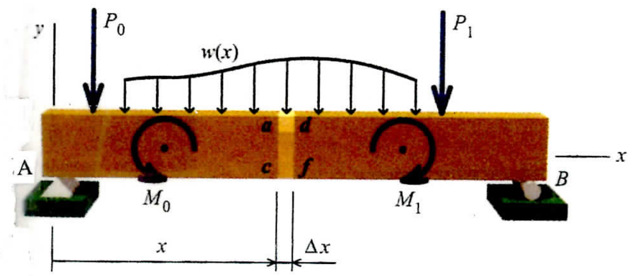
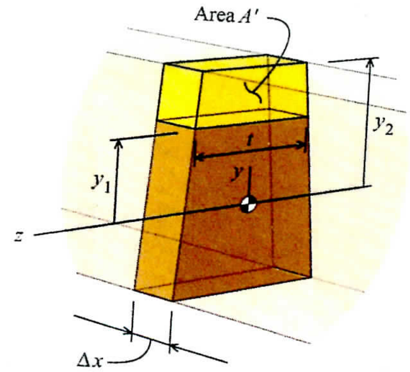
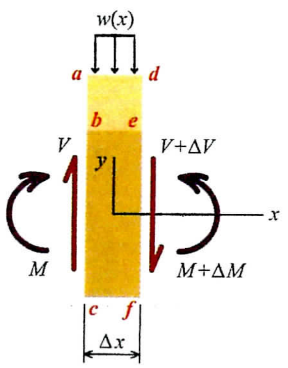


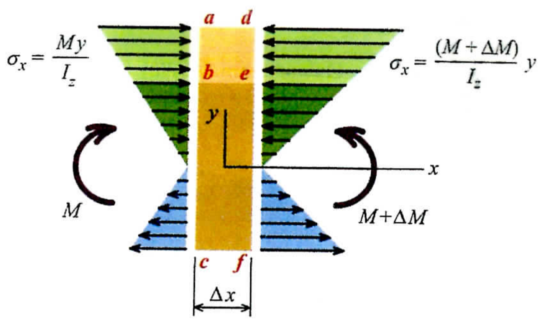
Prismatic beam subjected to non-uniform bending



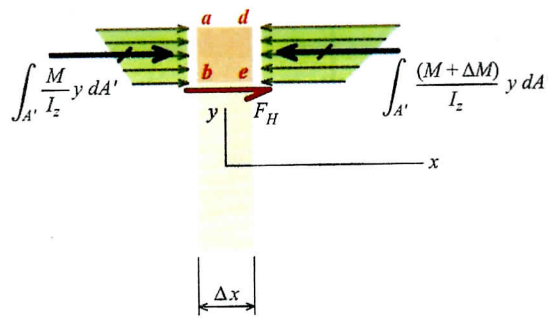
A slice of the beam with width Δx



Free-body-diagram of slice



Internal Bending Stresses



Free-body-diagram of area A'

In the last free-body-diagram set $\sum F_x = 0$ or : $F_{ab} - F_{de} + F_H = 0$ where:

$F_{ab} = \int_{A'} \sigma_{ab}\, dA'$ $\quad F_{de} = \int_{A'} \sigma_{de}\, dA'$ and F_H = Shear force over the area t by Δx

Therefore: $\boxed{\Sigma F_x = \int_{A'} \frac{M}{I_z} y\, dA' - \int_{A'} \frac{(M+\Delta M)}{I_z} y\, dA' + F_H = 0}$

Canceling terms and rearranging we get : $\boxed{F_H = \int_{A'} \frac{\Delta M}{I_z}\, y\, dA'}$

Since with respect to area A', ΔM and I are constant, then : $\boxed{F_H = \frac{\Delta M}{I_z} \int_{A'} y\, dA'}$

Note that the $\int_{A'} y\, dA'$ is the first moment of the area A' with respect to z-axis or Q, therefore: $\boxed{F_H = \frac{\Delta M Q}{I_z}}$

Since τ_{ave}= Force/Area, then $\boxed{\tau_{H,\text{avg}} = \frac{F_H}{t\,\Delta x} = \frac{\Delta M\, Q}{t\,\Delta x\, I_z} = \frac{\Delta M}{\Delta x}\frac{Q}{I_z\, t}}$ Letting $\Delta x \to 0$ then $\Delta M/\Delta x$ becomes dM/dx

However $dM/dx = V$ or shear force, then the horizontal shear stress $\boldsymbol{\tau}$ is : $\boxed{\tau_H = \frac{V\,Q}{I_z\, t}}$

Since $\boldsymbol{\tau_{xy} = \tau_{yx}}$, therefore $\boldsymbol{\tau_H = \tau_V = \tau}$ or: $\boxed{\tau = \frac{V\,Q}{I_z\, t}}$ or $\boxed{\boldsymbol{\tau = V\,Q / I_z\, t}}$

And if we define the amount of the shear force over one unit length of the beam as **q**, or **shear-flow**, then:

$$\boxed{\mathbf{q = \tau \times t = V\,Q / I_z}}$$

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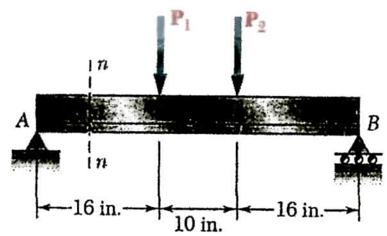
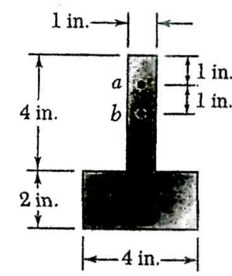
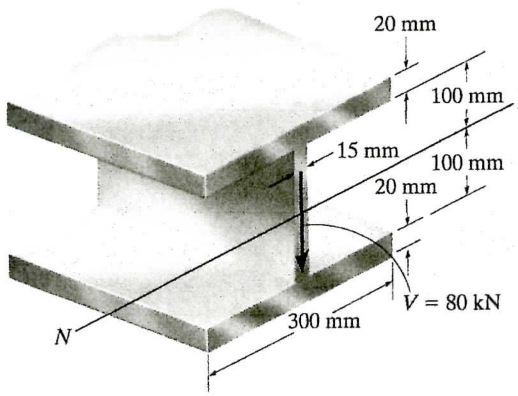
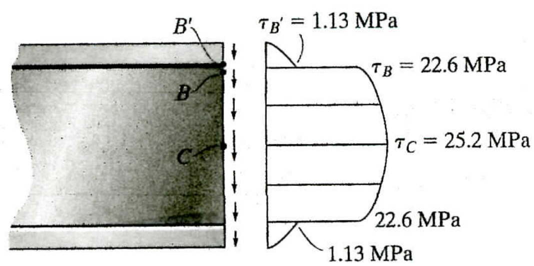
Question 1:
Given P_1 = 8-kips and P_2 = 18.5-kips, in the section n-n of the beam with cross section shown:
a- Find shear stresses at points a and b.
b-Find maximum shear stress in the section.
c- Plot the shear stress distribution over this section.





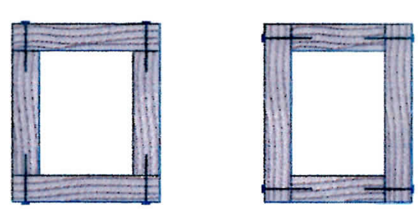
Question 2:
A steel wide-flange beam has the cross section shown. If it is subjected to a shear force V = 80-kN, plot the shear stress distribution over the beam's cross sectional area.





Result

Question 3:
Consider a box beam obtained by nailing four pieces of wood together in two different arrangements.
The height of the beam is 200-mm, the width 120-mm and the thickness of each wood 20-mm. If each section is subjected to a shear force of V = 3.5-kN and the spacing of the nails is 70-mm, determine the shear force in each nail.



(a) (b)

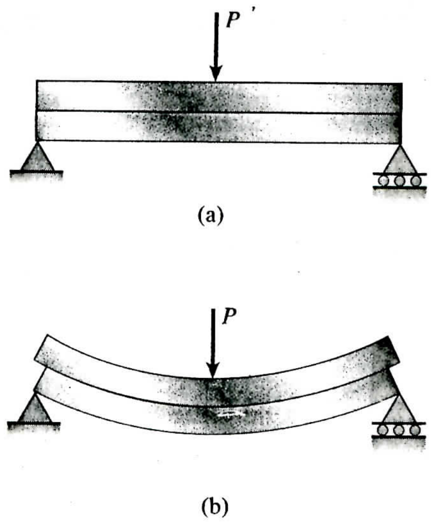


FIG. 5-27 Bending of two separate beams

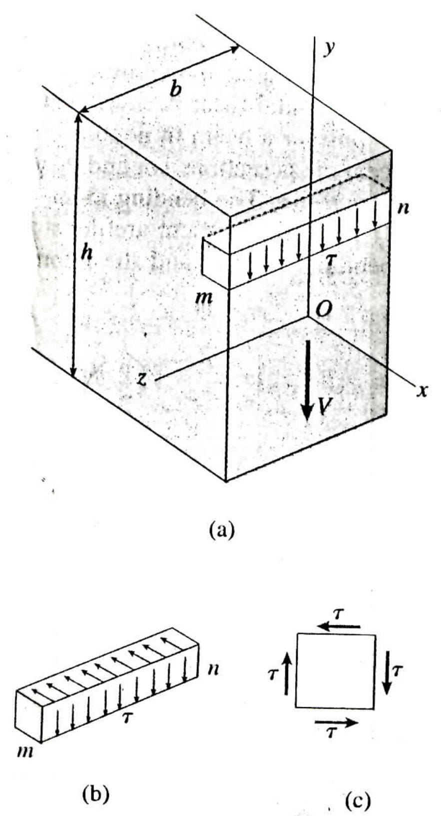
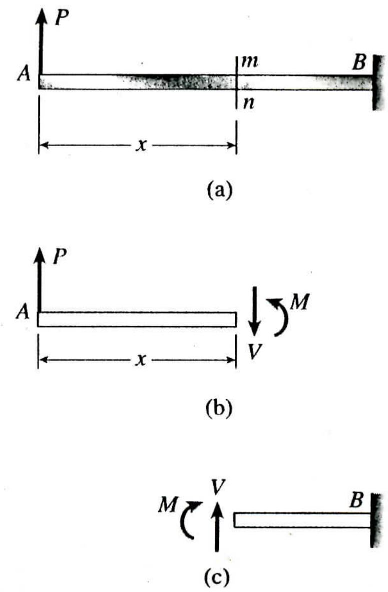


FIG. 5-26 Shear stresses in a beam of rectangular cross section



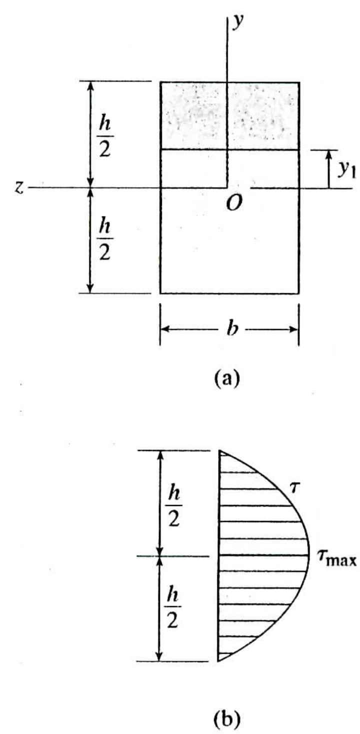
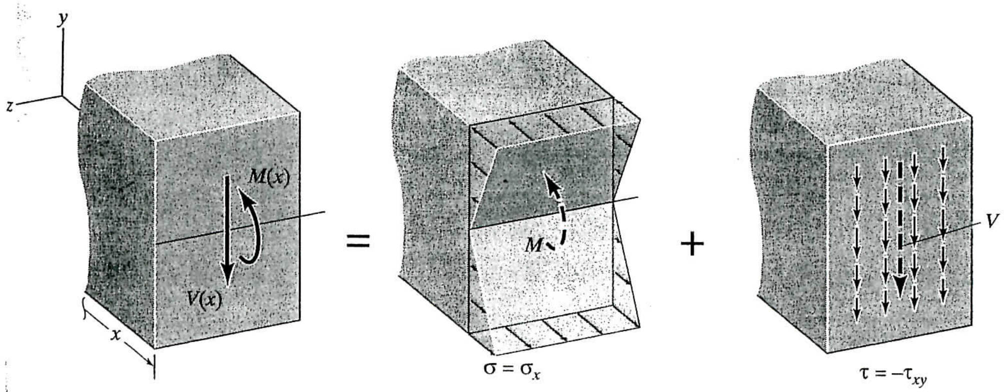
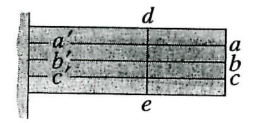


FIG. 5-30 Distribution of shear stresses in a beam of rectangular cross section: (a) cross section of beam, and (b) diagram showing the parabolic distribution of shear stresses over the height of the beam

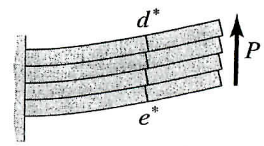


(a) Stress resultants. (*b*) Normal (flexural) stress. (*c*) Transverse shear stress.

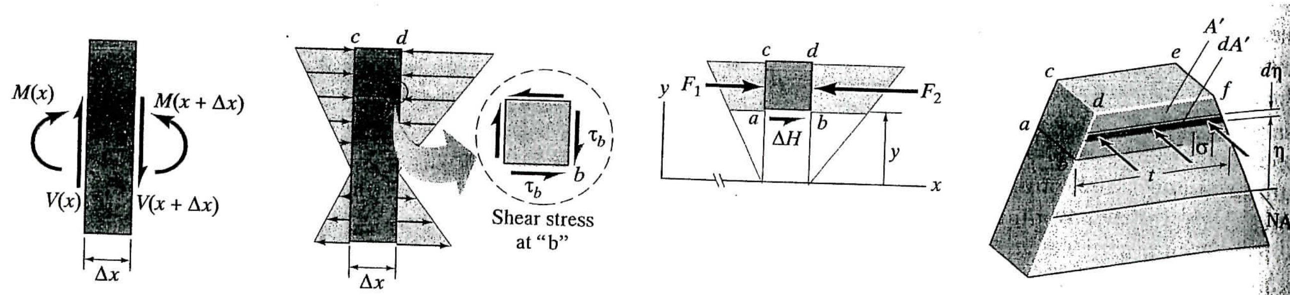
The stress resultants and stresses on a cross section.



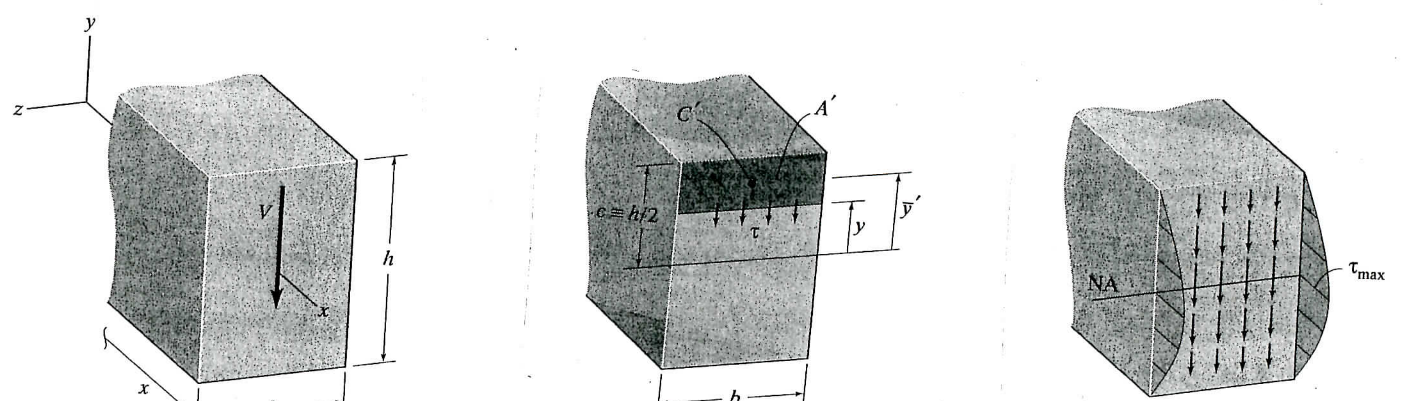
(*a*) A beam made of separate "planks."



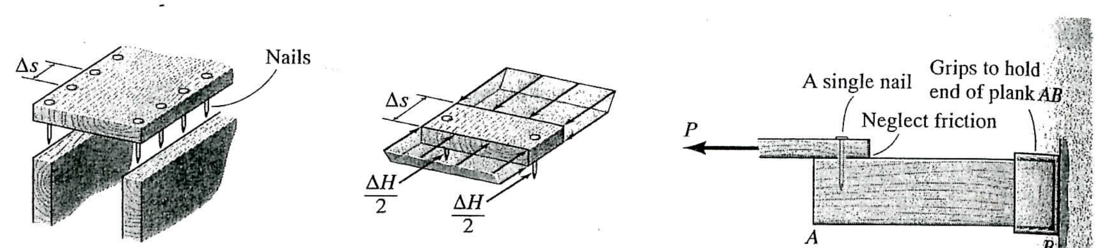
(*b*) Slip between non-bonded "planks."



(*a*) An element of length Δx. (*b*) The distribution of flexural stress. (*c*) A free body diagram (minus vertical shear on *ac* and *bd*). (*d*) The flexural stress contributing to F_2.



The shear-stress distribution on a rectangular cross section.



(*a*) Nails on a uniform spacing of Δs. (*b*) A free-body diagram for determining shear force in discrete connectors. (*c*) A direct-shear test to determine the shear capacity of a single nail.