\theta\, d\phi$$

A. The volume enclosed by a hemisphere of radius 9.

B. The volume enclosed by a hemisphere of radius 3.

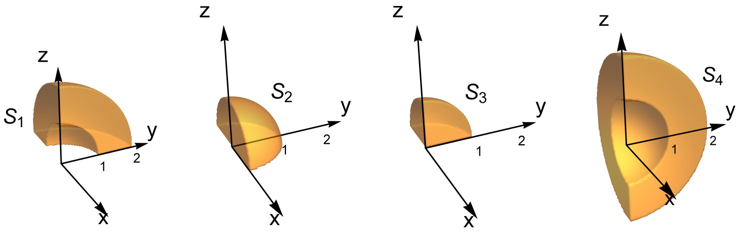
C. The average value of $\rho^2 \sin(\phi)$ over the lower hemisphere of radius 9 centered at the origin.

D. The average value of $\rho^2 \sin(\phi)$ over the upper hemisphere of radius 9 centered at the origin.

Spherical Coordinates

Question

For the solids S_k $(k = 1, 2, 3, 4)$ shown, rank the triple integrals $I_k = \iiint_{S_k} (1 - \rho)\, dV$ from smallest to largest.



A. $I_4 < I_1 < I_3 < I_2$

B. $I_3 < I_2 < I_1 < I_4$

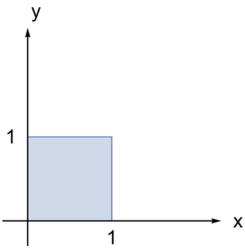
C. $I_3 < I_1 < I_2 < I_4$

D. $I_1 < I_3 < I_4 < I_2$

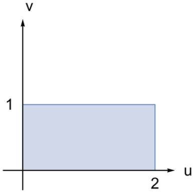
Change of Variables

Question

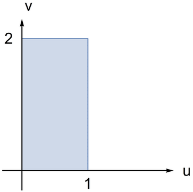
Which picture is the image of the region shown at right under the transformation $u = 2x,\ v = y$?



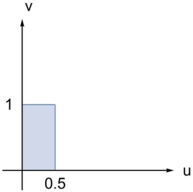
A.



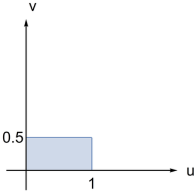
B.



C.



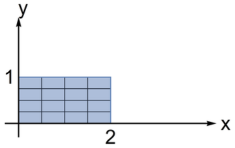
D.



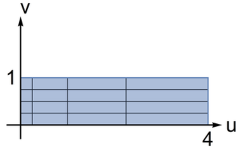
Change of Variables

Question

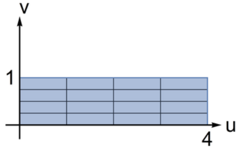
Which picture is the image of the grid region shown at right under the transformation $u = x^2$, $v = y$?



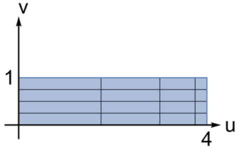
A.



B.



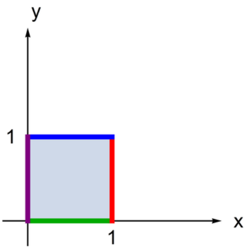
C.



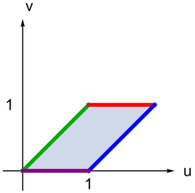
Change of Variables

Question

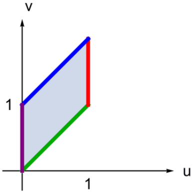
Which picture is the image of the region shown at right under the transformation $u = x + y$, $v = x$?



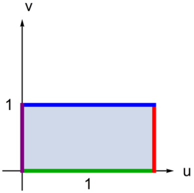
A.



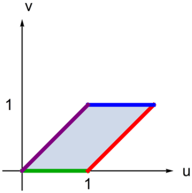
B.



C.

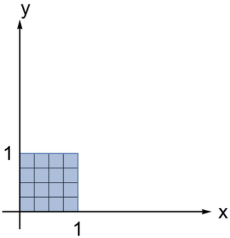


D.

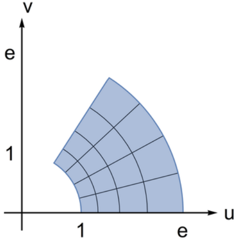


Question

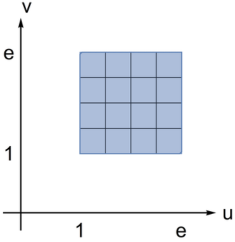
Which picture is the image of the region shown at right under the transformation $u = e^x \cos(y)$, $v = e^x \sin(y)$?



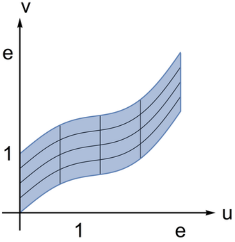
A.



B.



C.



Question

Compute the Jacobian $\left|\dfrac{\partial(x,y)}{\partial(r,\theta)}\right|$ of the transformation $x = r\cos\theta$, $y = r\sin\theta$.

A. r

B. r^2

C. $r(\cos^2\theta - \sin^2\theta)$

D. $r^2(\cos^2\theta - \sin^2\theta)$

Change of Variables

Question

True or False: The Jacobian $\left|\frac{\partial(x,y)}{\partial(u,v)}\right|$ of the transformation $u = 3x - y$, $v = 2x + 5y$ is

$$\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = 17.$$

A. True, and I am confident

B. True, but I am not confident.

C. False, but I am not confident.

D. False, and I am confident.

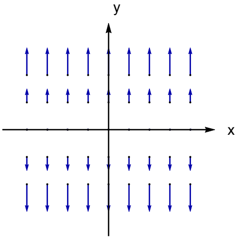
Vector Fields

Question

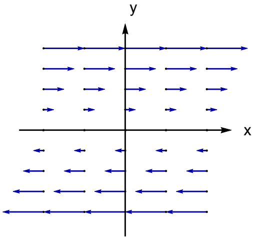
Which of these figures illustrates the vector field

$$\vec{F}(x, y) = y\vec{i}\ ?$$

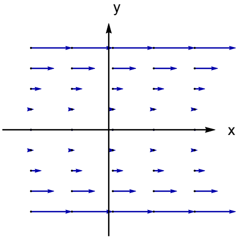
A.



B.



C.



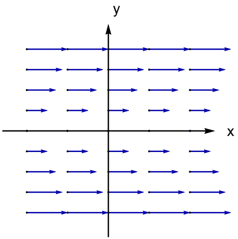
Vector Fields

Question

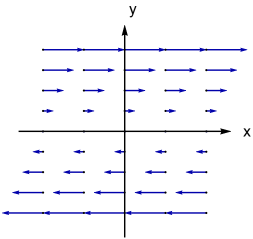
Which of these figures illustrates the vector field

$$\vec{F}(x,y) = y^2\vec{i}\ ?$$

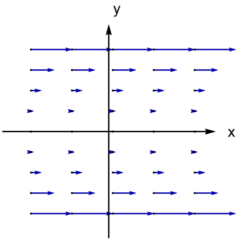
A.



B.



C.

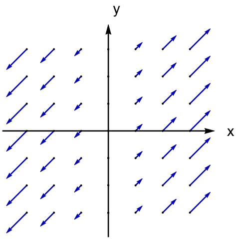


Question

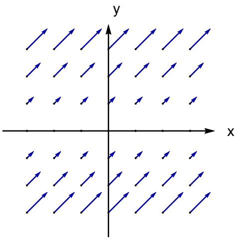
Which of these figures illustrates the vector field

$$\vec{F}(x,y) = y\vec{i} + y\vec{j}\ ?$$

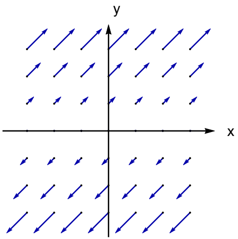
A.



B.



C.



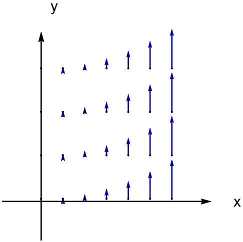
Vector Fields

Question

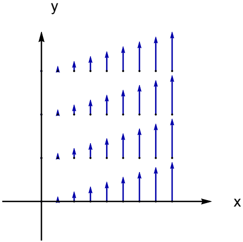
Which of these figures illustrates the vector field

$$\vec{F}(x, y) = x^2\vec{j}\ ?$$

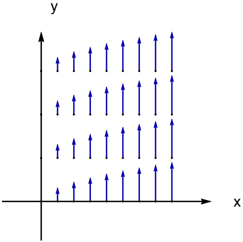
A.



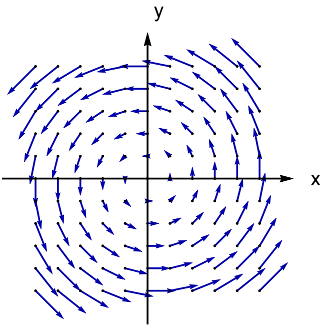
B.



C.



Vector Fields



Question

The figure illustrates a vector field in the plane. Which formula could give this picture?

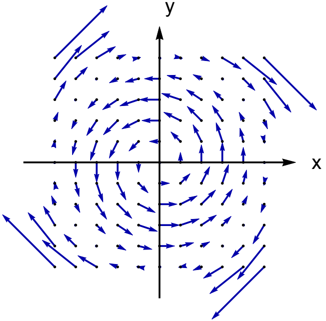
A. $\langle -x, y\rangle$

B. $\langle x, y\rangle$

C. $\langle y, x\rangle$

D. $\langle -y, x\rangle$

Vector Fields



Question

The figure illustrates a vector field in the plane. Which formula could give this picture?

A. $(x^2 + y^2)\langle -y, x\rangle$

B. $\dfrac{1}{x^2 + y^2 - 1}\langle -y, x\rangle$

C. $(x^2 + y^2 - 1)\langle -y, x\rangle$

D. $(1 - x^2 - y^2)\langle -y, x\rangle$

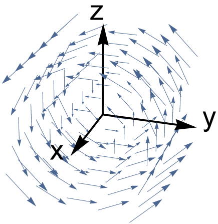
Vector Fields

Question

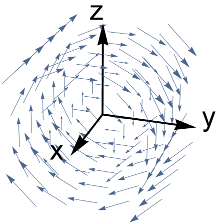
Which of these figures illustrates the vector field

$$\vec{F}(x, y, z) = -z\vec{j} + y\vec{k} \text{ ?}$$

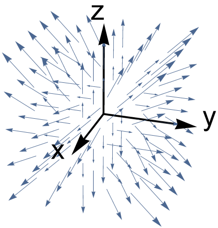
A.

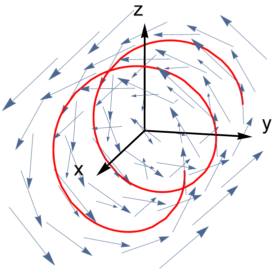


B.



C.





Question

The figure illustrates a vector field in space along with one of its trajectories. Which of these formulas could give this field?

A. $-z\vec{j} + y\vec{k}$

B. $\vec{i} - z\vec{j} + y\vec{k}$

C. $x\vec{i} - z\vec{j} + y\vec{k}$

D. $x(-z\vec{j} + y\vec{k})$

Vector Fields

Question

Which of the fields shown are gradient vector fields in the plane?

$$\vec{F} = x\vec{i} + y\vec{j} \quad \vec{G} = y\vec{i} \quad \vec{H} = \vec{i}$$

A. $\vec{F}$ only

B. $\vec{F}$ and $\vec{H}$ only

C. $\vec{G}$ only

D. All of these

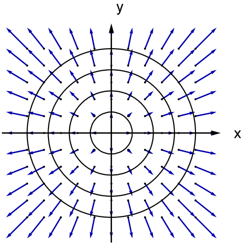
E. None of these

Vector Fields

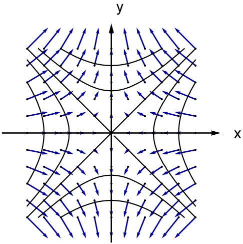
Question

Each figure illustrates a vector field together with a family of curves to which it is orthogonal. Which of these vector fields could NOT be the gradient of a function on the plane?

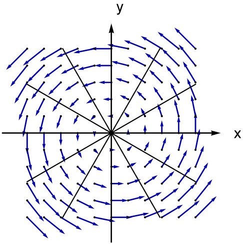
A.



B.



C.



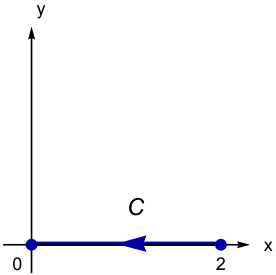
Vector Fields

Question

Consider the vector field $\vec{F}(x, y) = \langle 1 - \cos(y), 1 - \cos(y) \rangle$ in the plane. Which of these features does $\vec{F}$ have?

I. Every vector in the field has the same direction.
II. Every vector in the field points away from the origin.
III. The field is periodic in the y-direction.
IV. The field is zero along horizontal lines.

A. I and III only
B. II and IV only
C. I and IV only
D. All of them
E. None of these



Question

True or False? If C is the oriented curve in the plane shown and $f(x, y) = x$, then $\int_C f(x, y)ds$ is negative.

A. True, and I am confident

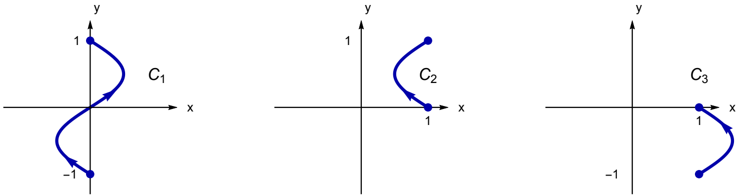
B. True, but I am not confident.

C. False, but I am not confident.

D. False, and I am confident.

Question

Rank the integrals $I_j = \int_{C_j} x\,ds$ over the three curves shown.



A. $I_3 < I_2 < I_1$

B. $I_3 = I_2 < I_1$

C. $I_3 < I_1 < I_2$

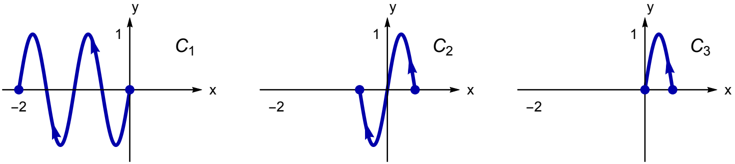
D. $I_1 < I_2 < I_3$

E. There is not enough information.

Line Integrals

Question

Rank the integrals $I_j = \int_{C_j} x\, ds$ over the three curves shown.



A. $I_3 < I_2 < I_1$

B. $I_1 < I_2 < I_3$

C. $I_1 = I_2 < I_3$

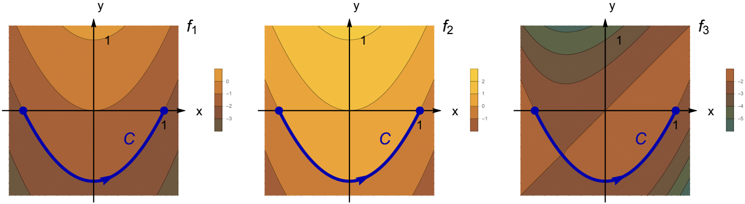
D. $I_2 < I_1 < I_3$

E. There is not enough information.

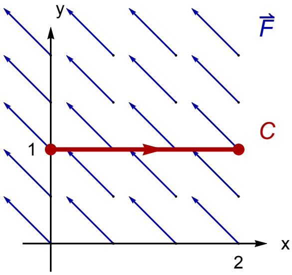
Line Integrals

Question

The figures show the same oriented curve C in the contour plots of functions f_1, f_2, and f_3. Lighter colors indicate higher values. Rank the integrals $I_j = \int_C f_j(x, y)\, ds$.



A. $I_1 = I_3 < I_2$ B. $I_1 = I_2 = I_3$
C. $I_2 < I_1 = I_3$ D. $I_1 < I_2 < I_3$
E. There is not enough information.



Question

The figure shows a constant vector field $\vec{F}$ with unit length and an oriented line segment C. Which of these numbers could be the value of $\int_C \vec{F} \cdot d\vec{r}$?

A. $\sqrt{2}$

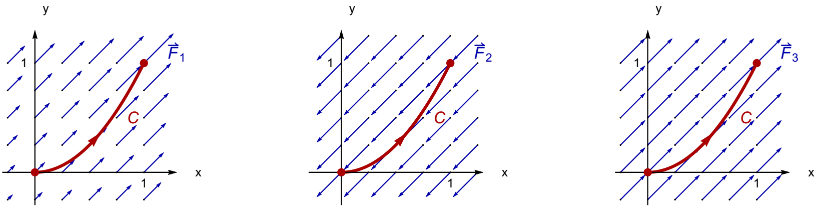
B. 2

C. -2

D. $-\sqrt{2}$

Question

The figures show the same oriented curve C in vector fields $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$. Rank the integrals $I_j = \int_C \vec{F}_j(x, y) \cdot d\vec{r}$.

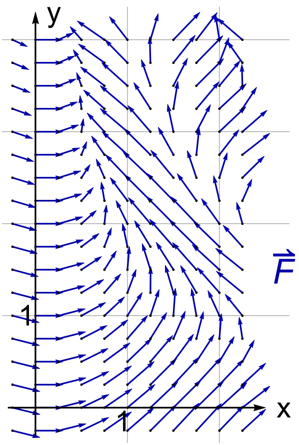


A. $I_1 < I_3 < I_2$ B. $I_2 = I_3 < I_1$

C. $I_2 < I_1 = I_3$ D. $I_2 < I_1 < I_3$

E. There is not enough information.

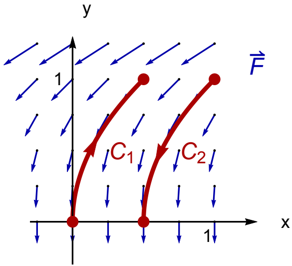
Line Integrals



Question

The figure shows a vector field $\vec{F}$ in the plane. If I_1 is the line integral of $\vec{F}$ over the curve $C_1(t, t^2)$, $1 \leq t \leq 2$, and I_2 is the line integral of $\vec{F}$ over the curve $C_2(t^2, t^4)$, $1 \leq t \leq \sqrt{2}$, rank the values of I_1 and I_2.

A. $I_1 = I_2$ B. $I_1 < I_2$ C. $I_1 > I_2$
D. There is not enough information.



Question

The vector field $\vec{F}$ and the oriented curves C_1 and C_2 are shown.

$$\text{True or False: } \int_{C_1} \vec{F} \cdot d\vec{r} < 0$$

$$\text{and } \int_{C_2} \vec{F} \cdot d\vec{r} > 0?$$

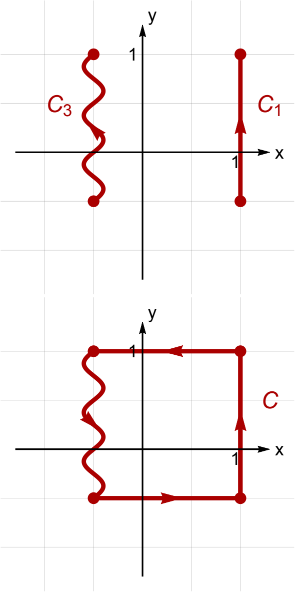
A. True, and I am confident

B. True, but I am not confident.

C. False, but I am not confident.

D. False, and I am confident.

Line Integrals



Question

If $\vec{F}$ is a vector field parallel to $\vec{j}$ and $\int_{C_3} \vec{F} \cdot d\vec{r} = \int_{C_1} \vec{F} \cdot d\vec{r} = 3$, then what is the value of $\int_C \vec{F} \cdot d\vec{r}$?

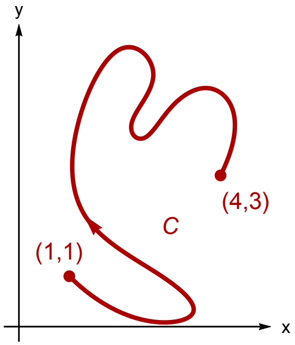
A. 6

B. 3

C. 0

D. There is not enough information.

The Fundamental Theorem

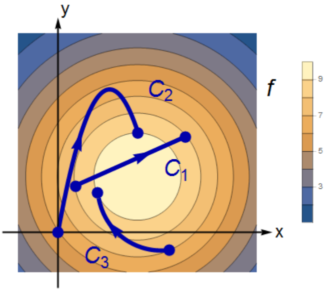


Question

What is the value of $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = 2y\vec{i} + 2x\vec{j}$ and C is the oriented curve in the plane shown?

A. $3\vec{i} + 2\vec{j}$ B. 22 C. 23
D. 24 E. 25
F. There is not enough information.

The Fundamental Theorem



Question

The contour map of a function $f(x, y)$ is shown, along with oriented curves C_1, C_2, and C_3. If $\vec{F} = \nabla f$, rank the integrals $I_j = \int_{C_j} \vec{F} \cdot d\vec{r}$.

A. $I_3 < I_1 < I_2$

B. $I_1 < I_3 < I_2$

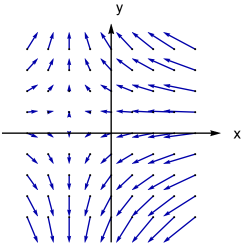
C. $I_1 < I_2 = I_3$

D. $I_2 = I_3 < I_1$

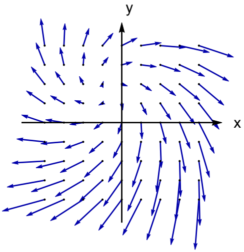
Question

Which of the vector fields shown is definitely NOT conservative?

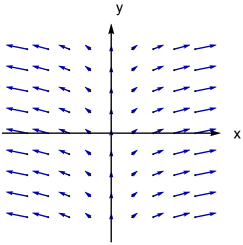
A.



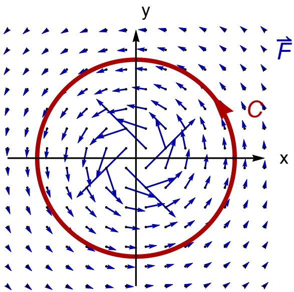
B.



C.



The Fundamental Theorem



Question

The vector field

$$\vec{F} = \frac{-y}{x^2+y^2}\vec{i} + \frac{x}{x^2+y^2}\vec{j} = P\vec{i} + Q\vec{j}$$

satisfies the condition $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ throughout its domain.

True or False: $\int_C \vec{F} \cdot d\vec{r} = 0$?

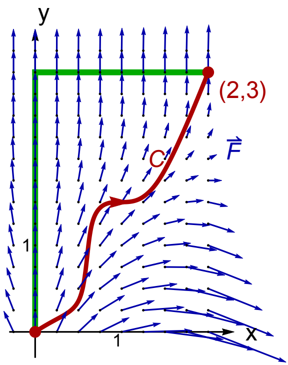
A. True, and I am confident

B. True, but I am not confident.

C. False, but I am not confident.

D. False, and I am confident.

The Fundamental Theorem



Question

If $\vec{F}$ is conservative but has unit length along the green lines, what is the value of $\int_C \vec{F} \cdot d\vec{r}$?

A. 0

B. 3

C. 5

D. 6

E. There is not enough information.

Question

Which of these is the general potential function of

$$\vec{F} = (2xy - 2)\vec{i} + (x^2 + 3y^2)\vec{j}$$

on the plane?

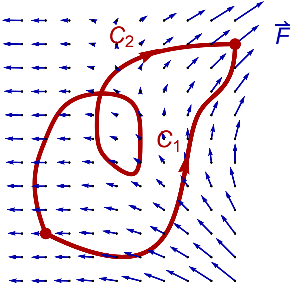
A. $f(x, y) = x^2 y - 2x + C$

B. $f(x, y) = x^2 y + y^3 + C$

C. $f(x, y) = x^2 y + y^3 - 2x + C$

D. None of these. $\vec{F}$ is not conservative.

The Fundamental Theorem



True or False?

If $\vec{F}$ is the conservative vector field shown, then

$$\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}$$

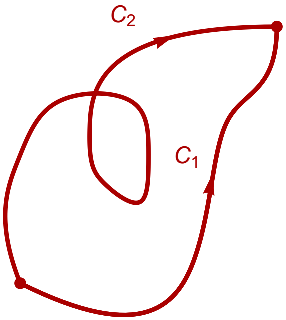
A. True, and I am confident

B. True, but I am not confident.

C. False, but I am not confident.

D. False, and I am confident.

The Fundamental Theorem



True or False?

If $\vec{F} = (e^{xy} + xye^{xy})\vec{i} + (x^2e^{xy})\vec{j}$, then

$$\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}$$

for the oriented curves shown.

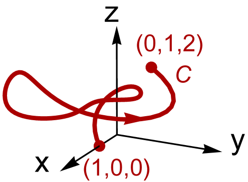
A. True, and I am confident

B. True, but I am not confident.

C. False, but I am not confident.

D. False, and I am confident.

The Fundamental Theorem



Question

Determine the sign of the line integral

$$\int_C 2xyz\,dx + x^2z\,dy + x^2y\,dz$$

over the space curve shown in the figure.

A. Positive

B. Zero

C. Negative

D. There is not enough information.

Green's Theorem

Question

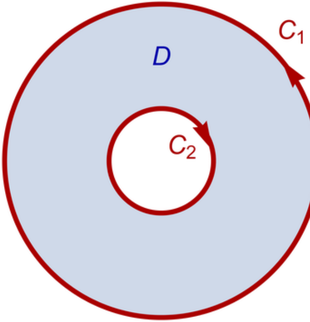
True or False? The curve C_1 has positive orientation with respect to the region D.

A. True, and I am confident

B. True, but I am not confident.

C. False, but I am not confident.

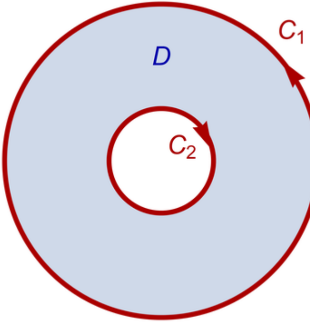
D. False, and I am confident.



Green's Theorem

Question

True or False? The curve C_2 has positive orientation with respect to the region D.



A. True, and I am confident

B. True, but I am not confident.

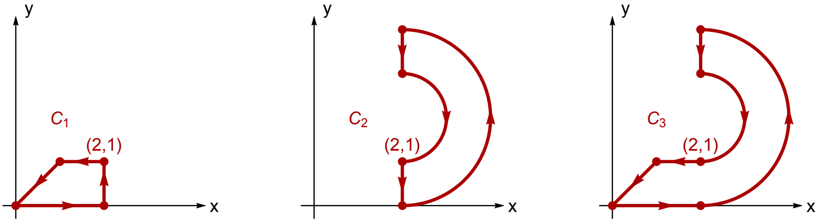
C. False, but I am not confident.

D. False, and I am confident.

Green's Theorem

Question

The figures show 3 curves in the plane. If L is the line segment from $(2,0)$ to $(2,1)$, which of the formulas computes $\int_{C_3} \vec{F} \cdot d\vec{r}$?



A. $\int_{C_1} \vec{F} \cdot d\vec{r} + \int_{C_2} \vec{F} \cdot d\vec{r}$

B. $\int_{C_1} \vec{F} \cdot d\vec{r} + \int_{C_2} \vec{F} \cdot d\vec{r} + \int_{L} \vec{F} \cdot d\vec{r}$

C. $\int_{C_1} \vec{F} \cdot d\vec{r} + \int_{C_2} \vec{F} \cdot d\vec{r} + 2\int_{L} \vec{F} \cdot d\vec{r}$

D. There is not enough enough information.

Question

Which of the following double integrals over the disk D inside the positively oriented circle C is equal to $\int_C \cos(xy)dx + \sin(x)dy$?

A. $\iint_D (x\sin(xy) + \cos(x))dA$

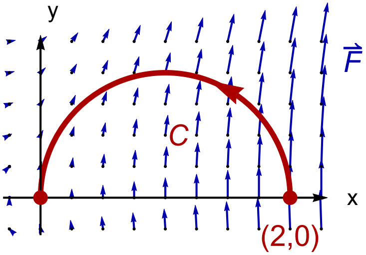
C. $\iint_D (-x\sin(xy) - \cos(x))dA$

B. $\iint_D (y\sin(xy))dA$

D. $\iint_D (-y\sin(xy))dA$

Question

The figure shows a semicircular curve C in a field $\vec{F} = P\vec{i} + Q\vec{j}$ with $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 2$ on $\mathbb{R}^2$. What is the value of $\int_C \vec{F} \cdot d\vec{r}$?



A. $-\pi$

B. 0

C. π

D. There is not enough information.

Green's Theorem

Question

True or False? If $\vec{F} = P\vec{i} + Q\vec{j}$ has scalar curl $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0$ in the first quadrant and C_1 and C_2 are the curves shown, then

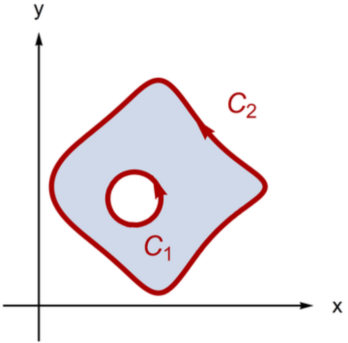
$$\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}.$$

A. True, and I am confident

B. True, but I am not confident.

C. False, but I am not confident.

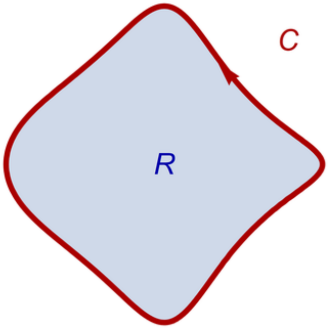
D. False, and I am confident.



Green's Theorem

Question

Which of the following is equal to $\frac{1}{2}\int_C xdy - ydx$?



A. $-\iint_R dA$

B. 0

C. $\iint_R dA$

D. There is not enough information.

Green's Theorem

Question

Which of these line integrals has the same value as the double integral $\iint_R dA$?

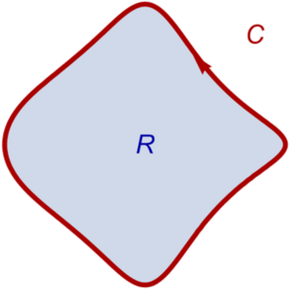
A. $\int_C x dy$

B. $\int_C dx + x dy$

C. $\int_C (x+y) dy$

D. All of the above.

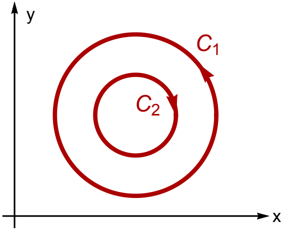
E. More than one but not all of the above.



Green's Theorem

Question

The circles C_1 and C_2 have radii 1 and 2, respectively, and the same center. If $\vec{F} = P\vec{i} + Q\vec{j}$ has $\frac{\partial Q}{\partial y} - \frac{\partial P}{\partial x} = 1$ throughout the first quadrant, then which of the following is equal to $\int_{C_1} \vec{F} \cdot d\vec{r}$?



A. $\displaystyle\int_{C_2} \vec{F} \cdot d\vec{r}$

B. $\displaystyle 3\pi + \int_{C_2} \vec{F} \cdot d\vec{r}$

C. $\displaystyle -\int_{C_2} \vec{F} \cdot d\vec{r}$

D. $\displaystyle 3\pi - \int_{C_2} \vec{F} \cdot d\vec{r}$

E. There is not enough information.

Green's Theorem

Question

The field $\vec{F} = \dfrac{-y\vec{i} + x\vec{j}}{x^2 + y^2}$ satisfies $\dfrac{\partial Q}{\partial x} = \dfrac{\partial P}{\partial y}$ throughout its domain.

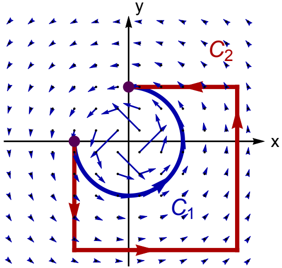
True or False: $\displaystyle\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}\,?$

A. True, and I am confident

B. True, but I am not confident.

C. False, but I am not confident.

D. False, and I am confident.



Green's Theorem

Question

The field $\vec{F} = \dfrac{-y\vec{i} + x\vec{j}}{x^2 + y^2}$ satisfies $\dfrac{\partial Q}{\partial x} = \dfrac{\partial P}{\partial y}$ throughout its domain.

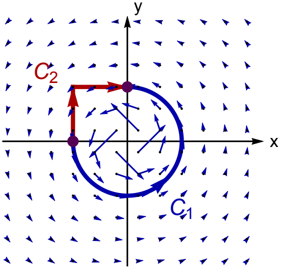
$$\text{True or False: } \int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}\,?$$

A. True, and I am confident

B. True, but I am not confident.

C. False, but I am not confident.

D. False, and I am confident.



Curl and Divergence

Question

For a scalar function f on $\mathbb{R}^3$ and a vector field $\vec{F}$ on $\mathbb{R}^3$, which of the formulas is meaningless?

A. $\text{div}(\vec{F}) + 2f$

B. $\text{curl}(\text{div}(\vec{F}))$

C. $\text{div}(\text{curl}(\vec{F})) + \vec{F} \cdot \vec{F}$

D. $\nabla f + \text{curl}(\vec{F})$

Curl and Divergence

Question

For a scalar function f on $\mathbb{R}^3$ and a vector field $\vec{F}$ on $\mathbb{R}^3$, which of these quantities is vector-valued?

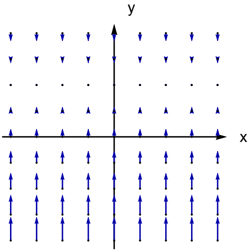
A. $\text{curl}(\nabla f) + f$
B. $f\text{div}(\vec{F}) + \nabla f \cdot \vec{F}$
C. $\text{div}(f) + \vec{F}$.
D. $\nabla(||\vec{F}||^2) + \nabla f$

Curl and Divergence

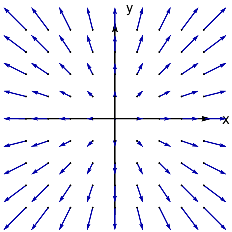
Question

For which of the vector fields $\vec{F}$ shown is $\mathrm{curl}(\vec{F})$ a non-zero function?

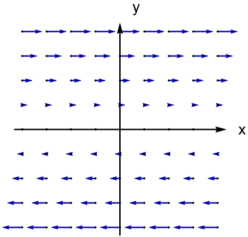
A.



B.



C.



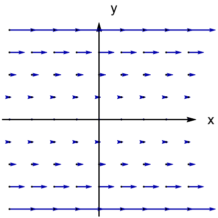
D. All of the above.

Curl and Divergence

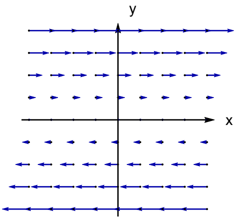
Question

For each vector field $\vec{F}$ shown, the curl is NOT the zero function. Which field has curl always pointing in the $\vec{k}$-direction?

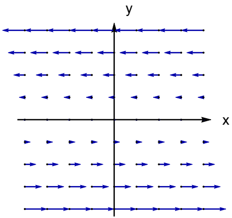
A.



B.



C.

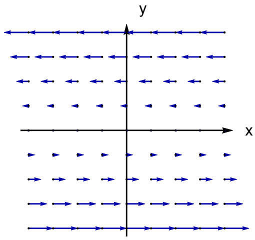


D. All of the above.

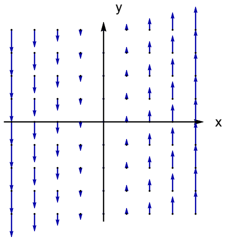
Question

For each vector field $\vec{F}$ shown, the curl is NOT the zero function. For which field is the curl always in the $\vec{k}$-direction?

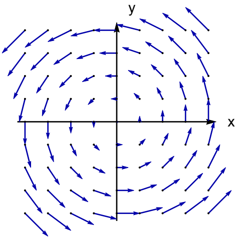
A.



B.



C.

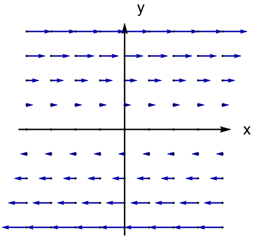


D. All of the above.

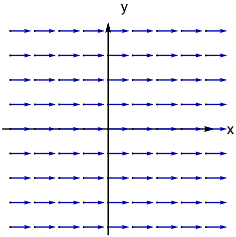
Question

For which of the following fields is the divergence a nonzero function?

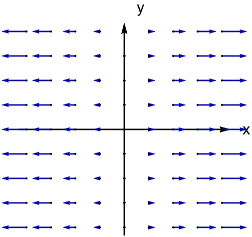
A.



B.



C.

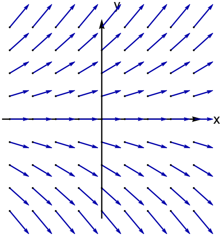


D. All of the above.

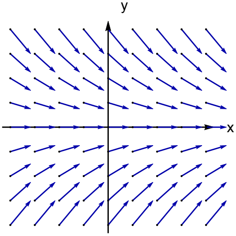
Question

For each of the vector fields $\vec{F}$ shown, the divergence is a nonzero function. For which is the divergence always positive?

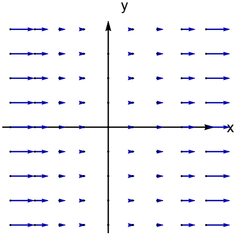
A.



B.



C.



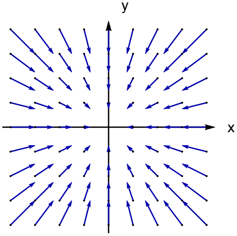
D. All of the above.

Curl and Divergence

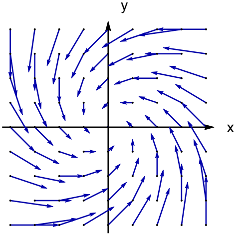
Question

For each of the vectors fields $\vec{F}$ shown, the divergence is a nonzero function. For which is the divergence always positive?

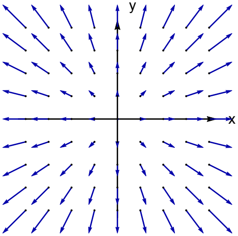
A.



B.



C.



D. All of the above.

Curl and Divergence

Question

True or False? If $\vec{F} = e^{\cos x}\vec{i} + \frac{1}{1+y^2}\vec{j} + z^3 e^{-z}\vec{k}$, then $\vec{F}$ is irrotational.

A. True, and I am confident

B. True, but I am not confident.

C. False, but I am not confident.

D. False, and I am confident.

Curl and Divergence

Question

True or False? If $\vec{F} = e^{yz}\vec{i} + \cos x \sin(z)\vec{j} + \frac{1}{1+x^2+y^2}\vec{k}$, then $\vec{F}$ is incompressible.

A. True, and I am confident

B. True, but I am not confident.

C. False, but I am not confident.

D. False, and I am confident.

Curl and Divergence

Question

If $\vec{G} = \text{curl}(\vec{H})$ and $\vec{F}$ is incompressible, which of these formulas computes $\text{div}(\vec{F} + \vec{G})$?

I. $\text{div}(\vec{F}) + \text{div}(\vec{G})$

II. $\text{div}(\vec{F})$

III. $\text{div}(\vec{G})$

IV. 0

A. I only.

B. I and II only.

C. I and III only.

D. All of I, II, III, and IV.

Curl and Divergence

Question

If f, g, and h are differentiable functions, which of these vector fields must have constant curl?

A. $(g(y)+2z)\vec{i}+(f(x)+y)\vec{j}+(h(z)-x)\vec{k}$

B. $(h(z)-x)\vec{i}+(g(y)+2z)\vec{j}+(f(x)+y)\vec{k}$

C. $(f(x)+y)\vec{i}+(h(z)-x)\vec{j}+(g(y)+2z)\vec{k}$

D. $(f(x)+y)\vec{i}+(g(y)+2z)\vec{j}+(h(z)-x)\vec{k}$

Parametric Surfaces

Question

If $x = u\cos(v), y = u\sin(v), z = u$ are the parametric equations of a surface S, which of the following best describes the surface?

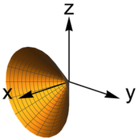
A. Plane
B. Cylinder
C. Cone
D. Paraboloid

Parametric Surfaces

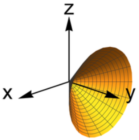
Question

If $x = u, y = u\cos(v), z = u\sin(v)$ with $0 \leq u \leq 1$ and $0 \leq v \leq 2\pi$, then which figure depicts this parametric surface?

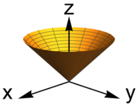
A.



B.



C.

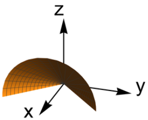


Parametric Surfaces

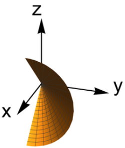
Question

If $x = u, y = u\cos(v), z = u\sin(v)$ with $0 \leq u \leq 1$ and $-\frac{\pi}{2} \leq v \leq \frac{\pi}{2}$, which figure depicts this parametric surface?

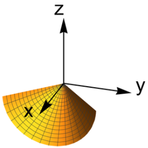
A.



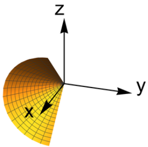
B.



C.



D.

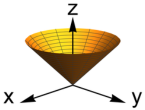


Parametric Surfaces

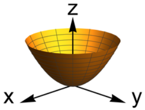
Question

If $x = u\cos(v), y = u\sin(v), z = u^2$ with $0 \leq u \leq 1$ and $0 \leq v \leq 2\pi$, which figure depicts the parametric surface?

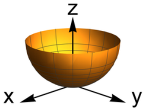
A.



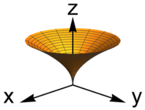
B.



C.



D.



Parametric Surfaces

Question

Which of these is *not* a parametrization of a sphere?

A. $\begin{cases} x = \cos(\theta)\sin(\phi) \\ y = \sin(\theta)\sin(\phi) \\ z = \cos(\phi) \end{cases}$

B. $\begin{cases} x = \cos(\theta)\cos(\phi) \\ y = \sin(\theta)\cos(\phi) \\ z = \sin(\phi) \end{cases}$

C. $\begin{cases} x = \cos(\phi)\sin(\theta) \\ y = \sin(\phi)\sin(\theta) \\ z = \cos(\theta) \end{cases}$

D. $\begin{cases} x = \cos(\theta)\cos(\phi) \\ y = \sin(\theta)\cos(\phi) \\ z = \cos(\phi) \end{cases}$

Question

Which of these vector-valued functions of u and v does *not* have range which is contained in a plane?

A. $\vec{r}(u,v) = \langle 3,1,2\rangle + u^2\langle 1,-1,0\rangle + v\langle 1,0,0\rangle$

B. $\vec{r}(u,v) = \langle 3,1,2\rangle + u\langle 1,-1,0\rangle + v\langle 1,0,0\rangle$

C. $\vec{r}(u,v) = (3+u+v)\vec{i} + (1-u)\vec{j} + 2\vec{k}$

D. $\vec{r}(u,v) = u\vec{i} + v\vec{j} + uv\vec{k}$

Parametric Surfaces

Question

If $\vec{r}(u, v) = u^2\vec{i} + (u + 2)v\vec{j} + v^2\vec{k}$ is a parametrization of a surface S, which values of the parameters correspond to the point $(4, 0, 1)$ on S?

A. $u = 2, v = 1$

B. $u = 2, v = -1$

C. $u = -2, v = 1$

D. $u = -2, v = -1$

E. More than one of the above.

Parametric Surfaces

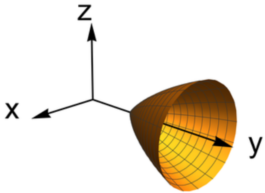
Question

The figure shows a surface obtained by revolving the curve $y = e^x$ for $0 \leq x \leq 1$ in the xy-plane about the y-axis. Which of the following could *not* be a parametrization of this surface?

A. $x\cos(\theta)\vec{i} + e^x\vec{j} + x\sin(\theta)\vec{k}$

B. $\ln(y)\cos(\theta)\vec{i} + y\vec{j} + \ln(y)\sin(\theta)\vec{k}$

C. $x\cos(\theta)\vec{i} + e^x\vec{j} + e^x\sin(\theta)\vec{k}$



Parametric Surfaces

Question

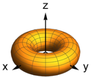
Which of these pictures is NOT the range of one of these three parametrizations?

I.
$$\begin{cases} x = (2+\cos\phi)\cos\theta \\ y = (2+\cos\phi)\sin\theta \\ z = \sin\phi \end{cases}$$

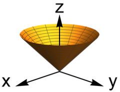
II.
$$\begin{cases} x = u \\ y = \sin u \cos\theta \\ z = \sin u \sin\theta \end{cases}$$

III.
$$\begin{cases} x = u\cos v \\ y = u\sin v \\ z = v \end{cases}$$

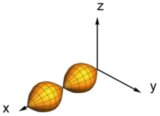
A.



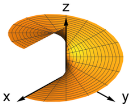
B.



C.



D.



Parametric Surfaces

Question

If $x = r\cos\theta, y = r\sin\theta, z = -\sqrt{r^2+1}$ are the parametric equations for a piece of a surface S, then what could the surface S be?

A. The hyperboloid of one sheet $x^2 + y^2 - z^2 = 1$.

B. The hyperboloid of two sheets $x^2 + y^2 - z^2 = -1$.

C. The sphere $x^2 + y^2 + z^2 = 1$.

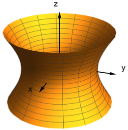
D. The paraboloid $x^2 + y^2 - z = 0$.

Parametric Surfaces

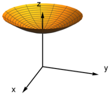
Question

If $x = r\cos\theta, y = r\sin\theta, z = -\sqrt{r^2+1}$ with $0 \le r \le 1$ and $0 \le \theta \le 2\pi$, then which figure shows the parametric surface?

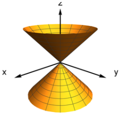
A.



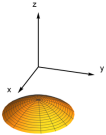
B.



C.

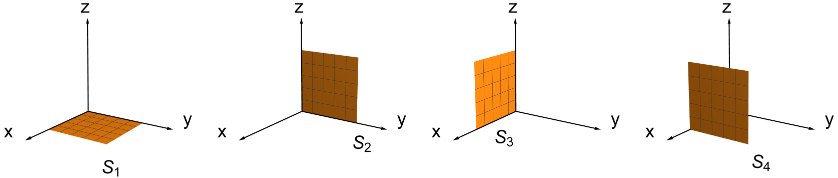


D.



Question

The figures show four congruent squares S_i in space. Rank the surface integrals $I_j = \iint_{S_i} x\,dS$.



A. $I_1 < I_2 = I_3 < I_4$

B. $I_2 < I_1 < I_3 < I_4$

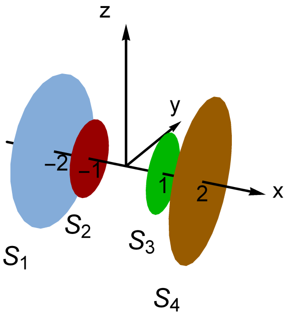
C. $I_2 < I_1 = I_3 < I_4$

D. $I_3 < I_1 = I_2 = I_4$

Surface Integrals

Question

The figures show four parallel disks S_i, two of radius 1 and two of radius 2. Rank the integrals $I_j = \iint_{S_j} x dS$.



A. $I_1 < I_2 < I_3 < I_4$

B. $I_4 < I_3 < I_2 < I_1$

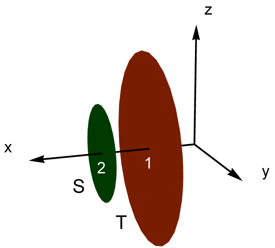
C. $I_2 = I_3 < I_1 = I_4$

D. $I_1 = I_4 < I_2 = I_3$

Question

The disks S and T are parallel to the yz-plane and the radius of T is double the radius of S.

True or False: $\iint_S x^3 dS < \iint_T x^3 dS$?



A. True, and I am confident

B. True, but I am not confident.

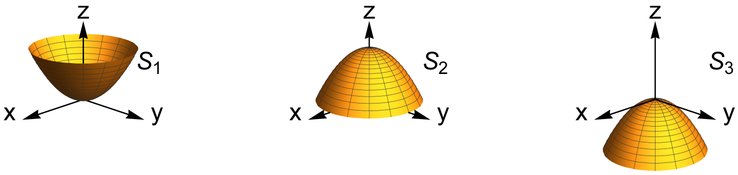
C. False, but I am not confident.

D. False, and I am confident.

Surface Integrals

Question

The figures show 3 congruent paraboloids S_i in space. If $f(x, y, z)$ computes the distance from (x, y, z) to the z-axis, rank the surface integrals $I_j = \iint_{S_i} f(x, y, z)dS$.



A. $I_1 = I_2 = I_3$

B. $I_3 < I_2 < I_1$

C. $I_3 < I_1 < I_2$

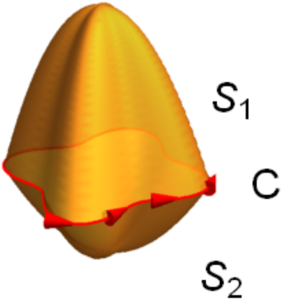
D. $I_3 < I_1 = I_2$

Surface Integrals

Question

The figure shows a closed surface S with the outward orientation consisting of two surfaces S_1 and S_2 meeting along a curve C.

True or False? The curve C has the positive orientation for S_2.



A. True, and I am confident

B. True, but I am not confident.

C. False, but I am not confident.

D. False, and I am confident.

Question

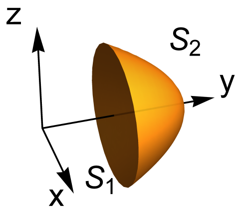
The figure shows a surface S consisting of a disk S_1 of radius 1 in the plane $y = 1$ and a paraboloid S_2. If $\iint_S y\, dS = 10$, what is the value of $\iint_{S_2} y\, dS$?

A. $10 - \pi$

B. $10 + \pi$

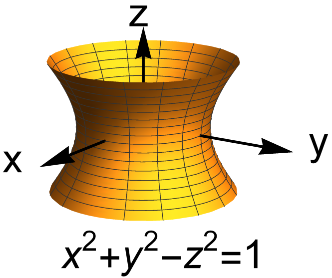
C. 10

D. There is not enough information.



Question

True or False? If $\vec{F} = \langle 2x, 2y, -2z \rangle$ and $\vec{n} = \dfrac{\vec{F}}{||\vec{F}||}$, then $\vec{n}$ is the outward normal to the surface shown.



A. True, and I am confident

B. True, but I am not confident.

C. False, but I am not confident.

D. False, and I am confident.

Question

The figure shows a surface S bounded by a circle of radius 1 in the plane $z = 1$. If $\vec{F} = -y\vec{i} + x\vec{j}$ on $\mathbb{R}^3$ and S is oriented away from the origin, compute

$$\iint_S \text{curl}(\vec{F}) \cdot d\vec{S}.$$

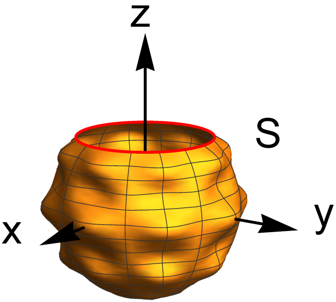
A. -2π

B. 0

C. 2π

D. 1

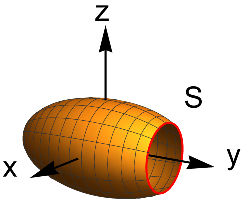
E. There is not enough information.



Stokes' Theorem

Question

True or False? Suppose $\vec{F}$ is a conservative vector field on $\mathbb{R}^3$. Then $\iint_S \operatorname{curl}(\vec{F}) \cdot d\vec{S} = 0$ for the surface S with the outward orientation.



A. True, and I am confident

B. True, but I am not confident.

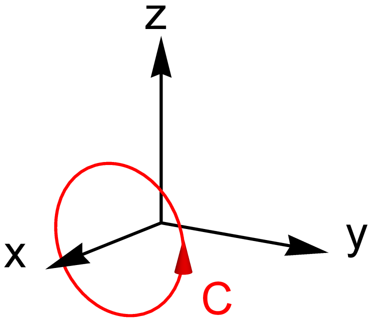
C. False, but I am not confident.

D. False, and I am confident.

Question

If $\text{curl}(\vec{F}) = 2\vec{i} + (2x+1)\vec{j} - e^x\vec{k}$, compute $\int_C \vec{F} \cdot d\vec{r}$, where C is the circle of radius 1 in the plane $x = 1$ oriented as shown.

A. -2π
B. 0
C. 2π
D. There is not enough information.



Stokes' Theorem

Question

The vector field $\vec{F} = \dfrac{-y\vec{i} + x\vec{j}}{(x^2+y^2)}$ has $\text{curl}(\vec{F}) = 0$ everywhere that $\vec{F}$ is defined and C is the oriented circle in the plane $z = 0$ shown.

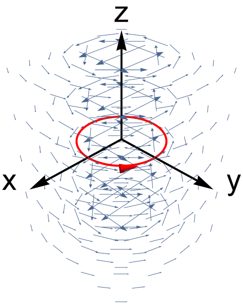
$$\text{True or False: } \int_C \vec{F} \cdot d\vec{r} = 0 \text{ ?}$$

A. True, and I am confident

B. True, but I am not confident.

C. False, but I am not confident.

D. False, and I am confident.



Stokes' Theorem

Question

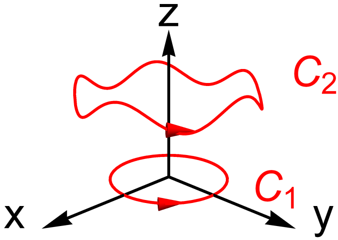
The vector field $\vec{F} = \dfrac{-y\vec{i} + x\vec{j}}{(x^2 + y^2)}$ has $\text{curl}(\vec{F}) = 0$ everywhere that $\vec{F}$ is defined. How do $\int_{C_1} \vec{F} \cdot d\vec{r}$ and $\int_{C_2} \vec{F} \cdot d\vec{r}$ compare?

A. $\int_{C_1} \vec{F} \cdot d\vec{r} < \int_{C_2} \vec{F} \cdot d\vec{r}$

B. $\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}$

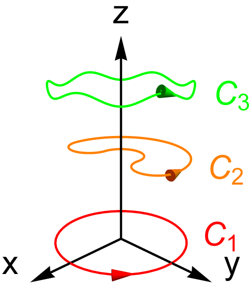
C. $\int_{C_1} \vec{F} \cdot d\vec{r} > \int_{C_2} \vec{F} \cdot d\vec{r}$

D. There is not enough information.



Question

The vector field $\vec{F} = \dfrac{-y\vec{i} + x\vec{j}}{(x^2+y^2)}$ has $\mathrm{curl}(\vec{F}) = 0$ everywhere that $\vec{F}$ is defined. Rank the integrals $I_j = \int_{C_j} \vec{F} \cdot d\vec{r}$, where the curves C_j are shown.

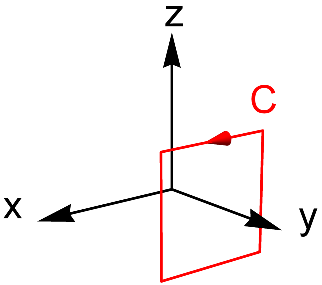


A. $I_3 < I_2 < I_1$
B. $I_2 < I_1 = I_3$
C. $I_1 = I_2 = I_3$
D. $I_1 < I_2 < I_3$

Stokes' Theorem

Question

Suppose $\vec{F}$ is a vector field with $\text{curl}(\vec{F}) = \langle 1, 3, 2 \rangle$ on $\mathbb{R}^3$. Compute $\int_C (\vec{F} + \nabla(y^2)) \cdot d\vec{r}$ where C is the square with side length 2 centered on the y-axis in the plane $y = 1$, oriented as shown.



A. 2π

B. 4

C. 12

D. 3

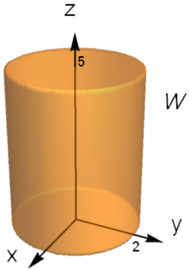
E. There is not enough information.

Question

If $\vec{F} = \dfrac{x\vec{i} + y\vec{j}}{1 + x^2 + y^2}$ on $\mathbb{R}^3$ and W is the solid cylinder shown, compute

$$\iiint_W \operatorname{div}(\vec{F})dV.$$

A. 0
B. 8π
C. 12π
D. 16π
E. There is not enough information.



The Divergence Theorem

Question

If $\vec{G} = \text{curl}(\vec{F})$ throughout the region W shown, then

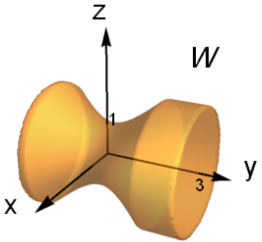
$$\text{True or False: } \iiint_W \text{div}(\vec{G})dV = 0 \text{ ?}$$

A. True, and I am confident

B. True, but I am not confident.

C. False, but I am not confident.

D. False, and I am confident.



Question

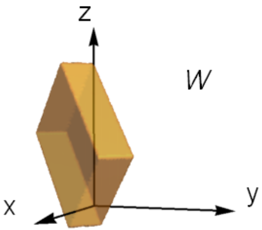
If $\vec{F} = x\vec{i} + y\vec{j} - z\vec{k}$, then compute $\iint_S \vec{F} \cdot \vec{n}\, dS$ over the surface of the rectangular box with side lengths 1, 2, and 3 shown with the inward orientation.

A. -6

B. 0

C. 6

D. There is not enough information.



The Divergence Theorem

Question

The vector field

$$\vec{F} = \frac{x\vec{i} + y\vec{j} + z\vec{k}}{(x^2 + y^2 + z^2)^{3/2}}$$

has $\mathrm{div}(\vec{F}) = 0$ everywhere it is defined. If S is the unit sphere with the outward orientation, then

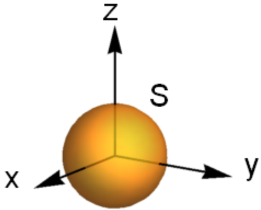
$$\text{True or False: } \iint_S \vec{F} \cdot \vec{n}\, dS = 0?$$

A. True, and I am confident

B. True, but I am not confident.

C. False, but I am not confident.

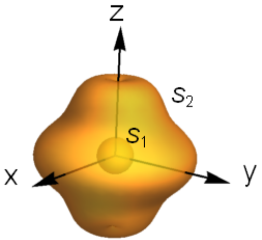
D. False, and I am confident.



The Divergence Theorem

Question

The vector field $\vec{F} = \dfrac{x\vec{i} + y\vec{j} + z\vec{k}}{(x^2+y^2+z^2)^{3/2}}$ has $\operatorname{div}(\vec{F}) = 0$ everywhere it is defined. How do $\iint_{S_1} \vec{F} \cdot d\vec{S}$ and $\iint_{S_2} \vec{F} \cdot d\vec{S}$ compare if both surfaces are oriented outward?



A. $\iint_{S_1} \vec{F} \cdot d\vec{S} < \iint_{S_2} \vec{F} \cdot d\vec{S}$

B. $\iint_{S_1} \vec{F} \cdot d\vec{S} = \iint_{S_2} \vec{F} \cdot d\vec{S}$

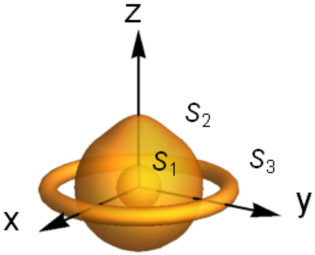
C. $\iint_{S_1} \vec{F} \cdot d\vec{S} > \iint_{S_2} \vec{F} \cdot d\vec{S}$

D. There is not enough information.

The Divergence Theorem

Question

The field $\vec{F} = \dfrac{x\vec{i} + y\vec{j} + z\vec{k}}{(x^2+y^2+z^2)^{3/2}}$ has $\text{div}(\vec{F}) = 0$ everywhere it is defined. Rank the integrals $I_j = \iint_{S_j} \vec{F} \cdot d\vec{S}$ if S_1 and S_2 are oriented outward while S_3 is oriented inward.



A. $I_3 < I_2 < I_1$

B. $I_2 < I_1 = I_3$

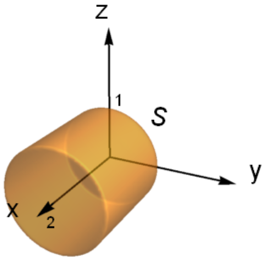
C. $I_1 = I_2 = I_3$

D. $I_3 < I_1 = I_2$

The Divergence Theorem

Question

If $\vec{F} = 2x\vec{i} + x\vec{j}$ and $\vec{G}$ is a vector field on $\mathbb{R}^3$, compute $\iint_S (\vec{F} + \text{curl}(\vec{G})) \cdot \vec{n}\, dS$ over the outward oriented closed cylinder shown.



A. 2π

B. 4π

C. -2π

D. There is not enough information.