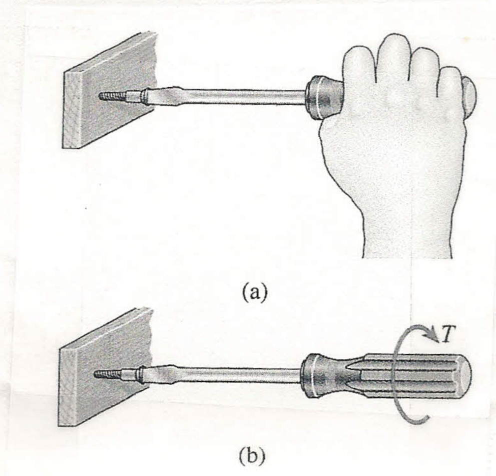
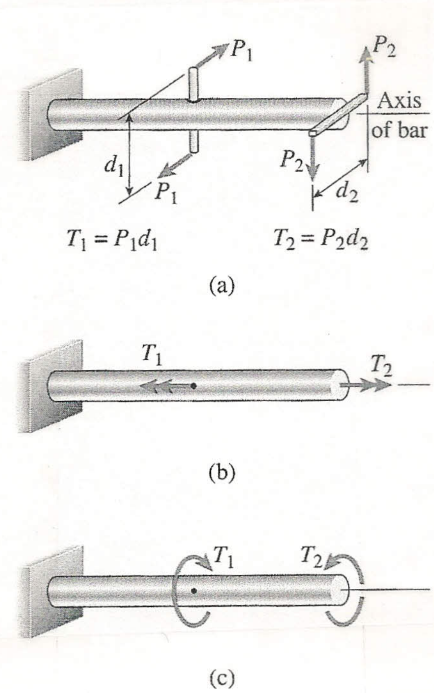
Introduction:

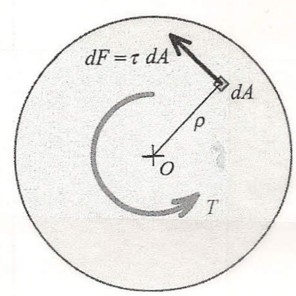


Torsion in a Screwdriver due to a torque T applied to the handle



Circular bar subjected to torsion by torques T_1 and T_2

Stress:



At any point in a cross section the shear stress is:

$$\tau = \text{Shear Force/Area} = dF/dA \quad \text{or} \quad dF = \tau\, dA$$

The resultant moment of the force dF about the shaft center O is:

$$dM_x = dT = \rho\, dF = \rho\, \tau\, dA$$

The sum of the shear forces dF over entire cross sectional area is zero.
(dF 's are in pairs, equal and opposite).

The integral of dT over entire cross sectional area must be equal to applied torque T or:

(1) $$T = \int_A dT = \int_A \rho\, \tau\, dA$$

Other relevant formulas:

Shear Strain: $\gamma = \tau / G$ or $\tau = G\gamma$

Polar Moment of Inertia of a circular cross section: $J_o = \int_A \rho^2 dA$

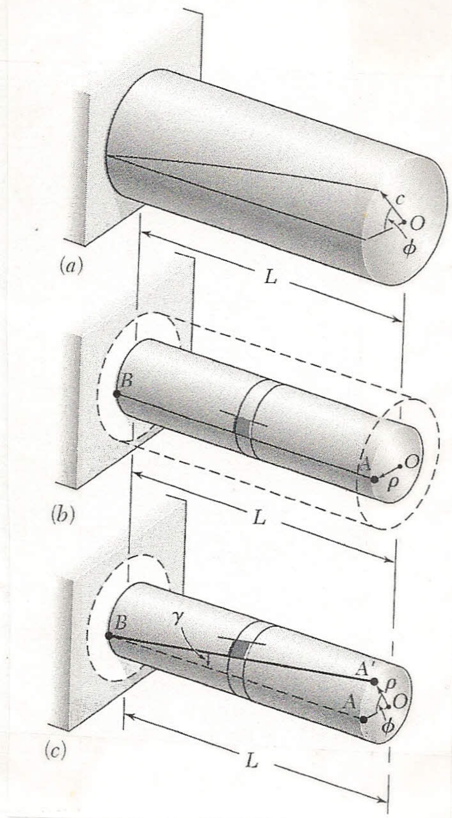
When a torque T is applied to a circular shaft of length L and radius C, the shaft will go through an angular deformation or angular twist φ. The relations between shearing strain γ and angular twist φ can be written as follows:

in triangle BAA' : $\tan\gamma = \gamma = AA'/L$

in triangle OAA' : $\tan\varphi = \varphi = AA'/\rho$

Therefore: $\gamma L = \varphi\rho$ or:

(2) $\gamma = \varphi\rho/L$



Note at the center of the shaft where $\rho = 0$,
$\gamma = 0$ Therefore no Strain and no Stress

At the outer surface of the shaft where $\rho = C$
γ is maximum or:

(3) $\gamma_{max} = \varphi C/L$

Dividing equation (2) by equation (3) we obtain:

$\gamma/\gamma_{max} = \rho/C$ or $\gamma = \gamma_{max}\,\rho/C$.

Multiply both sides of this equation by G, modulus of rigidity, we get: $\gamma G = \gamma_{max} G\,\rho/C$ or :

(4) $\tau = \tau_{max}\,\rho/C$.

Again note at the center of the shaft $\rho = 0$, $\tau = 0$ no Shear Stress.
At the outer surface of the shaft $\rho = C$, $\tau = \tau_{max}$

Replace τ in equation (1) by equation (4):

$$T = \int_A \rho\,\tau\,dA = \int_A \rho\,(\tau_{max}\,\rho/C)\,dA = (\tau_{max}/C)\int_A \rho^2 dA = \tau_{max} J_o/C$$

Or τ_{max} Stress on the outer surface of the shaft is: $\boxed{\tau_{max} = TC/J}$

From equation (4), the stress τ at any other point in the cross section: $\boxed{\tau = T\rho/J}$

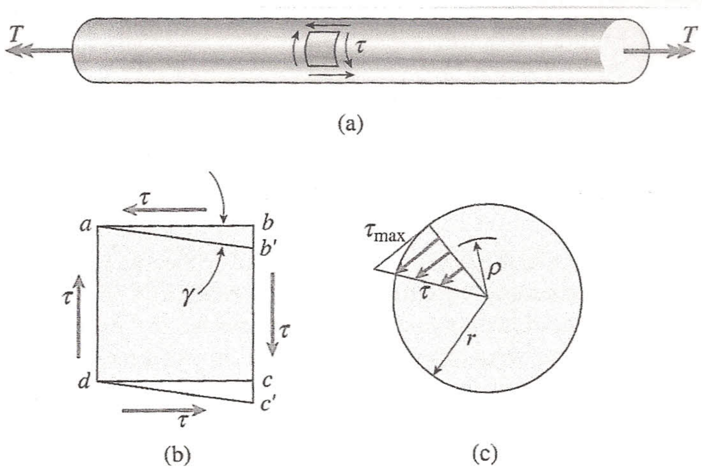
Multiplying both sides of equation (3) by G we get: $G\gamma_{max} = G\varphi C/L$ or $\tau_{max} = TC/J = G\varphi C/L$

Or φ Angular twist at the end of the shaft is: $\boxed{\varphi = TL/GJ}$

Result:

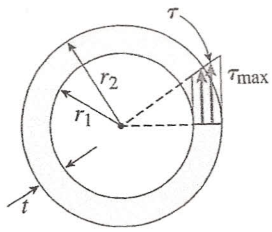
Shear stress in a circular bar in torsion

$J = \pi r^4/2$



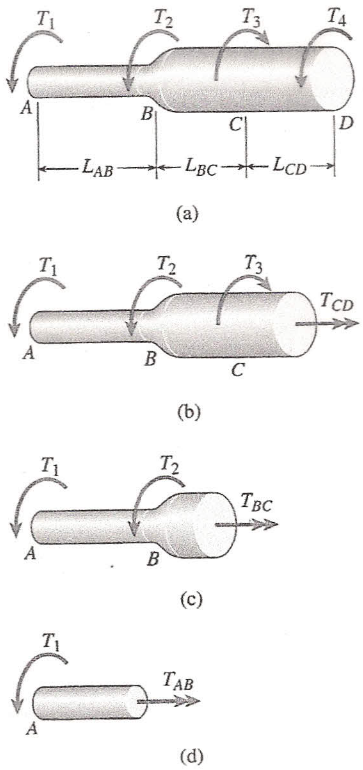
Shear stress in a circular tube in torsion

$$J = \frac{\pi}{2}(r_2^4 - r_1^4)$$



Statics:

Bar in non-uniform torsion
T_1, T_2, T_3 and T_4 are external torques
T_{AB}, T_{BC} and T_{CD} are internal torques, or the reaction of the bars AB, BC and CD to external loading.



$T_1 + T_2 - T_3 + T_{CD} = 0 \qquad T_{CD} = -T_1 - T_2 + T_3$

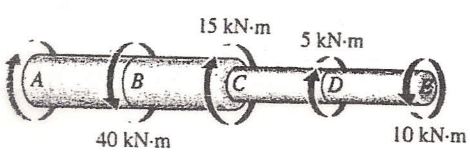
$T_1 + T_2 + T_{BC} = 0 \qquad T_{BC} = -T_1 - T_2$

$T_1 + T_{AB} = 0 \qquad T_{AB} = -T_1$

Example:

Statics:

Questions 1 and 2:
A Shaft is loaded with several torques as shown:
a- Find required torque reaction at support A for equilibrium.
b-Determine the maximum torque transmitted by any transverse cross section of the shaft (torque in AB, BC, etc.)



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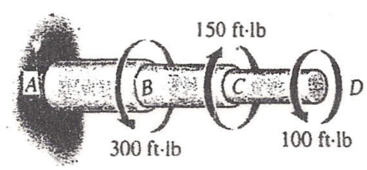
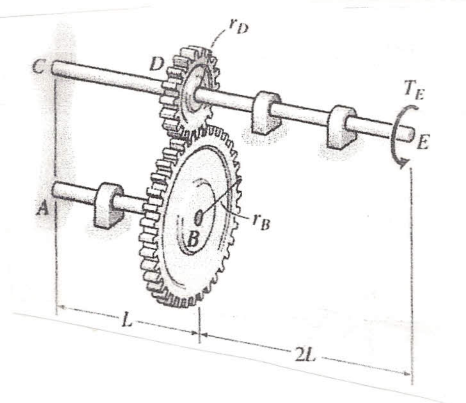


(1) (2)

Gear Systems:

Question 3: Sample Quiz

A torque T_E = 4,300 lb-in is applied at the end E of the two shaft and gear system shown. Find:
a-Maximum value of shear stress in shafts DE and AB.
b-The relative angle of rotation of point E with respect to A.
Assume: r_D = 3-in r_B = 5-in d_{AB} = 1.8-in d_{DE} = 1.4-in L = 2-ft G = $12x10^6$-psi



Sample Quizzes

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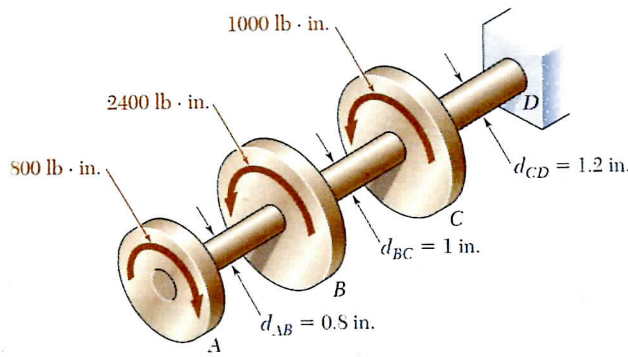
Question 1: Gear System

Three torques are applied to the 3-parts solid circular shaft ABCD as shown.
All parts of the shaft are made of aluminum with G = 11,000-ksi.
Given lengths AB = BC = CD = 10-in, determine:

a- The maximum shear stress in the shaft.

b-The relative angular twist of point A with respect to D.

c- Show state of stress for each part.

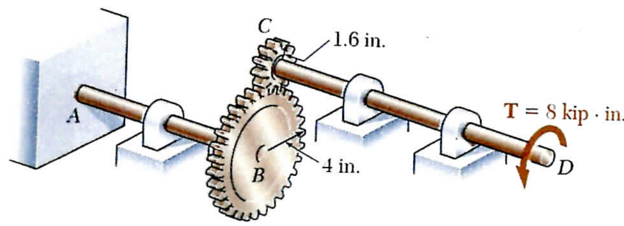


Question 2: Gear System

A torque of T = 8-kips-in applied at the end of the shaft CD. Both shafts are made of steel with G = 10,000-ksi. Knowing shaft AB has diameter of d = 2.25-in and length of L = 1.5-ft and shaft CD has diameter of d = 1.75-in and length of L = 2-ft, determine:

a- The shear stress in each shaft.

b-The relative angular deformation of point D with respect to A.

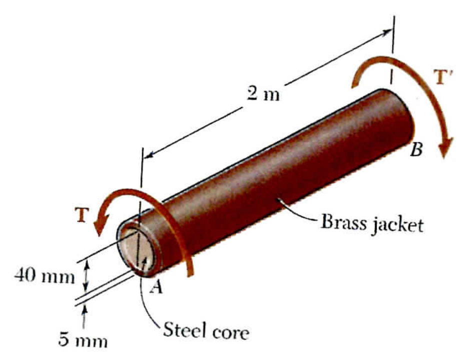


Question 3: Composite Shaft

The composite shaft shown consists of a 5-mm thick brass jacket (G_{brass} = 39-GPa) bonded to a 40-mm diameter steel core (G_{Steel} = 77.2-Gpa).
When torque T = 600-N-m is applied at A, determine:

a- The shear stress in steel and brass.

b-The relative angular deformation of point B with respect to A.



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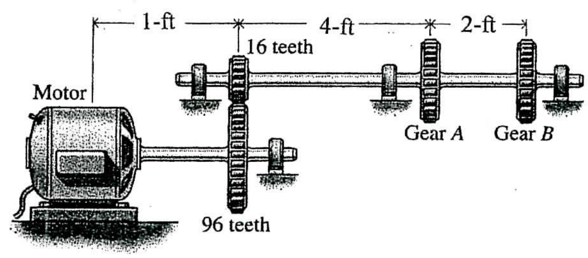
Question 4:

The motor shown develops 120-HP at a speed of 360-rpm. Gears A and B each delivers 50% of the power to the respective operating units in the factory. If maximum shearing stress τ in the shafts is limited to 12-ksi, determine:

a-The minimum required diameter of the motor shaft.

b-The minimum required diameter of the power shaft.

c-The relative angle of rotation at B with respect to motor, if shafts are made of steel with G = 12,000 ksi.



Question 5:

A motor applies a torque T to the shafts shown through gears D and C. Gears A and B each delivers 12 kips-in and 48 kips-in power to the respective operating units in the factory. Determine:

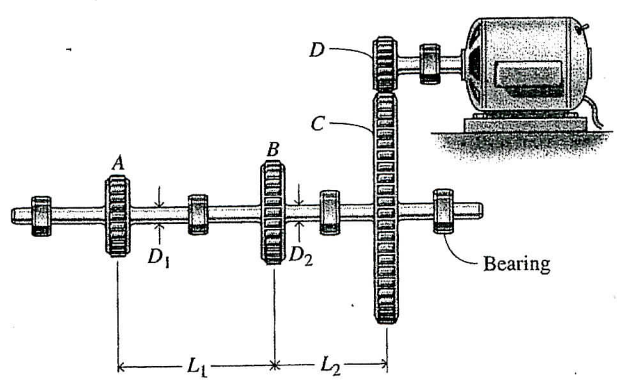
a-The maximum shearing stresses in the shafts AB and BC.

b-The relative rotation of gear A with respect to Gear C.

c-The required power in HP generated by the motor if it is operating at 240-rpm, and gear D has 1/3 diameter of gear C.

Assume: L_1 = 4-ft , D_1 = 2-in L_2 = 2-ft , D_2 = 3-in

G = 12,000 ksi



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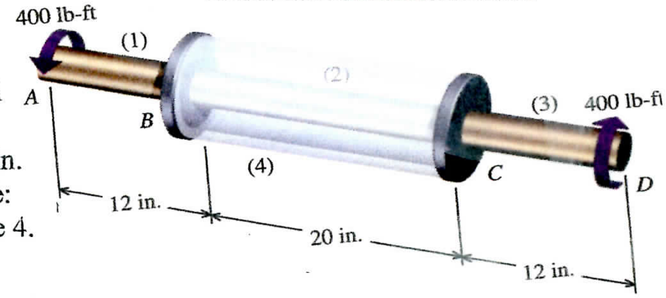
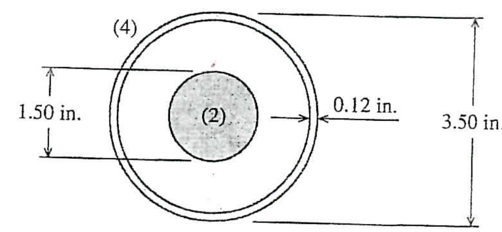


Question 6: 20 Points)
A solid 1.5-in diameter brass shaft ABCD (G = 6,000-ksi) is stiffened between B to C by a stainless steel tube (4) with G = 12,000-ksi.
The tube has outside diameter of 3.5-in and the thickness of t = 0.12-in.
The tube is attached to the brass by means of rigid flanges. Determine:
a-The max.shear stress in segments 1, 2 and 3 of the shaft, and in tube 4.
b-The angular rotation of D relative to A.

$\tau = TC/J \quad \varphi = TL / GJ \quad J = \pi c^4/2$



Cross Section Through Tube