SUMMARY:

A- State of Stress

B- Stress Transformation Equations

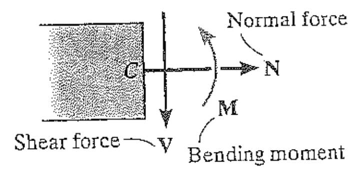
C- Mohr's Circle, Equations

D- Mohr's Circle, Graphical Method

A- State of Stress

Plane State of Stress on the surface of any structural member can be found using principals and concepts presented in Chapters 1 to 8. Examples are followed.

Internal Actions
From Statics Equilibrium



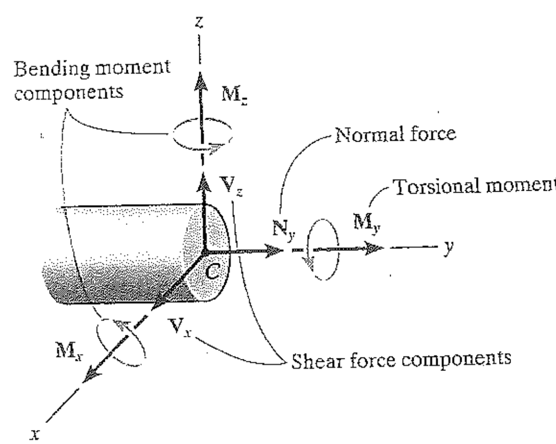
Possible State of Stress in 2-D:

$\sigma = \pm N/A \pm M_z C / I_z$

$\tau = \pm V\, Q/I\, t$

Where N is Axial Force, M_z is Bending Moment, and V is Transverse Shear Force.

Internal Actions
From Statics Equilibrium

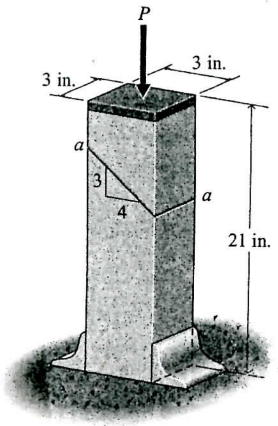


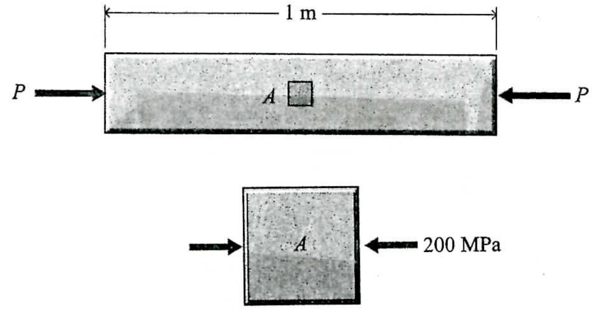
Possible State of Stress in 3-D:

$\sigma = \pm N/A \pm M_z C / I_z \pm M_x C / I_x$

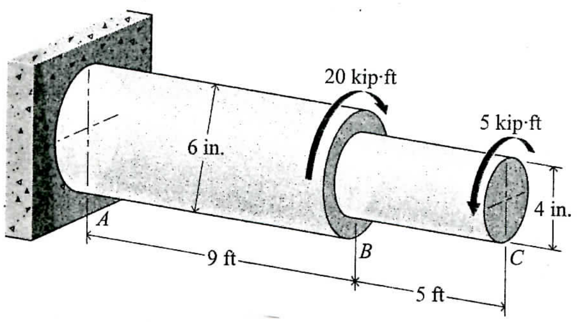
$\tau = \pm TC / J \pm V_z Q / I_z\, t \pm V_x Q / I_x\, t$

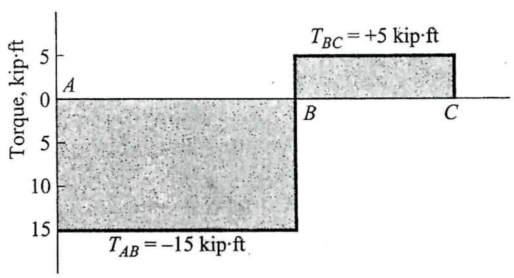
Where N is Axial Force, M_x and M_z are Bending Moments, $T = M_y$ is Torque, and V_z and V_x are Transverse Shear Forces.

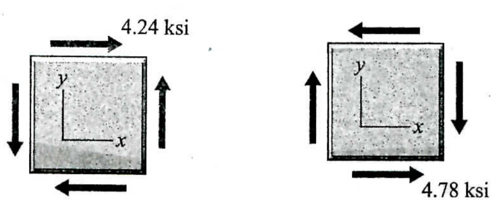




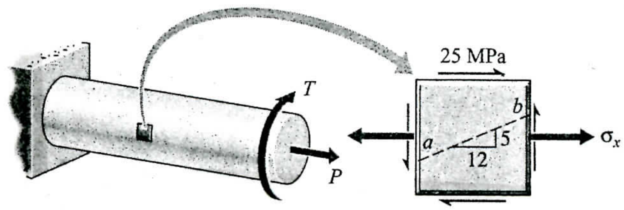
Axial



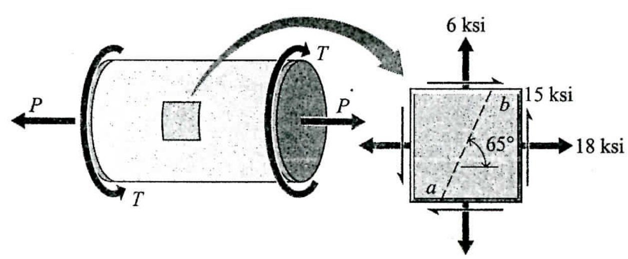




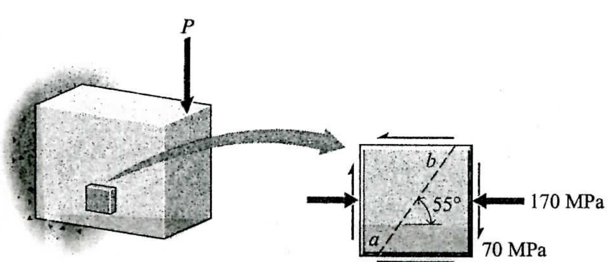
Torque/Torsion



Axial and Torsion Combined

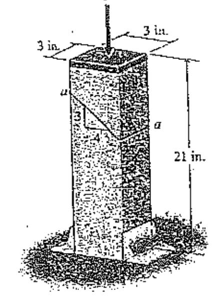


Plane Stress

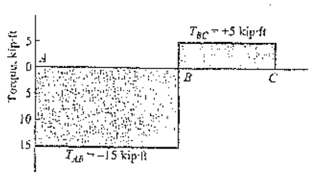


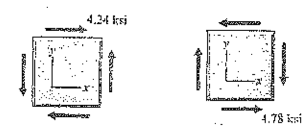
Bending

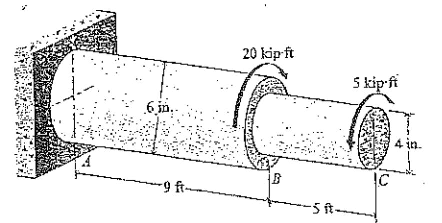
Question 1:
For P = 135-kips determine the state of stress on inclined surface a-a.



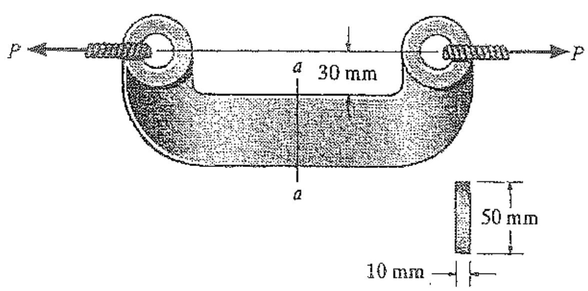
Question 2:
Determine the state of stress on the outside surface of shaft AB and BC.



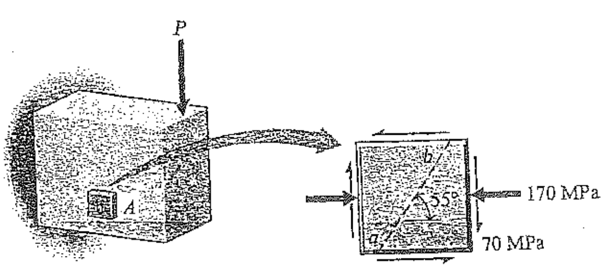




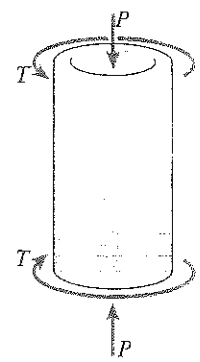
Question 3:
A strain-gage is installed on the top surface of a steel link at section a-a as shown. When load P is applied the reading of the gage is $\varepsilon = +1{,}000 \times 10^{-6}$ mm/mm. Given E = 200-GPa, determine the value of P. Also draw the normal stress distribution in this section.

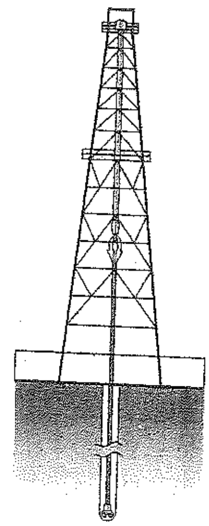


Question 4:
In the cantilever beam shown, for P = 80-kN determine the state of stress at point A 16-mm below the neutral surface and 102-mm from load P. Figure not in scale. Assume height and width of the beam 80-mm and 18-mm respectively.



Question 5:
The hollow drill pipe for a soil auger has 2.5-in outer diameter and ¼ -in thickness. The compressive force P is 5,000-lb and the drilling torque T is 500-lb-ft. Determine the State of Stress on a point on the outside surface of the pipe.



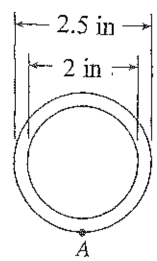


$A = \pi\,(1.25)^2 - \pi\,(1.00)^2 = 1.77\text{-in}^2$

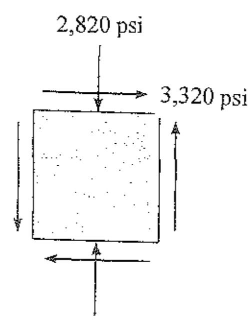
$J = \pi\,(1.25)^4 - \pi\,(1.00)^4 = 2.26\text{-in}^4$

$\sigma_y = -\,(5{,}000) / 1.77 = -\,2{,}820\text{-psi}$

$\tau_{xy} = (\,500 \times 12) \times 1.25 / 2.26 = 3{,}320\text{-psi}$



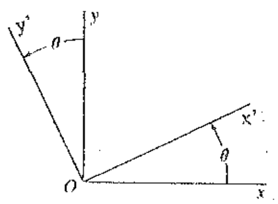
Section

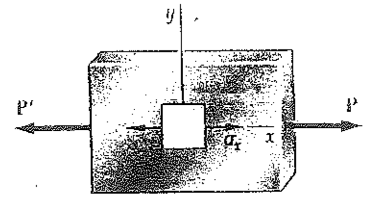


B- Stress Transformation Equations

1- The state of Stress on an inclined surface for an axially loaded member, when the load is along it's x-axis, was driven to be:

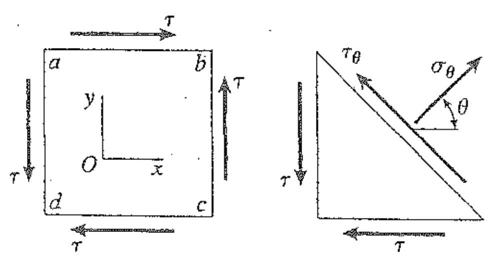
$$\sigma_{x'} = \sigma_{\theta} = \sigma_{x} \cos^{2} \theta \quad \text{and} \quad \tau_{x'y'} = \tau_{\theta} = \sigma_{x} \sin \theta \cos \theta$$

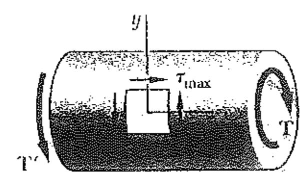




2- Similarly the state of stress on an inclined surface for a circular member under torsion or pure shear stress can be obtained as:

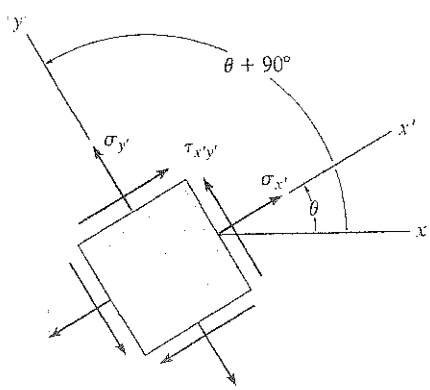
$$\sigma_{x'} = \sigma_{\theta} = \tau_{xy} \sin 2\theta \quad \text{and} \quad \tau_{x'y'} = \tau_{\theta} = \tau_{xy} \cos 2\theta$$

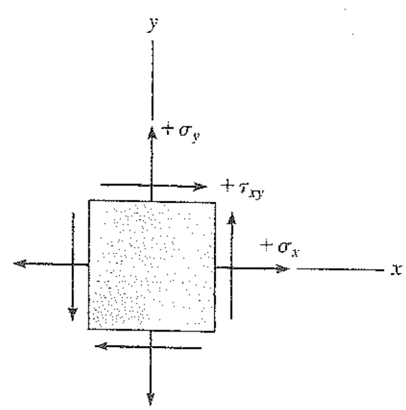




3- Finally for a General Plane Stress using similar approach, the state of stresses along planes perpendicular to the x' and y' axes will be driven to be:

$$\sigma_{x'} = \sigma_{x} \cos^{2} \theta + \sigma_{y} \sin^{2} \theta + 2\tau_{xy} \sin \theta \text{ con } \theta \quad \text{and}$$
$$\tau_{x'y'} = -(\sigma_{x} - \sigma_{y}) \sin \theta \text{ con } \theta + \tau_{xy} (\cos^{2} \theta - \sin^{2} \theta)$$





4- By using trigonometric formulas changing all θ angles to 2θ, the Stress Transformation Equations for a general Plane Stress will be changed to the following format (Equations 7-5, 7-6 and 7-7 in the text). These equations can be used for finding stresses on any inclined surface perpendicular to x' and y' axes at an angle of θ° measured positively from the x-axis

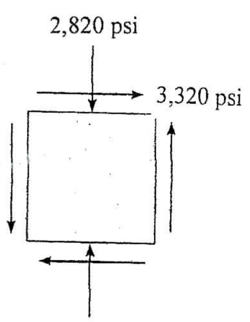
$$\sigma_{x'} = (\sigma_x + \sigma_y)/2 + (\sigma_x - \sigma_y)/2 \ \text{con } 2\theta + \tau_{xy} \sin 2\theta \qquad (7\text{-}5)$$

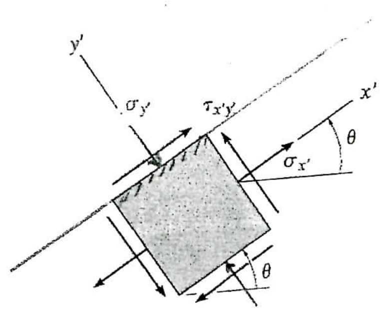
$$\tau_{x'y'} = -(\sigma_x - \sigma_y)/2 \ \sin 2\theta + \tau_{xy} \cos 2\theta \qquad (7\text{-}6)$$

and

$$\sigma_{y'} = (\sigma_x + \sigma_y)/2 - (\sigma_x - \sigma_y)/2 \ \text{con } 2\theta - \tau_{xy} \sin 2\theta \qquad (7\text{-}7)$$

5- Example: Find stresses along the weld line for the auger pipe given θ = 22.5° C.C.W. from the x-axis.





$\sigma_x = 0$
$\sigma_y = -2{,}820$ psi
$\tau_{xy} = 3{,}320$ psi

$\sigma_{x'} = 1{,}934$ psi (increased)
$\sigma_{y'} = -4{,}754$ psi (increased)
$\tau_{x'y'} = 1{,}350$ psi (decreased)

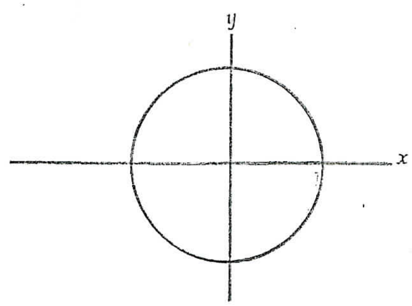
The results show the normal stress along the y' direction or normal to the weld line has increased compared to the original normal stress along the y-axis, therefore it is more critical for the pipe. At the same time the shear stress has decreased. Since the state of stress will change for different value of θ:

<u>We must find the highest value of normal stress σ and highest value of shear stressτ, and the planes associated with those values.</u>

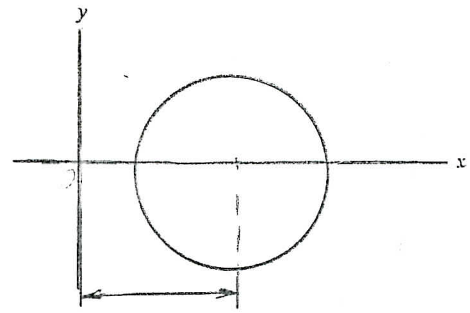
C- Mohr's Circle, Equations

If we are interested to find the highest values of normal and shear stresses in previous example and their planes, we can plot Equations 7-5 and 7-6 for θ from 0 to 180° with increments of 1° or less. Not a practical approach.

Equation of a circle in x-y co-ordinate system is:



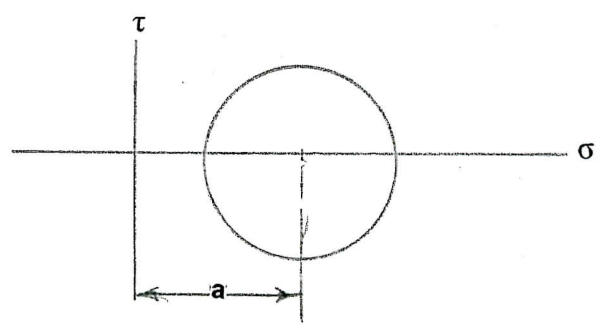
$$x^2 + y^2 = R^2$$



$$(x - a)^2 + y^2 = R^2$$

If we change the x and y axes to σ and τ respectively, the equation for the second circle would be:

$$(\sigma - a)^2 + \tau^2 = R^2$$



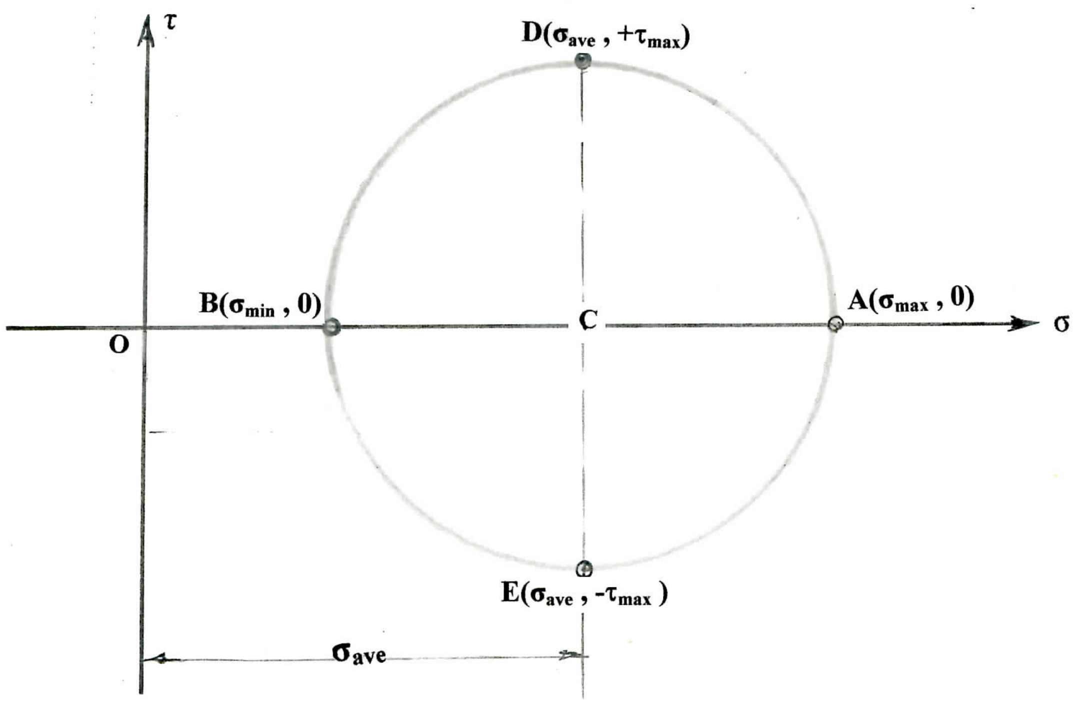
Equations 7-5 and 7-6 represent such a circle. To eliminate the angle θ from these equations we need to square both equations and add them together. The result is:

$$(\sigma - \sigma_{ave})^2 + \tau^2 = R^2$$

where

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} \quad \text{and} \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

This circle represents all the stresses at a point on the surface of a structure for every possible inclined surface or planes specified by a set of axes such as x' and y'. To plot the circle we need only the center of the circle at $\sigma = \sigma_{ave}$ and the radius R. Points A and B show two perpendicular planes with the maximum and minimum values of normal stress σ, which are called Principal Plans. Points D and E represents two perpendicular planes with the highest possible numerical value for the in-plane shear stress τ. Since points A and B correspond to zero shear stress, the direction of the principal planes or θ_P can be determine by setting Equation 7-6 equal to zero. Similarly for points D and E we need to set Equation 7-5 equal to $\sigma_{ave} = (\sigma x + \sigma y)/2$, which will give us the θ_S or the directions of the maximum shear planes.



$$\sigma_A = \sigma_{max} = \sigma_{ave} + R$$

$$\sigma_B = \sigma_{min} = \sigma_{ave} - R$$

$$\tan 2\theta_P = 2\tau_{xy} / (\sigma_x - \sigma_y)$$

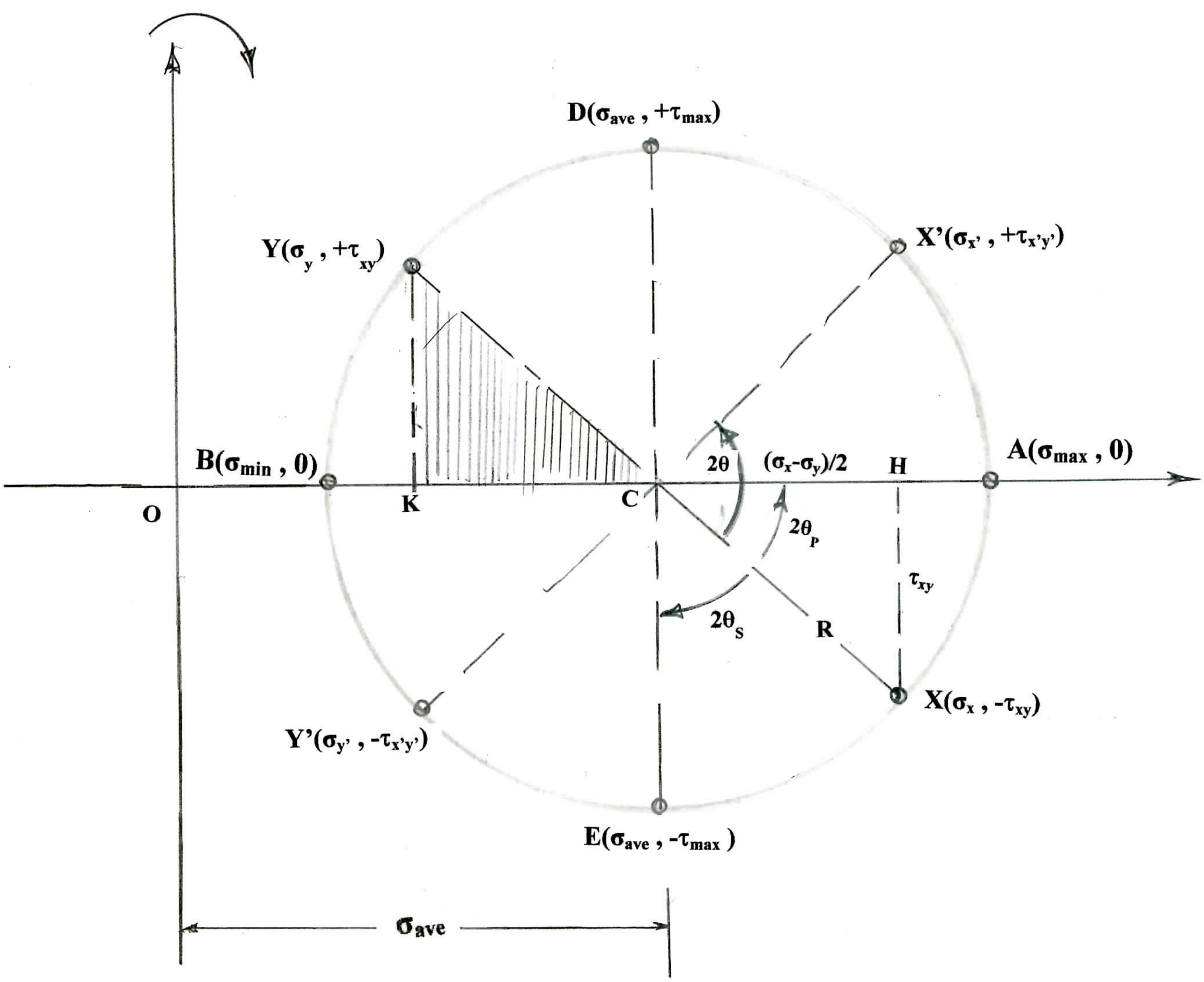
$$\tan 2\theta_S = -(\sigma_x - \sigma_y) / 2\tau_{xy}$$

Mohr's Circle – Graphical Method:

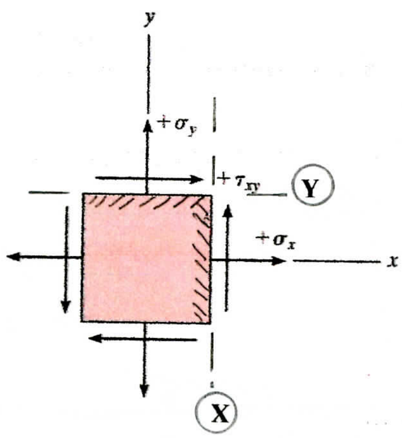
To plot the Mohr's Circle we can use equations given previously or use the following graphical method.

First we plot the stresses on the plane perpendicular to the x-axis on the circle(x-plane), and do the same for the plane perpendicular to the y-axis(y-plane). For the shear stress we need to consider the direction of arrow as positive and oppose to that as negative. We connect these two points to find the center of the circle C. The property of triangles CXH or CYK will give us values of R, θ_P , θ_S , σ_{max} , σ_{min} and τ_{max} without need for any equations.

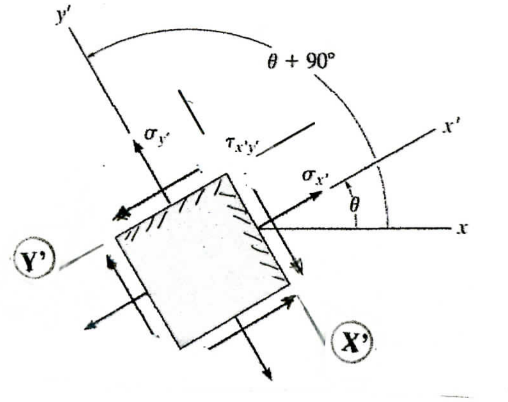
From the circle we can find stresses on any other plane perpendicular to x' and y' axis, when θ is given, by calculating σ and τ for points X' and Y' in place of Equations 7-5 to 7-7.



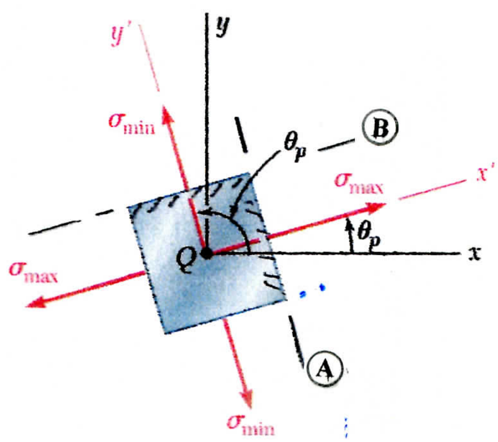
Stresses on different planes from the Mohr's Circle:



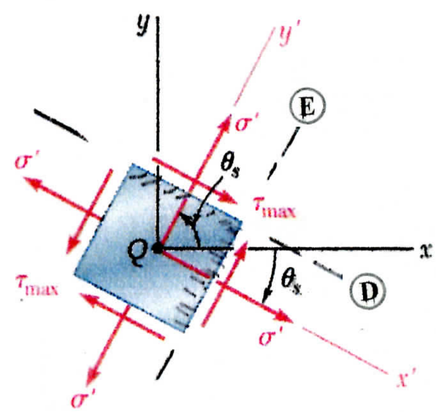
Original State of Stress
x-y Plane



State of Stress at θ° ccw
x'-y' Plane

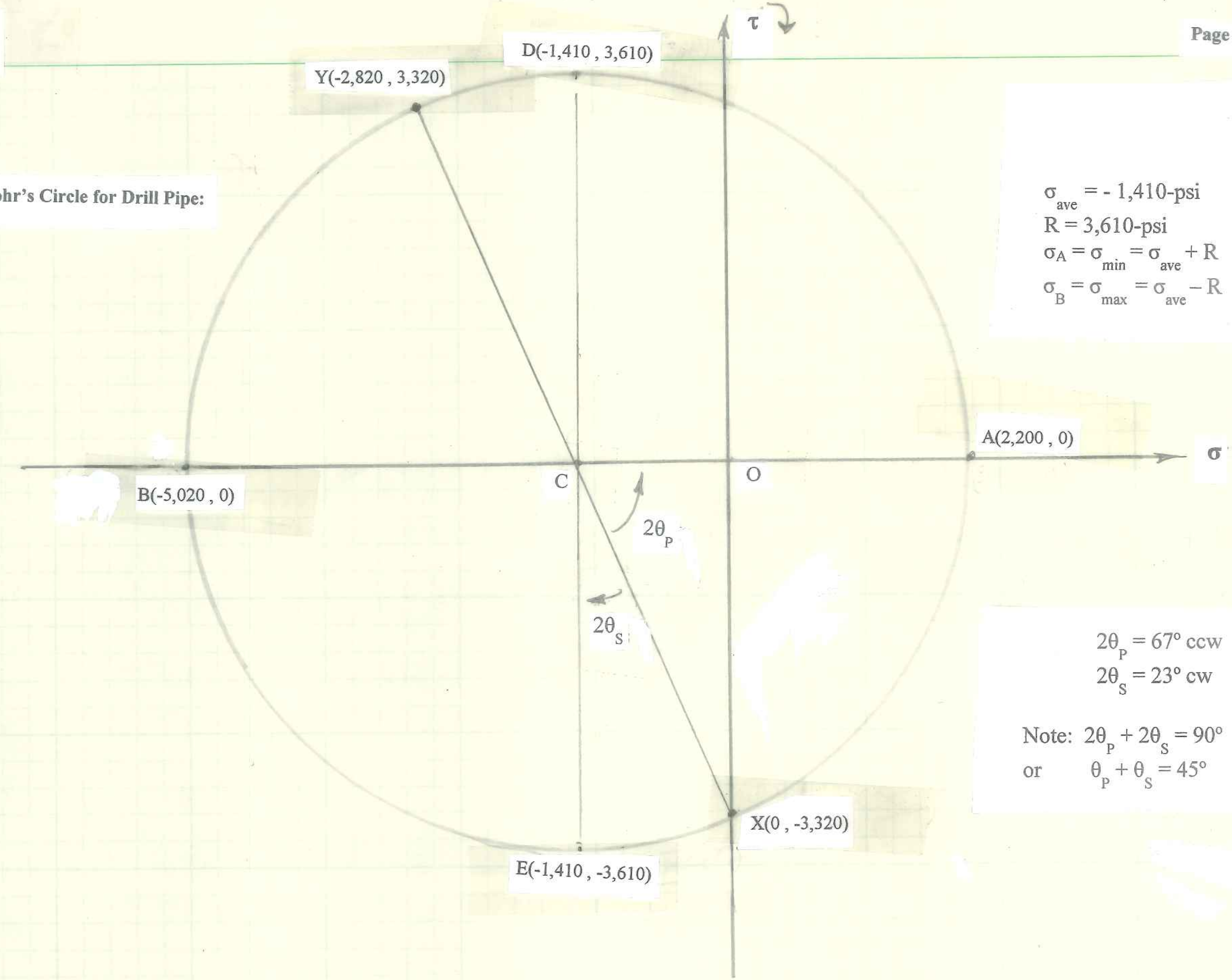


Principal Stresses
x'-y' Plane at θ_p
A-B Plane



Maximum in-Plane Shear
x'-y' Plane at θ_s
D-E Plane

Mohr's Circle for Drill Pipe:

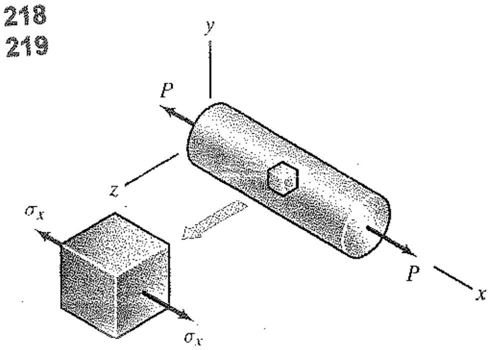


$\sigma_{ave} = -\ 1{,}410\text{-psi}$
$R = 3{,}610\text{-psi}$
$\sigma_A = \sigma_{min} = \sigma_{ave} + R$
$\sigma_B = \sigma_{max} = \sigma_{ave} - R$

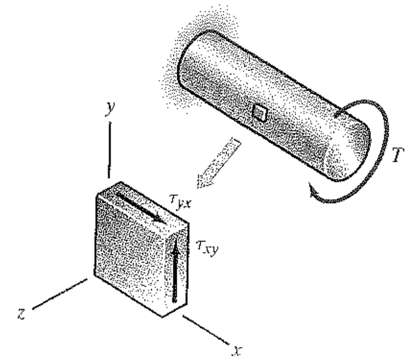
$2\theta_P = 67°$ ccw
$2\theta_S = 23°$ cw

Note: $2\theta_P + 2\theta_S = 90°$
or $\theta_P + \theta_S = 45°$

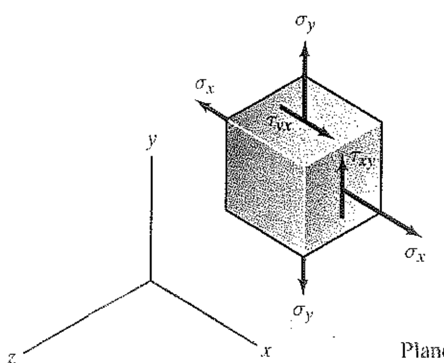
SUMMARY

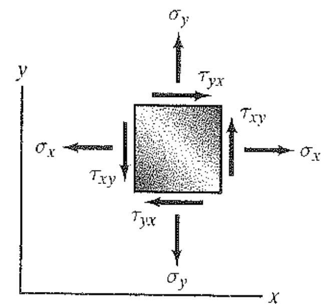


Stresses on an element of a bar subjected to axial forces.

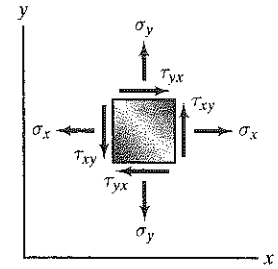


Stresses on an element of a bar subjected to torsion.

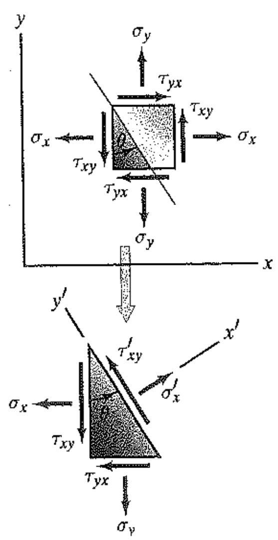




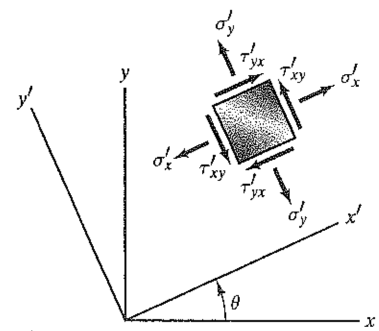
Plane stress.



(a)



Free-body diagram for determining σ_x' and τ_{xy}'.



(b)

Stress components (a) in terms of the xyz coordinate system; (b) in terms of the $x'y'z'$ coordinate system.

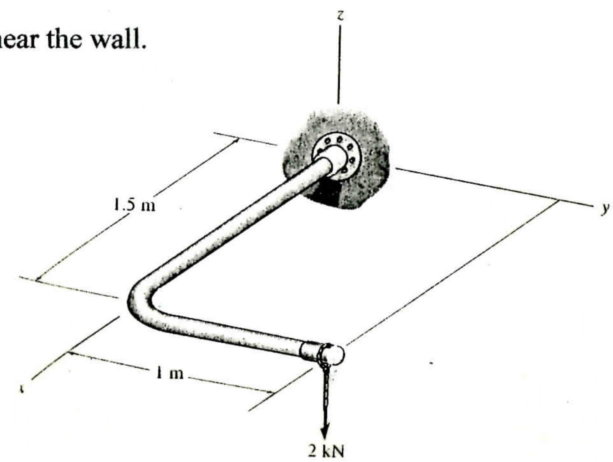
$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta,$$

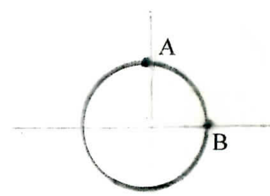
$$\tau_{xy}' = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta.$$

$$\sigma_y' = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta.$$

NAME:_______________________________ **SID #**________________________

Question 1:
The bent solid rod has a radius of 40-mm and is fixed to the wall.
a-The state of the stress at points A and B outside surface of the pipe near the wall.
Show the results on a volume element at each points.
b-The principal stresses and maximum shear stress.
c-The principal planes and plane of maximum shear.





Question 2:
Several forces are applied to the pipe assembly shown. If the inner and outer diameters of the pipe are equal to 1.50-in. and 1.75-in. respectively, determine:
a-The state of the stress at point H on the top outside surface of the pipe.
Show the results on a volume element.
b-The principal stresses and maximum shear stress.
c-The principal planes and plane of maximum shear.