Kalman Filter — Part 1

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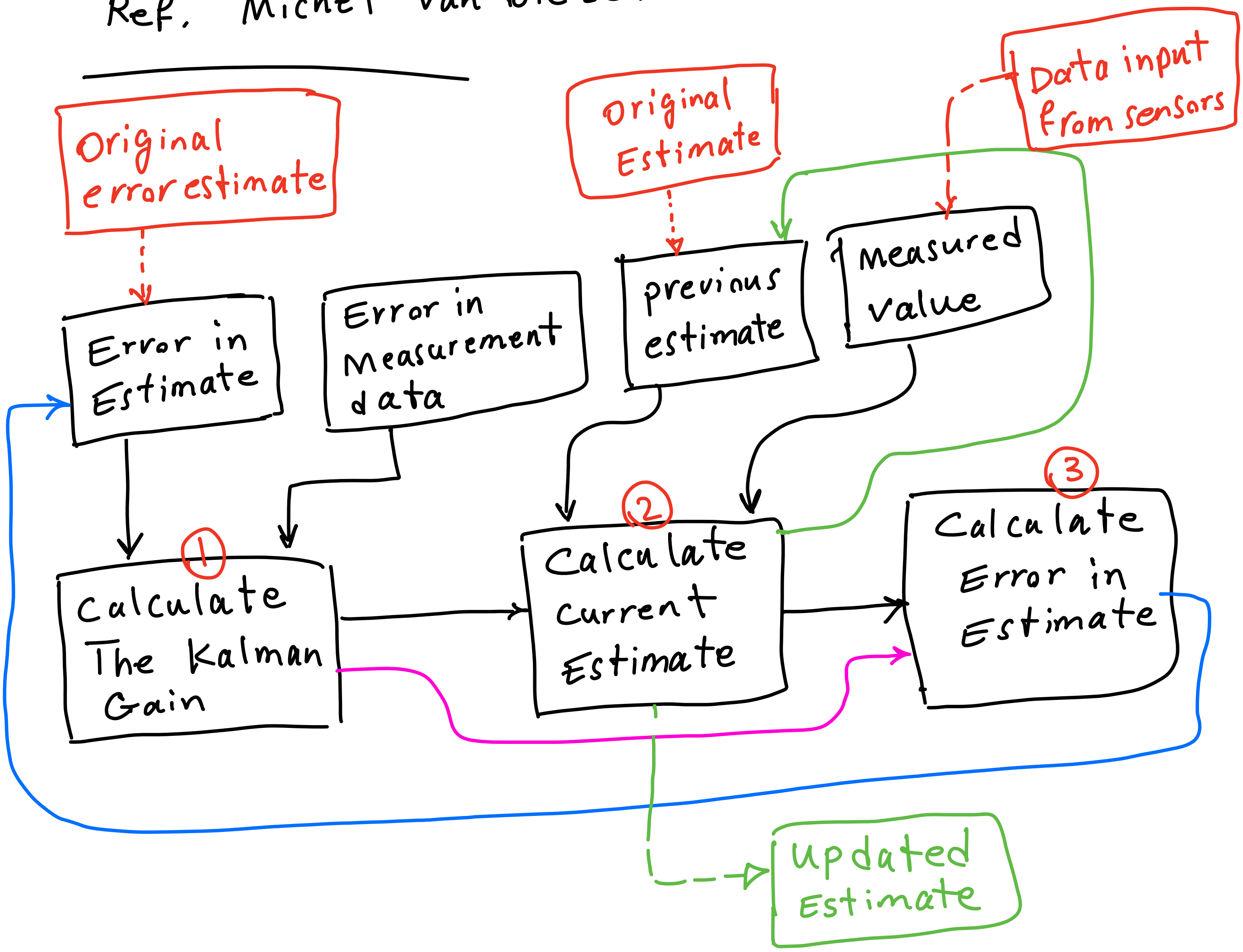


KG : Kalman gain

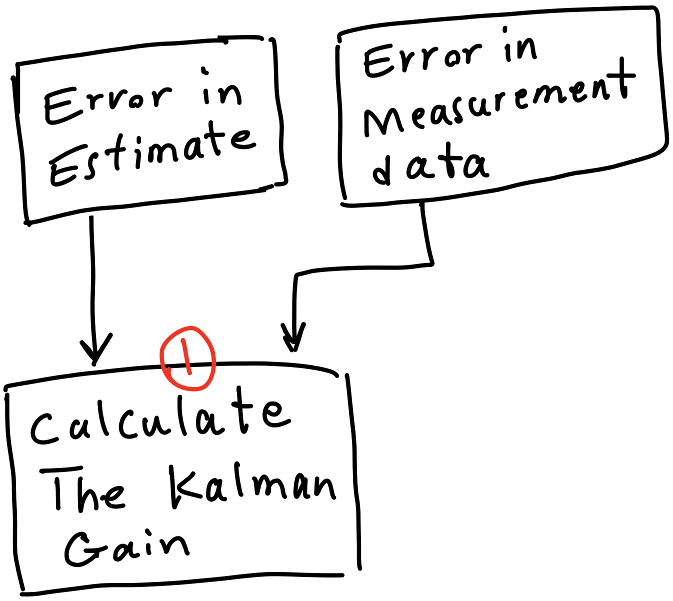
E_{Est} : Error in Estimate

E_{Meas} : Error in measurement

EST_t : current Estimate (time t is current time)

EST_{t-1} : previous Estimate (time t-1 is previous time)

Meas. : Measurement data.



Kalman Gain $$KG = \frac{E_{EST}}{E_{EST} + E_{Meas}}$$

$$0 \leqslant KG \leqslant 1$$

when E_{Meas} is large $E_{Meas} \gg E_{EST}$

Then KG is small ($KG \to 0$).

Therefore smaller value of KG corresponds to larger error in the sensor measurement data.

If E_{EST} is larger than E_{meas}

$(E_{EST} \gg E_{meas})$, then $KG \longrightarrow 1$

$$EST_t = EST_{t-1} + KG\,[\,Meas. - EST_{t-1}\,]$$

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1 Calculate The Kalman Gain $\longrightarrow$ 2 Calculate current Estimate $\longrightarrow$ 3 Calculate Error in Estimate

1 $$KG = \frac{E_{EST}}{E_{EST} + E_{Meas}}$$

2 $$EST_t = EST_{t-1} + KG\,[\,Meas. - EST_{t-1}\,]$$

3

$$E_{EST_t} = \frac{(E_{Meas.})(E_{EST_{t-1}})}{(E_{Meas.}) + (E_{EST_{t-1}})}$$

$$(1-KG)(E_{EST_{t-1}}) = \left(1 - \frac{E_{EST_{t-1}}}{E_{EST_{t-1}} + E_{Meas}}\right) E_{EST_{t-1}}$$

$$= E_{EST_{t-1}} - \frac{E_{EST_{t-1}} \, E_{EST_{t-1}}}{E_{EST_{t-1}} + E_{Meas}}$$

$$\frac{\cancel{E_{EST_{t-1}} \, E_{EST_{t-1}}} + E_{EST_{t-1}} \, E_{Meas.} - \cancel{E_{EST_{t-1}} \, E_{EST_{t-1}}}}{E_{EST_{t-1}} + E_{meas.}}$$

$$= \frac{E_{EST_{t-1}} \, E_{meas.}}{E_{EST_{t-1}} + E_{meas.}}$$

$$(1-KG)(E_{EST_{t-1}}) = \frac{E_{EST_{t-1}} \, E_{meas.}}{E_{EST_{t-1}} + E_{meas.}}$$

$$\Rightarrow E_{EST_t} = (1-KG)(E_{EST_{t-1}})$$

The Kalman filter algorithm

Initial state

X_0

P_0

previous state $K-1$

X_{K-1}

P_{K-1}

New or current state predicted (by mathematical model)

$$X_{K_p} = F X_{K-1} + B u_K + w_K$$

$$P_{K_p} = F P_{K-1} F^T + Q_K$$

For The true model not used in kalman

update with new measurement and Kalman Gain

$$K = \frac{P_{K_p} H}{H P_{K_p} H^T + R}$$

$$X_K = X_{K_p} + KG\left[Y - H X_{K_p}\right]$$

current step becoms previous step

$$P_K = (I - KG.H) P_{K_p}$$

X_p

output of updated state

X_K

P_K

w_K is used in the true model only but not in the actual Kalman Filter algorithm.
w_K simulates the noise in in the model of the system (that includes noise).
It is only part of the concept of noisy state. w_K is implemented through Q in the Kalman Filter algorithm.

$$\boxed{Y_K = C X_{K_{Meas.}} + Z_K}$$

Measurement input

Y_K: measurement of the State

measurement Noise (uncertainty)

X_{KP} that includes w_K is only used for simulation and comparison between the Kalman Filter results and X_{KP}(with w_K) simulated model.
In fact, w_K is never used in the hardware, w_K is the actual noise that exists inherently in the physical system. But when we need to model the system, we include W_K to model the noise in the system refered to as the True Model.

Predicted state is based on physical model of the system (and previous state)

U_K is control variable matrix at step K

W_K is the predicted state nois matrix corresponding to the uncertainty in the mathematical model.

Q is the process noise matrix

X is the state matrix, for example

Example $X = (x, \dot{x})$

position velocity

KG : Kalman Gain

R : sensor noise covariance matrix (measurement error / uncertainty)

I : Identity matrix

P : process covariance matrix (process error in the estimate)

Example

$$X_k = F X_{k-1} + B u_k + w_k$$

$$X = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \begin{matrix} \rightarrow \text{position} \\ \rightarrow \text{velocity} \end{matrix}$$