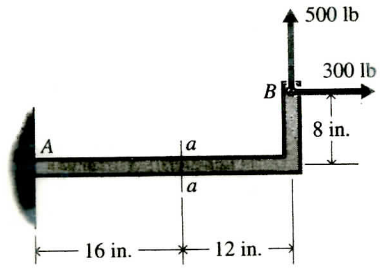
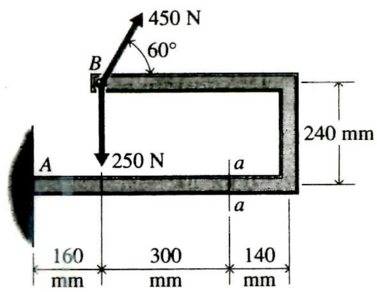
NAME:__________________________ **SID #**____________________

Questions 1 & 2 :
For the beam and loading shown:
a- Find the reaction at support A and draw Free-Body-Diagram(FBD).
b- Find ***internal forces*** and ***moment*** at section "a-a" *(bonding actions transmitted between two sections of the beam).*

Q-1

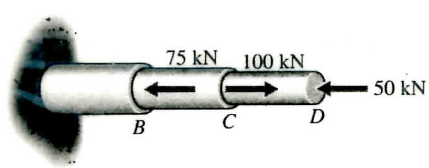


Q-2

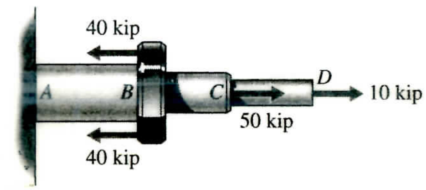


Questions 3 & 4 :
A bar is loaded and supported as shown.
a- Find the reaction at support A and draw FBD.
b- Find ***internal axial forces*** in member AB, BC and CD.

Q-3

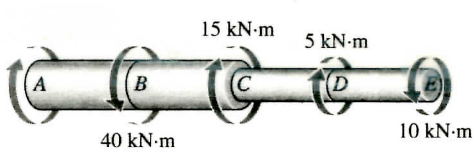


Q-4

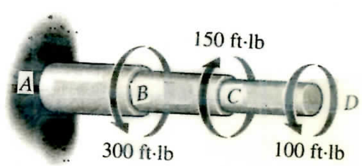


Questions 5 & 6 :
A shaft is loaded with several torque as shown.
a- Find required torque at support A for equilibrium.
b- Determine the maximum torque transmitted by any transverse cross section of the shaft (torque in segment AB, BC , etc.)

Q-5

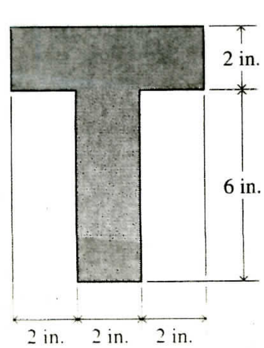


Q-6

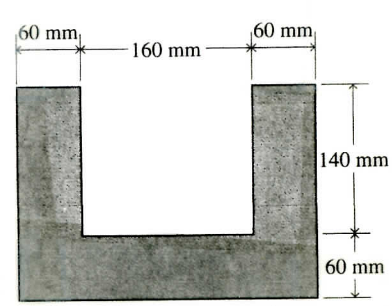


Questions 7 & 8 :
The cross section of a beam is shown.
a- Find **Moment of Inertias** with respect to x and y axes passing thru the **centroid** of the section.
b- Prove that the **first static moment** (**Q_x**) of the areas above and below x-axis are equal in magnitudes.

Q-7

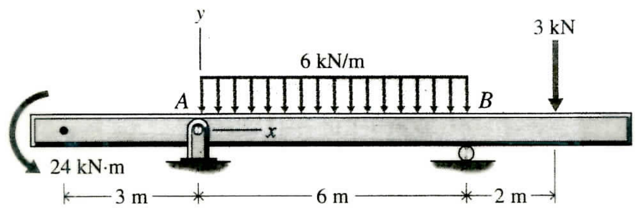


Q-8

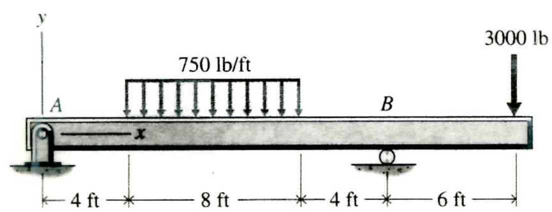


NAME:_____________________________ **SID #**____________________

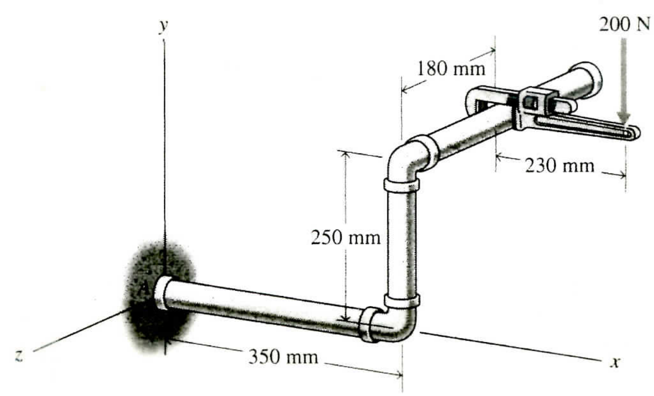
Question 9 :
For the beam and loading shown:
a- Find the reaction at supports A and B. Draw Free-Body-Diagram(FBD).
b- Find ***internal forces*** and ***moment*** at a section 2-m to the right of support A.



Question 10 :
For the beam and loading shown:
a- Find the reaction at supports A and B. Draw Free-Body-Diagram(FBD).
b- Find ***internal forces*** and ***moment*** at a section 6-ft to the right of support A.



Question 11 :
For the pipe assembly shown find the reaction at support A and draw Free-Body-Diagram(FBD). The 200-N force is parallel to y-axis. Neglect the weight of the pipe and the wrench.



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Answers:

Q1: a: $A_x = 300$ lbs (←) $A_y = 500$ lbs (↓) $M_A = 11{,}600$ lb-in CW
b: $N_x = 300$ lbs (→) $V_y = 500$ lbs (↑) $M_z = 3{,}600$ lb-in CCW shown on left FBD

Q2: a: $A_x = 225$ N (←) $A_y = 140$ N (↓) $M_A = 31.6$ N-m CCW
b: $N_x = 225$ N (→) $V_y = 140$ N (↑) $M_z = 96$ N-m CW shown on left FBD

Q3: a: $A_x = 25$ kN (→)
b: $N_{AB} = 25$ kN (C) $N_{BC} = 50$ kN (T) $N_{CD} = 50$ kN (C)

Q4: a: $A_x = 20$ kips (→)
b: $N_{AB} = 20$ kips (C) $N_{BC} = 60$ kips (T) $N_{CD} = 10$ kips (T)

Q5: a: $T_A = -30$ kN-m or CW
b: $T_{AB} = 30$ kN-m $T_{BC} = -10$ kN-m $T_{CD} = 5$ kN-m $T_{DE} = -5$ kN-m Torques on left FBD's

Q6: a: $T_A = -250$ lb-ft or CW
b: $T_{AB} = 250$ lb-ft $T_{BC} = -50$ lb-ft $T_{CD} = 100$ lb-ft Torques on left FBD's

Q7: a: $\hat{y} = 5$-in $I_{xc} = 136\ in^4$ $I_{yc} = 40\ in^4$
b: $Q_{x\text{-above}} = Q_{x\text{-below}} = 25\ in^3$

Q8: a: $\hat{y} = 80$-mm $I_{xc} = 116.5\times10^6\ mm^4$ $I_{yc} = 318\times10^6\ mm^4$
b: $Q_{x\text{-above}} = Q_{x\text{-below}} = 864\times10^3\ mm^3$

Q9: a: $R_A = 21$ kN $R_B = 18$ kN
b: $N_x = 0$ kN $V_y = -9$ kN (↓) $M_z = 6$ kN-m CCW Actions shown on left FBD

Q10: a: $R_A = 1{,}875$ lbs $R_B = 7{,}125$ lbs
b: $N_x = 0$ lbs $V_y = -375$ lbs (↓) $M_z = 9{,}750$ lb-ft CCW Actions shown on left FBD

Q11: a: Ax = Az = 0 N Ay = 200-N (↑)
$M_{xA} = 36$ N-m $M_{yA} = 0$ N-m $M_{zA} = 116$ N-m

7.1 Internal Forces Developed in Structural Members

The design of any structural or mechanical member requires an investigation of the loading acting within the member in order to be sure the material can resist this loading. These internal loadings can be determined by using the *method of sections*. To illustrate this method, consider the "simply supported" beam shown in Fig. 7–1*a*, which is subjected to the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ and the *support reactions* $\mathbf{A}_x$, $\mathbf{A}_y$, and $\mathbf{B}_y$, Fig. 7–1*b*. If the *internal loadings* acting on the cross section at C are to be determined, then an

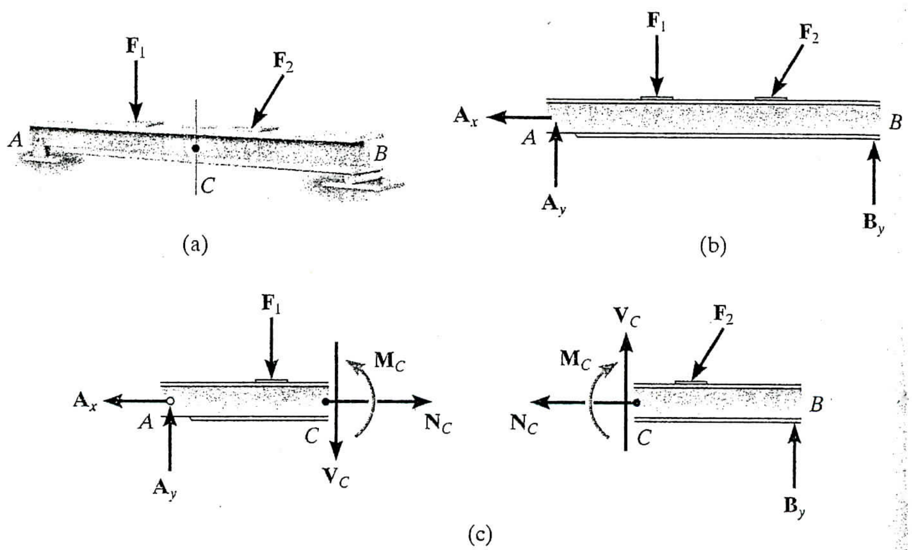


Fig. 7–1

imaginary section is passed through the beam, cutting it into two segments. By doing this the internal loadings at the section become *external* on the free-body diagram of each segment, Fig. 7–1*c*. Since both segments (AC and CB) were in equilibrium *before* the beam was sectioned, equilibrium of each segment is maintained provided rectangular force components $\mathbf{N}_C$ and $\mathbf{V}_C$ and a resultant couple moment $\mathbf{M}_C$ are developed at the section. Note that these loadings must be equal in magnitude and opposite in direction on each of the segments (Newton's third law). The magnitude of each of these loadings can now be determined by applying the three equations of equilibrium to either segment AC or CB. A *direct solution* for $\mathbf{N}_C$ is obtained by applying $\Sigma F_x = 0$; $\mathbf{V}_C$ is obtained directly from $\Sigma F_y = 0$; and $\mathbf{M}_C$ is determined by summing moments about point C, $\Sigma M_C = 0$, in order to eliminate the moments of the unknowns $\mathbf{N}_C$ and $\mathbf{V}_C$.

In mechanics, the force components $\mathbf{N}$, acting normal to the beam at the cut section, and $\mathbf{V}$, acting tangent to the section, are termed the *normal or axial force* and the *shear force*, respectively. The couple moment $\mathbf{M}$ is referred to as the *bending moment*, Fig. 7–2*a*. In three dimensions, a general internal force and couple moment resultant will act at the section. The x, y, z components of these loadings are shown in Fig. 7–2*b*. Here $\mathbf{N}_y$ is the *normal force*, and $\mathbf{V}_x$ and $\mathbf{V}_z$ are *shear force components*. $\mathbf{M}_y$ is a *torsional or twisting moment*, and $\mathbf{M}_x$ and $\mathbf{M}_z$ are *bending moment components*. For most applications, these *resultant loadings* will act at the geometric center or centroid (C) of the section's cross-sectional area. Although the magnitude for each loading generally will be different at various points along the axis of the member, the method of sections can always be used to determine their values.

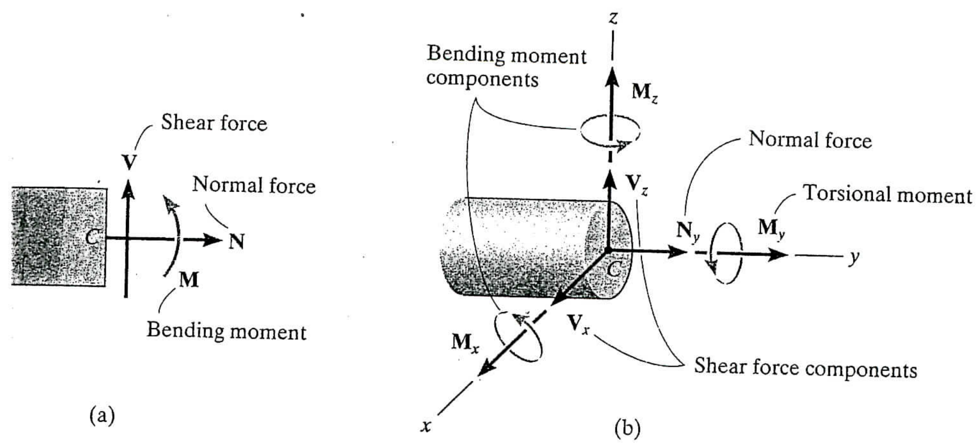


Fig. 7–2

Free-Body Diagrams. Trusses are composed of two-force members that only support normal loads. On the other hand, frames and machines are composed of *multiforce members*, and so each of these members will generally be subjected to internal normal, shear, and bending loadings. For example, consider the frame shown in Fig. 7–3*a*. If the blue section is passed through the frame to determine the internal loadings at points *H*, *G*, and *F*, the resulting free-body diagram of the top portion of this section is shown in Fig. 7–3*b*. At each point where a member is sectioned there is an unknown normal force, shear force, and bending moment. As a result, we cannot apply the *three* equations of equilibrium to this section in order to obtain these *nine unknowns*. Instead, to solve this problem we must *first dismember* the frame and determine the reactions at the connections of the members using the techniques of Sec. 6.6. Once this is done, *each member* may then be sectioned at its appropriate point, and the three equations of equilibrium can be applied to determine **N**, **V**, and **M**. For example, the free-body diagram of segment *DG*, Fig. 7–3*c*, can be used to determine the internal loadings at *G* provided the reactions of the pin, $\mathbf{D}_x$ and $\mathbf{D}_y$, are known.

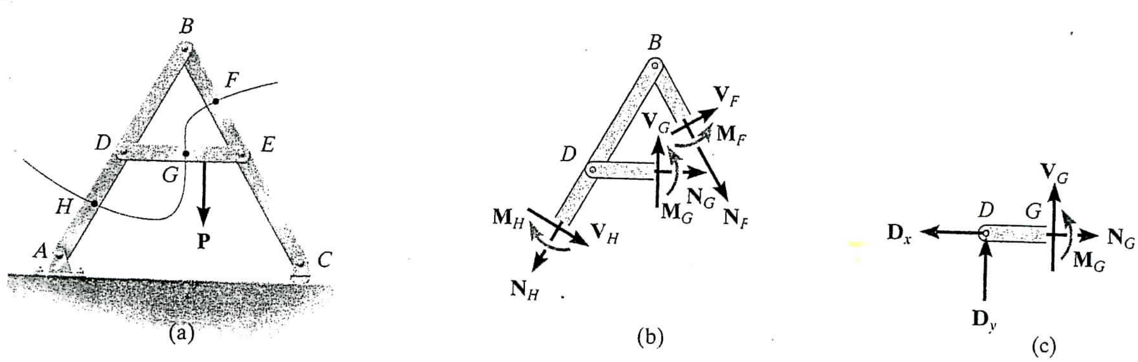
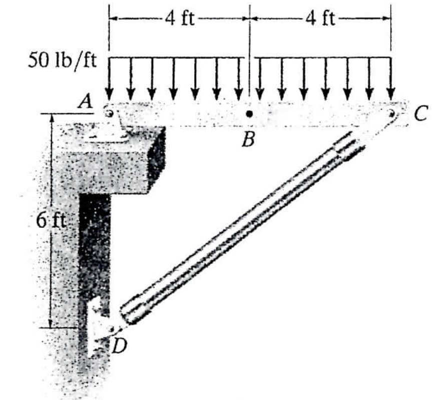


Fig. 7–3

EXAMPLE 7.4

Determine the internal normal force, shear force, and bending moment acting at point B of the two-member frame shown in Fig. 7–7*a*.



(a)

SOLUTION

Support Reactions. A free-body diagram of each member is shown in Fig. 7–7*b*. Since CD is a two-force member, the equations of equilibrium need to be applied only to member AC.

$$\circlearrowleft + \Sigma M_A = 0; \quad -400 \text{ lb } (4 \text{ ft}) + \left(\tfrac{3}{5}\right) F_{DC}(8 \text{ ft}) = 0 \qquad F_{DC} = 333.3 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad -A_x + \left(\tfrac{4}{5}\right)(333.3 \text{ lb}) = 0 \qquad A_x = 266.7 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 400 \text{ lb} + \tfrac{3}{5}(333.3 \text{ lb}) = 0 \qquad A_y = 200 \text{ lb}$$

Free-Body Diagrams. Passing an imaginary section perpendicular to the axis of member AC through point B yields the free-body diagrams of segments AB and BC shown in Fig. 7–7*c*. When constructing these diagrams it is important to keep the distributed loading exactly as it is until *after* the section is made. Only then can it be replaced by a single resultant force. Why? Also, notice that $\mathbf{N}_B$, $\mathbf{V}_B$, and $\mathbf{M}_B$ act with equal magnitude but opposite direction on each segment—Newton's third law.

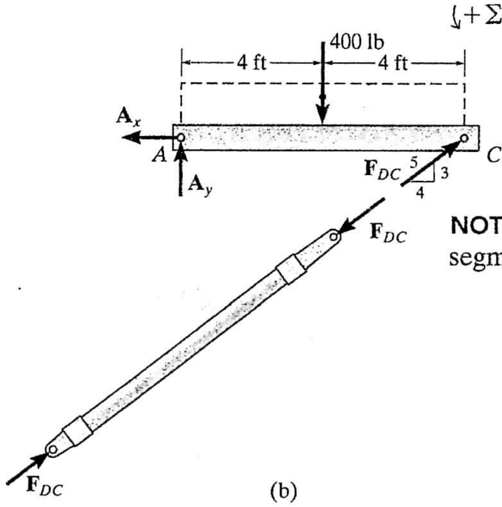
Equations of Equilibrium. Applying the equations of equilibrium to segment AB, we have

$$\xrightarrow{+} \Sigma F_x = 0; \quad N_B - 266.7 \text{ lb} = 0 \qquad N_B = 267 \text{ lb} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad 200 \text{ lb} - 200 \text{ lb} - V_B = 0 \qquad V_B = 0 \qquad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_B = 0; \quad M_B - 200 \text{ lb } (4 \text{ ft}) + 200 \text{ lb } (2 \text{ ft}) = 0$$

$$M_B = 400 \text{ lb} \cdot \text{ft} \qquad \textit{Ans.}$$



(b)

NOTE: As an exercise, try to obtain these same results using segment BC.

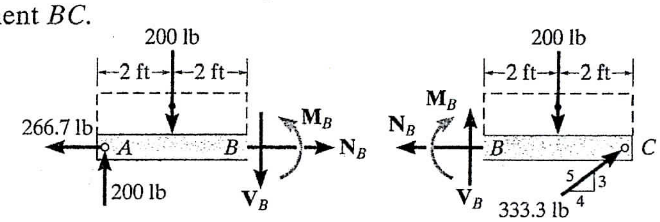


Fig. 7–7

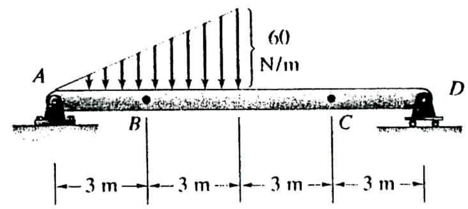
(c)

Question : 6

For the beam and loading shown:

a- Find reactions at supports A and D.

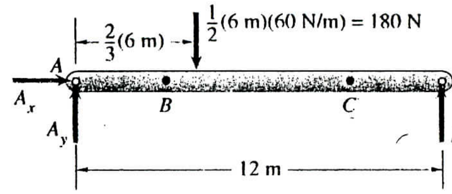
b- Find ***internal forces*** and ***moment (bonding actions)*** at sections B and C.



SOLUTION

Determine the External Forces and Moments

Free-body diagram of the entire beam with the distributed load represented by an equivalent force.



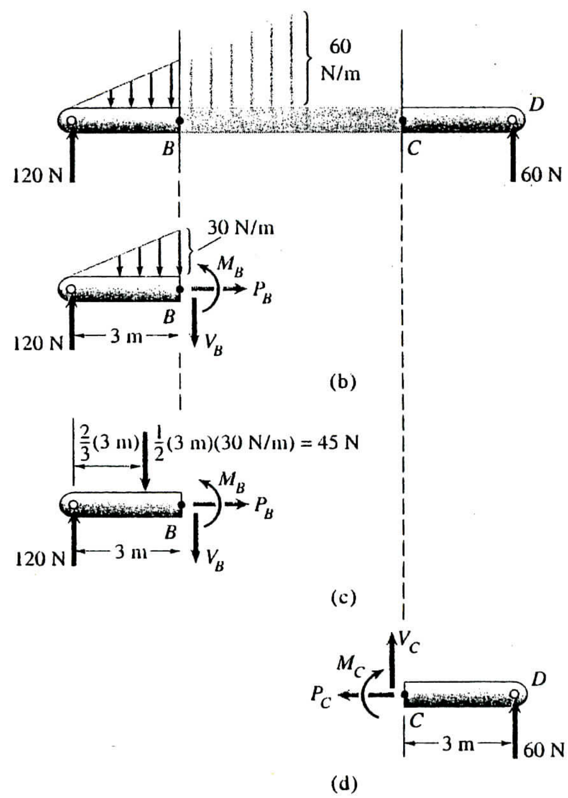
$$\Sigma F_x = A_x = 0,$$

$$\Sigma F_y = A_y + D - 180 = 0,$$

$$\Sigma M_{(\text{point } A)} = 12D - (4)(180) = 0.$$

Solving them, we obtain $A_x = 0$, $A_y = 120$ N, and $D = 60$ N.

Draw the Free-Body Diagram of Part of the Beam



$$\Sigma F_x = P_B = 0,$$

$$\Sigma F_y = 120 - 45 - V_B = 0,$$

$$\Sigma M_{(\text{point } B)} = M_B + (1)(45) - (3)(120) = 0,$$

we obtain $P_B = 0$, $V_B = 75$ N, and $M_B = 315$ N-m.

$$\Sigma F_x = -P_C = 0,$$

$$\Sigma F_y = V_C + 60 = 0,$$

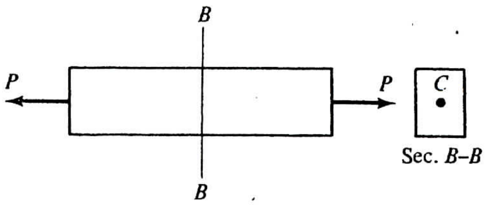
$$\Sigma M_{(\text{point } C)} = -M_C + (3)(60) = 0,$$

we obtain $P_C = 0$, $V_C = -60$ N, and $M_C = 180$ N-m.

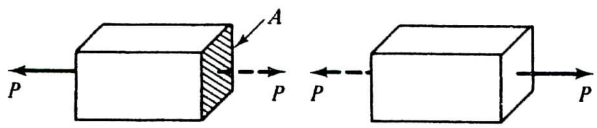
(b), (c) Free-body diagram of the part of the beam to the left of B.

(d) Free-body diagram of the part of the beam to the right of C.

STRESS IN AN AXIALLY LOADED MEMBER

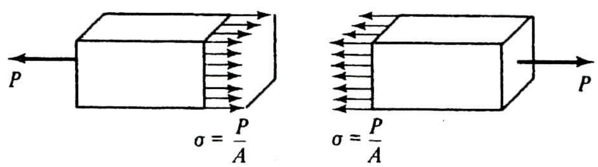


(a)



(b) (c)

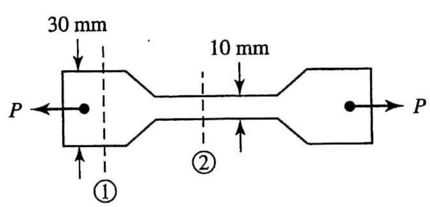
$$\sigma = \frac{P}{A} \quad \text{or} \quad \frac{\text{force}}{\text{area}}\left[\frac{\text{lb}}{\text{in.}^2} = \text{psi}\right] \quad \text{or} \quad \left[\frac{\text{N}}{\text{m}^2} = \text{Pa}\right]$$



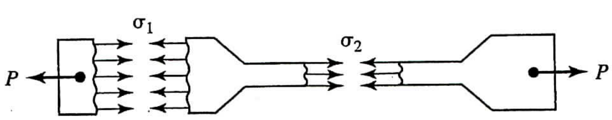
(d) (e)

XAMPLE

The tensile link shown is flat, 5 mm thick, and carries an axial load of $P = 1350\,\text{N}$.



(a)



(b)

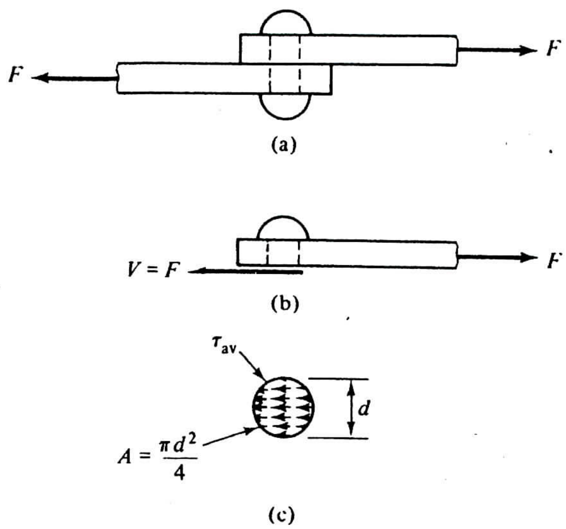
$$A_1 = 30\,\text{mm} \times 5\,\text{mm} = 150\,\text{mm}^2 = 150 \times 10^{-6}\,\text{m}^2$$

$$A_2 = 10\,\text{mm} \times 5\,\text{mm} = 50\,\text{mm}^2 = 50 \times 10^{-6}\,\text{m}^2$$

$$\sigma_1 = \frac{P}{A_1} = \frac{1350\,\text{N}}{150 \times 10^{-6}\,\text{m}^2} = 9 \times 10^6\,\text{Pa} = 9\,\text{MPa}$$

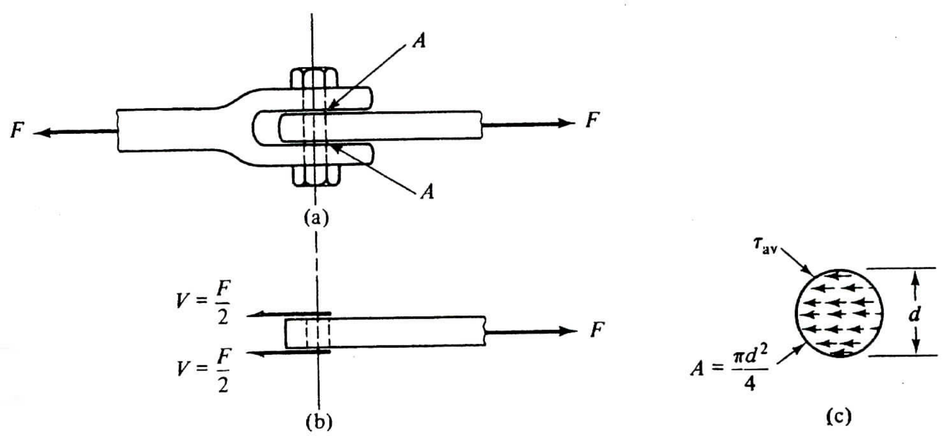
$$\sigma_2 = \frac{P}{A_2} = \frac{1350\,\text{N}}{50 \times 10^{-6}\,\text{m}^2} = 27 \times 10^6\,\text{Pa} = 27\,\text{MPa}$$

AVERAGE SHEAR STRESS



$$\tau_{av} = \frac{V}{A_s} \quad \text{or} \quad \frac{\text{force}}{\text{area}}\left[\frac{\text{lb}}{\text{in.}^2} = \text{psi}\right] \quad \text{or} \quad \left[\frac{\text{N}}{\text{m}^2} = \text{Pa}\right]$$

$$\tau_{av} = \frac{V}{A} = \frac{F}{\pi d^2/4}$$

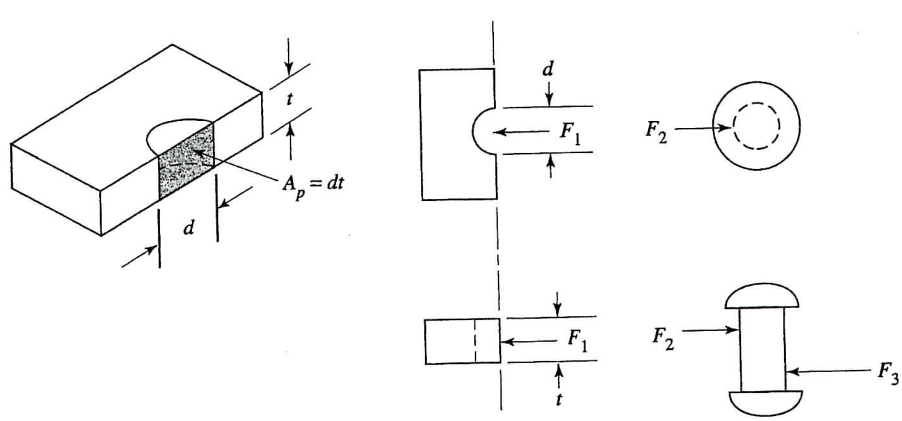


$$\tau_{av} = \frac{V}{A} = \frac{F/2}{\pi d^2/4}$$

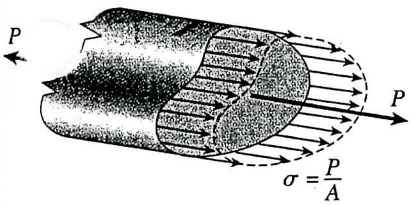
BEARING STRESS

$$\sigma_b = \frac{P}{A_{contact}}$$

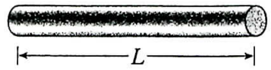
Pins, Rivets, or Bolts



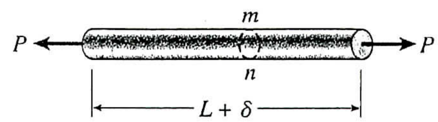
$$\sigma_b = \frac{F}{A_p} = \frac{F}{dt}$$



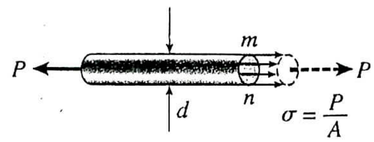
(a)



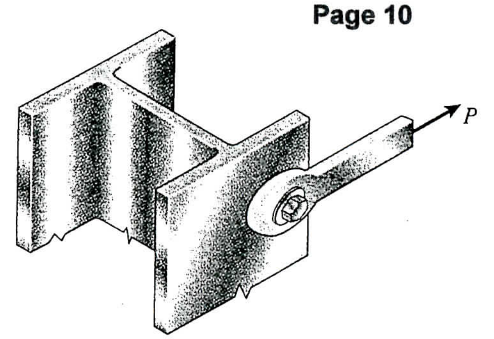
(b)



(c)

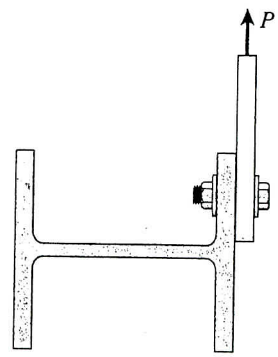


(d)

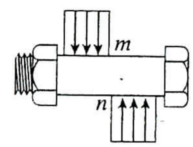


(a)

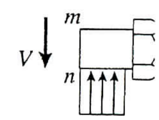
I - Normal Stress



(b)

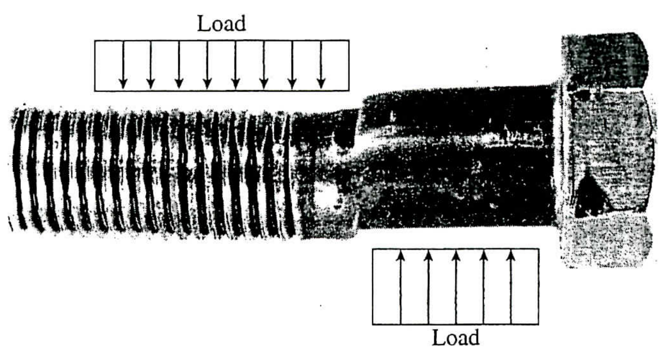


(c)

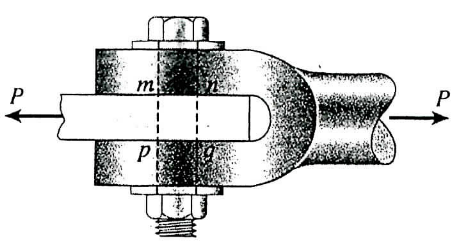


(d)

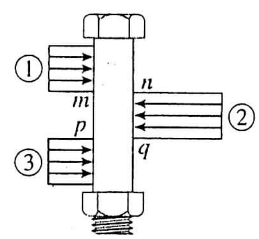
II - Shear Stress
Single Shear



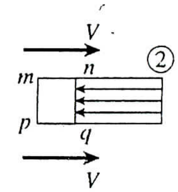
III – Double Shear



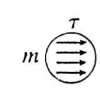
(b)



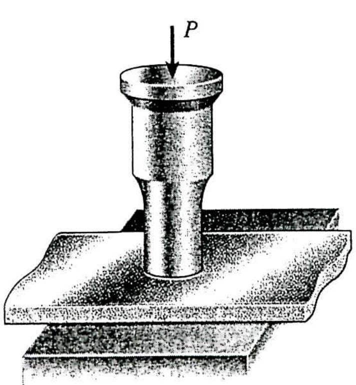
(c)

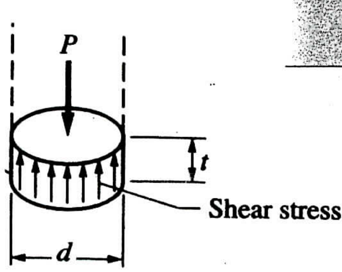


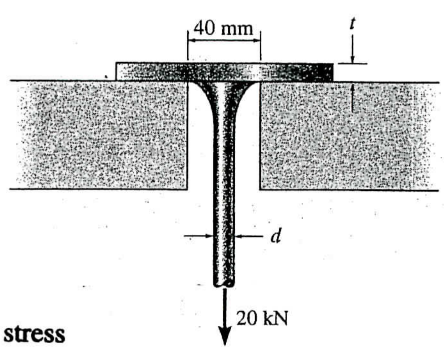
(d)

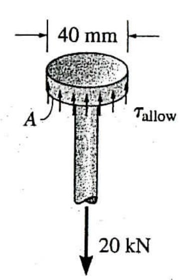


(e)









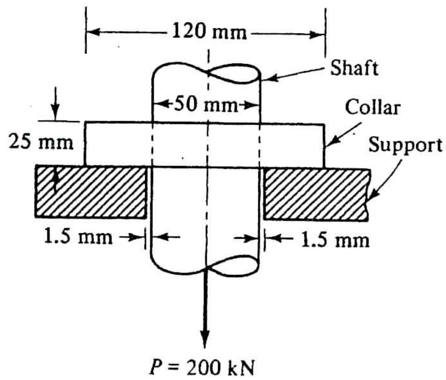
(b)

$\sigma_{allow} = 60$ MPa

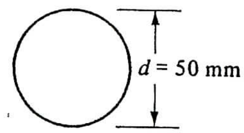
$\tau_{allow} = 35$ MPa.

EXAMPLE

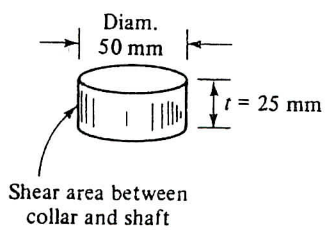
The collar bearing supports a load $P = 200$ kN   Find (a) the tensile stress in the shaft, (b) the shearing stress between the collar and the shaft, and (c) the bearing stress between the collar and the support.



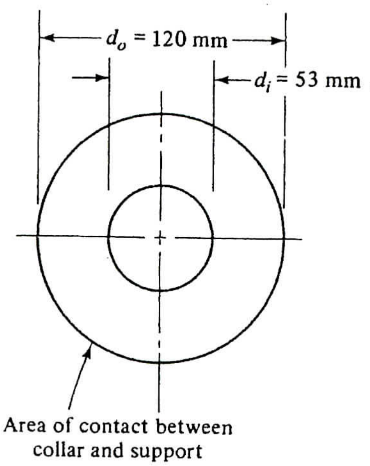
(a)



(b)



(c)



(d)

Solution

(a) *Tensile stress:* The area of the shaft is $A = \pi d^2/4 = \pi(50)^2/4 = 1.963 \times 10^3\ \text{mm}^2$. The tensile stress

$$\sigma = \frac{P}{A} = \frac{200 \times 10^3\ \text{N}}{1.963 \times 10^3\ \text{mm}^2} = 101.9\ \text{N/mm}^2 = 101.9\ \text{MPa}$$

(b) *Shear stress:* The shear area between the collar and the shaft is shown in Fig. (c) The area $A = \pi dt = \pi(50)(25) = 3.927 \times 10^3\ \text{mm}^2$. The average shear stress

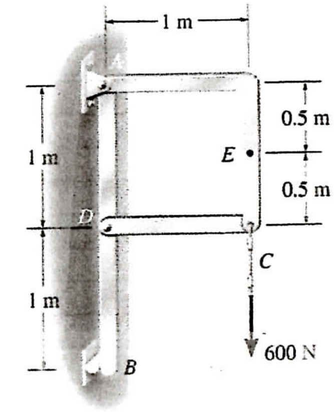
$$\tau_{av} = \frac{V}{A} = \frac{200 \times 10^3\ \text{N}}{3.927 \times 10^3\ \text{mm}^2} = 50.93\ \text{N/mm}^2 = 50.9\ \text{MPa}$$

(c) *Bearing stress:* The bearing area between the collar and the support is shown in Fig. (d) area $A = \pi(d_o^2 - d_i^2)/4 = \pi[(120)^2 - (53)^2]/4 = 9.104 \times 10^3\ \text{mm}^2$. The bearing stress

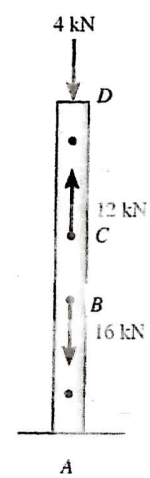
$$\sigma_b = \frac{P}{A} = \frac{200 \times 10^3\ \text{N}}{9.104 \times 10^3\ \text{mm}^2} = 21.97\ \text{N/mm}^2 = 22.0\ \text{MPa}$$

Internal Loads / Normal and Shear stress.

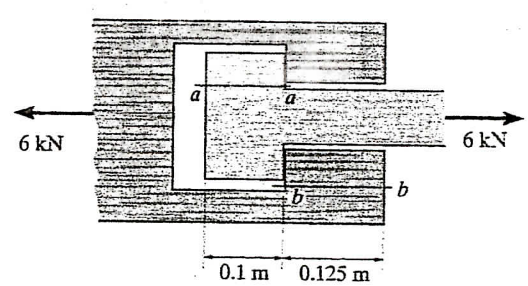
Question 1::
Determine the Normal Force, Shear Force, and Bending moment acting at point E of the frame shown.



Question 2::
Bar ABCD is made of steel with a uniform square cross sectional area of 10-mm by 10-mm .
Three external loads of 16-kN, 12-kN and 4-kN applied to points B, C and D respectively.
a- Determine the internal force action in segments AB, BC and CD of the bar and state whether each segment is in Tension (T) or Compression (C).
b- Find the normal stress σ in each segment.



Question 3::
If the wood joint shown has a width of 150-mm, determine the average shear stress developed along shear plane a-a and b-b. For each plane represent the state of stress on an element of the materials.



NAME:________________________ SID #________________

Class Exercise:

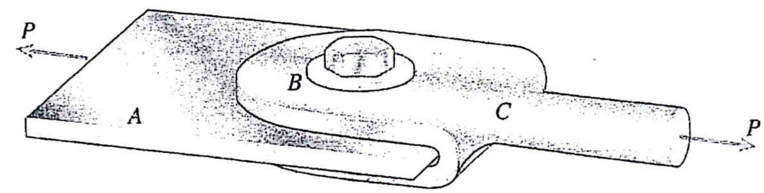
Bolt connection of a flat bar with a clevis.

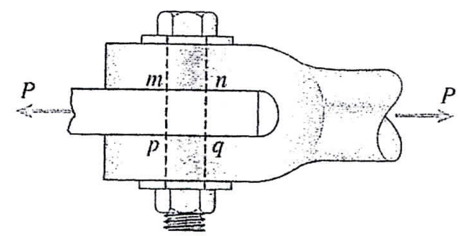
For the given data:
1- Determine **Normal Stress** in the flat bar and the **Factor of Safety**.
2- Determine **Normal Stress** in the rod and the **Factor of Safety**.
3- Determine **Shear Stress** in the bolt and the **Factor of Safety**.
4- Determine **Bearing Stress** between bolt and the flat bar.
5- Is there a possibility of **Punching Shear**?

P = 40,000 lbs

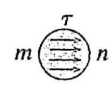
σ_{ult} = 36,000-psi τ_{ult} = 18,000-psi

Diameter of rod d_r = 2-inch
Diameter of bolt d_b = 1.6-inch
Bar width w = 4-inch, thickness t = 1 - inch

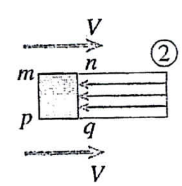




(b)



(c)

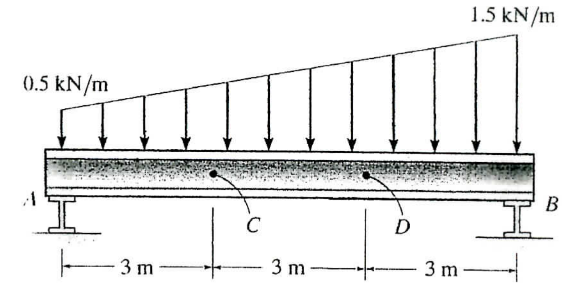


(d)

Internal Bounding forces and Couples:

Date 9-30-'10

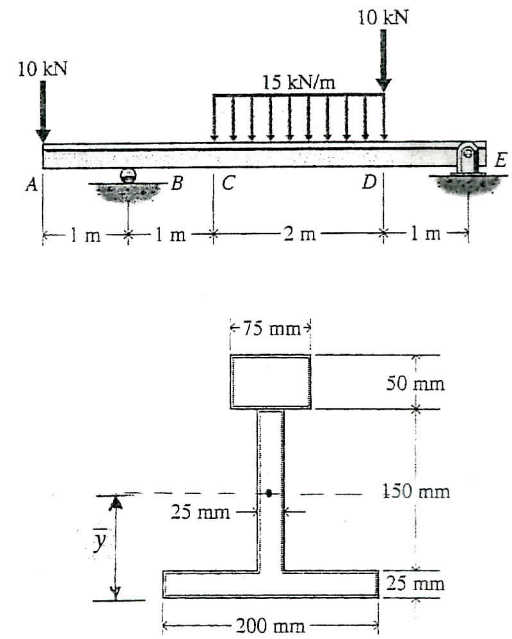
Question 1: (20 Points)
The beam supports the distributed load as shown.
Determine the resultant internal loadings acting on the cross section through point C. Assume the reactions at the supports A and B are vertical only.



$C_X = N_C = 0.00$ kN
$C_Y = V_C = 1.75$ kN (↓)
$M_C = 8.50$ kN-m CCW

Date 4-07-'11

Question 1: (10 Points)
For the beam and loading shown:
a-Find internal forces and couples acting on a section halfway between C and D.
b-If beam has a cross section as shown, find Moment of Inertia of this section about a horizontal axis through the centroid of the section.

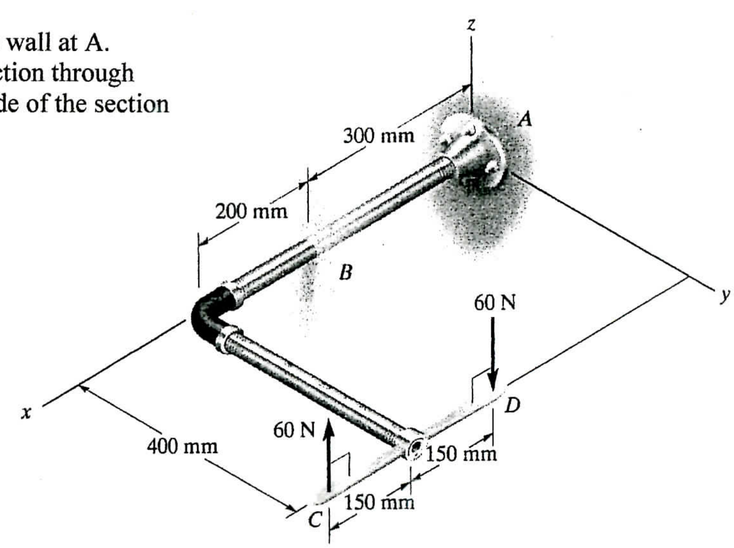


For point G, 1-m right of C:
$G_X = N_G = 0.00$ kN
$G_Y = V_G = 5.00$ kN (↓)
$M_G = 22.5$ kN-m CCW

$I_x = 83.5 \times 10^6\ mm^4$

Date 9-30-'10

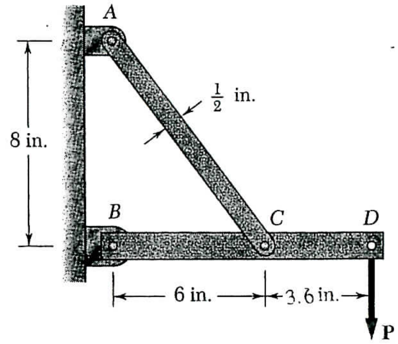
Question 1: (20 Points)
The pipe has a uniform weight of 100-N/m and it is fixed to the wall at A.
Determine the resultant internal loadings acting on the cross section through point B. Show Free-Body-Diagram and be specific on which side of the section your internal actions being calculated.



$B_X = 0.00$ N
$B_Y = 0.00$ N
$B_Z = 60.0$ N
$M_{BX} = 8.00$ N-m
$M_{BY} = 8.00$ N-m
$M_{BZ} = 0.00$ N-m

Date: 6-25-96

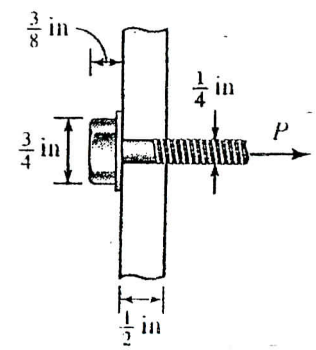
Question 1 : (20 Points)
Bar AC is made of steel with uniform rectangular cross sectional area of 1/4 by 1/2 inch. Rod BCD is rigid. All joints are pin connected with pin-diameter of 5/16-in. Pins A and C are in single shear and pin B is in double shear. If ultimate normal stress in the steel bar is 50-ksi, and the ultimate shear stress in pins is 25-ksi, determine the largest allowable load P when a factor of safety of 2.5 is required.



Answer: P = 383 lb

Date: 1-11-'07

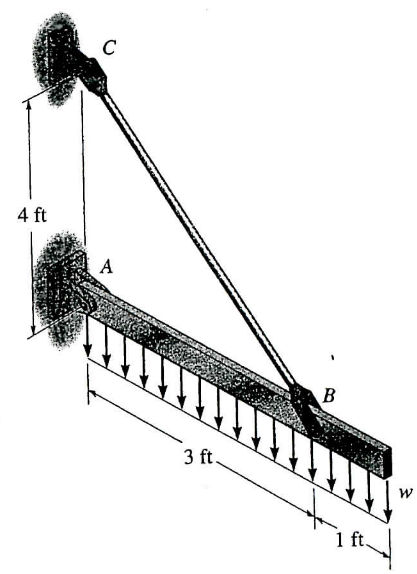
Question 2: (20 Points)
A steel bolt passes through a piece of wood as shown. Find the largest value of axial load P such as the stresses in the wood and bolt are not to exceed the followings:
Maximum allowable **normal stress of steel** = 24 ksi
Maximum allowable **bearing stress of wood** = 3 ksi
Maximum allowable **shear stress of steel** = 12 ksi
Maximum allowable **shear stress of wood** = 1.5 ksi
For each case draw a sketch showing the areas under stress.



Answer: P = 1,178 lbs

Date: 4-11-'02

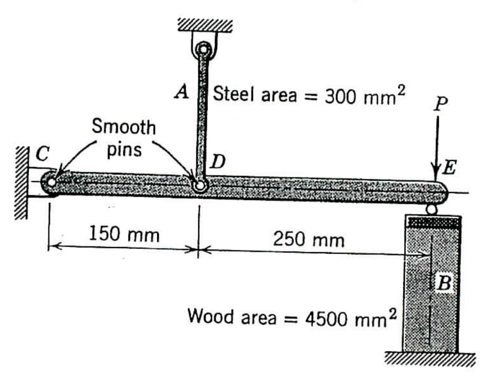
Question 3: (20 Points)
In the assembly shown the intensity of the uniform distributed load is w = 600 lb/ft. If allowable shear stress in the pins A, B and C is not to exceed 12-ksi, and the allowable normal stress in the rod BC is not to exceed 18-ksi, determine:
a- The minimum required diameter of pins A, B and C.
b- The minimum required diameter of rod BC.



Answers : $d_{rod} = 0.376$ in.
$d_A = 0.277$ in.
$d_B = d_C = 0.326$ in.

Date: 9-26-'02

Question 4: (20 Points)
In the assembly shown rod CDE is rigid. After load P is applied, the axial compressive stress in post B is 10-MPa and axial tensile stress in link AD is 200-MPa.
a- Determine magnitude of load P.
b- Find diameters of pins C and D, knowing the allowable shearing stress in the pins is limited to 80-MPa.



Answers : P = 67.5 kN
$d_C = 24.4$ mm
$d_D = 30.9$ mm