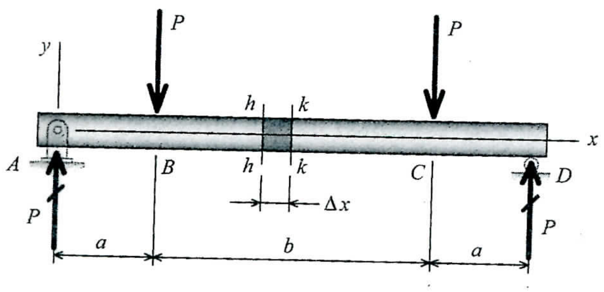
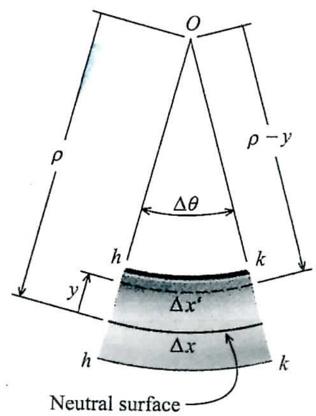
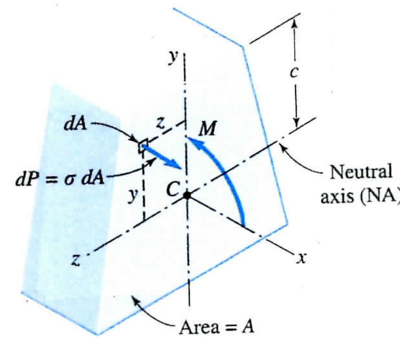
Introduction:



Pure bending in a region of a beam





Flexural deformation of a beam segment (Δx)
Upper parts in Compression, lower parts in Tension

Definitions:

Neutral Surface: Neither shorten nor elongation. Stress and Strain both zero.
Neutral axis: Where neutral surface intersects with the cross section of the beam.
O is Center of Curvature, ρ Radius of Curvature and $1/\rho$ is Curvature.

$$\boldsymbol{\sigma} = \text{Normal Force/Area} = dF/dA \quad \text{or} \quad dF = \boldsymbol{\sigma}\, dA$$

The sum of the normal forces dF over entire cross sectional area is zero. Therefore:

(1) $\int_A dF = \int_A \boldsymbol{\sigma}\, dA = 0$

The moment of the force dF about the neutral axis is: $dM_z = y\, dF = y\, \boldsymbol{\sigma}\, dA$

The integral of dM_z over the entire cross sectional area must be equal to applied moment M:

(2) $M = \int_A y\, dF = \int_A y\, \boldsymbol{\sigma}\, dA$

Other relevant formulas:

Normal Strain: $\varepsilon = \sigma / E$ or $\sigma = \varepsilon E$
Moment of Inertia about the x-axis, in this case z-axis, of the cross section area: $I_{NA} = \int_A y^2 dA$
Q The First moment of an area about the axis that passes through the centroid of that area is zero or

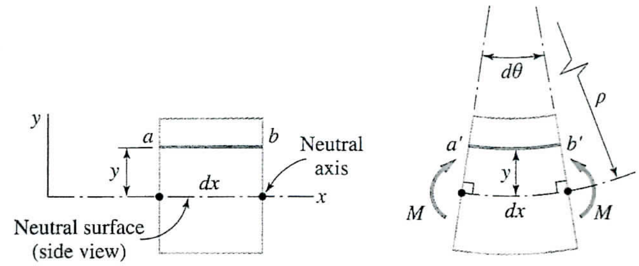
(3) $\int_A y dA = 0$

When moment M is applied to a segment of a beam, the relationship between the normal stress σ and the normal strain ε can be written as follows:

Δx the length before deformation becomes $\Delta x'$ after deformation and the strain ε becomes:

$\varepsilon = \delta/L$ = change in length/ original length = $(\Delta x' - \Delta x) / \Delta x$

However in triangle Oa'b': $\tan \Delta\theta = \Delta\theta = \Delta x' / (\rho - y)$ or
$\Delta x' = (\rho - y)\,\Delta\theta$, Similarly $\Delta x = \rho\,\Delta\theta$.
Therefore: $\varepsilon = [(\rho - y)\,\Delta\theta - \rho\,\Delta\theta] / \rho\,\Delta\theta$ or:



Before deformation After deformation

(4) $\varepsilon = - y / \rho$

Note that on the neutral axis where $y = 0$, $\varepsilon = 0$ Therefore no Strain and no Stress and at the top and the bottom of the beam where $y = C$, ε is maximum equal to:

(5) $\varepsilon_{max} = - C/ \rho$

Dividing equation (4) by equation (5) we obtain $\varepsilon / \varepsilon_{max} = y/C$ or $\varepsilon = \varepsilon_{max}\, y/C$.
Multiply both sides of this equation by E, modulus of elasticity, we get: $E\,\varepsilon = E\,\varepsilon_{max}\, y/C$ or :

(6) $\sigma = \sigma_{max}\, y/C$.

Again note that on the neutral axis $y = 0$, $\sigma = 0$ no Normal Stress.
At the top or the bottom of the beam where $y = C$, $\sigma = \sigma_{max}$
Replacing σ in equation (1) by equation (6), we get:

$$\int_A dF = \int_A \sigma\, dA = \int_A \sigma_{max}\, y/C\, dA = (\sigma_{max} / C) \int_A y\,dA = 0$$

Since σ_{max} can not be zero, then: $\int_A y\,dA = Q = 0$ or

Neutral axis passes through the centroid of the cross section.

Replacing σ in equation (2) by equation (6):

$$M = \int_A y\, dF = \int_A y\, \sigma\, dA = = \int_A y\, \sigma_{max}\, y/C\ dA = (\sigma_{max} / C) \int_A y^2 dA = (\sigma_{max} / C)\, I_{NA} \text{ or}$$

Maximum normal stress at the top or the bottom of the beam is :

$$\boxed{\sigma_{max} = \frac{|M_{max}|c}{I}}$$

Substituting Equation (6) into this equation, the

Normal stress at any other point in the cross section is:

$$\boxed{\sigma = -\frac{My}{I}}$$

Substituting Equation (5) into this equation, **The curvature of the beam is:**

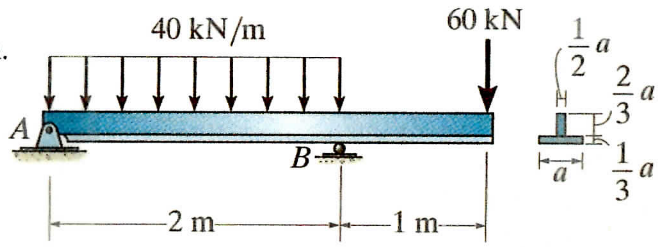
$$\boxed{\frac{1}{\rho} = \frac{M}{EI}}$$

Stresses in beams – Composite Section

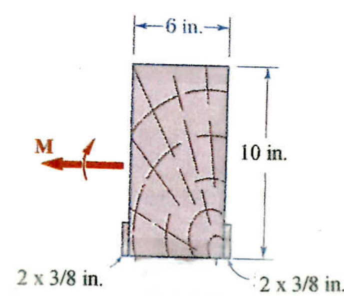
NAME:______________________________ **SID #**______________________

Question 1:
For the beam and loading shown:
a-Draw shear and Moment diagrams.
b-Find maximum tensile and compressive stresses in the cross section shown.
c-Write equations of Shear and Moments for both segments of the beam.

Assume a = 180-mm



Question 2:
The cross sectional area of a wood beam is 6" wide with the height of 10". The beam is reinforced with two 2" height and 3/8" thick steel plates attached to the lower sides of the beam. A positive moment of M = 200-kips-in is applied to this cross section. Knowing E_{wood} = 1,500-ksi and E_{steel} = 30,000-ksi, determine the maximum tensile and compressive stresses in each materials.



Sign Conventions for Shear-Force and Bending Moment Diagrams

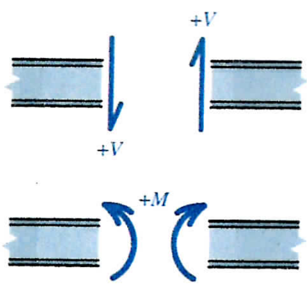


FIGURE 7.6 Sign conventions for internal shear force V and bending moment M.

> **A positive internal shear force V**
> - acts downward on the right-hand face of a beam.
> - acts upward on the left-hand face of a beam.
>
> **A positive internal bending moment M**
> - acts counterclockwise on the right-hand face of a beam.
> - acts clockwise on the left-hand face of a beam.

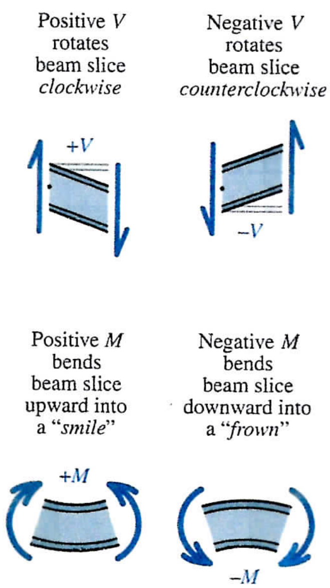


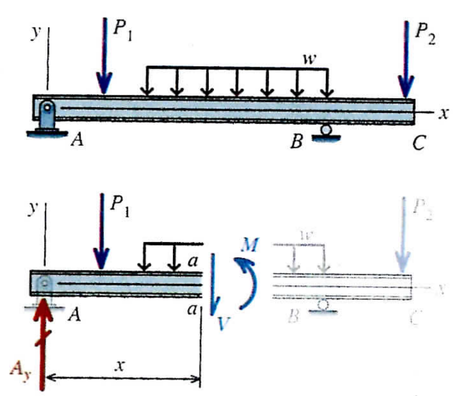
FIGURE 7.7 Sign conventions for V and M shown on beam slice.

> **A positive internal shear force V causes a beam element to rotate clockwise.**
> **A positive internal bending moment M bends a beam element concave upward.**

Free-Body-Diagram for a section of the beam

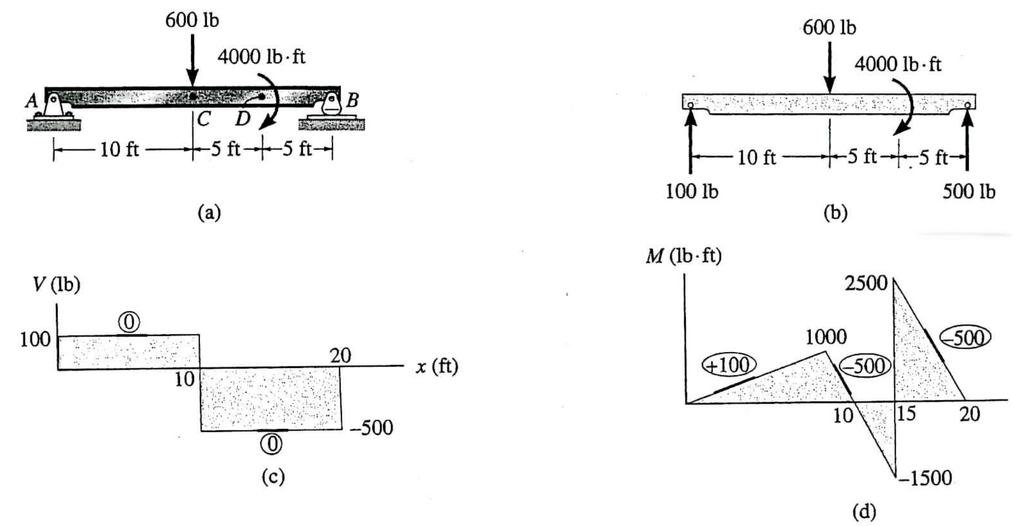
The internal Shear-Force V and Bending-Moment M must be drawn in **positive deformation format** according to the above sign convention.

The values of V and M vary along the length of the beam. They depend on the distance x, reaction at A, load P_1 and a portion of w load.



Examples:

Draw shear and moment diagrams for the beams shown.



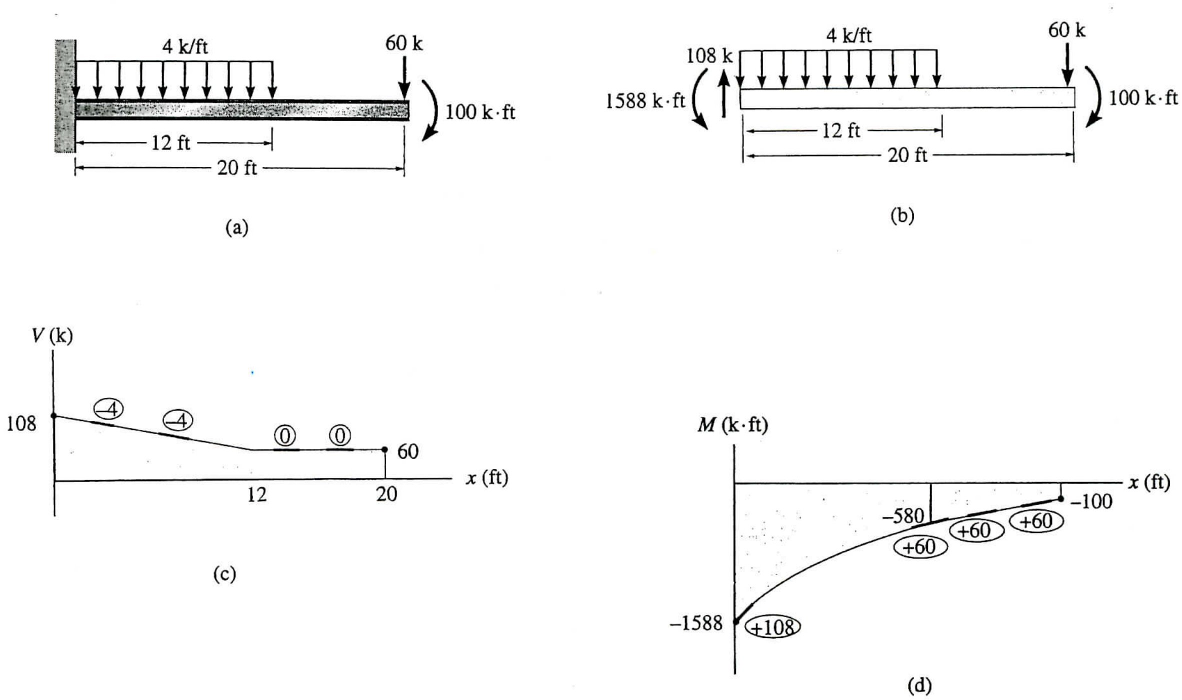
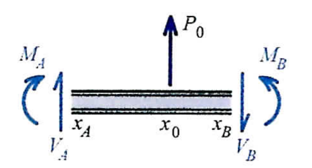
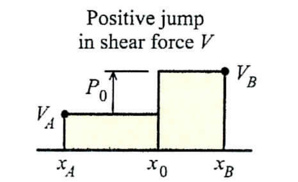
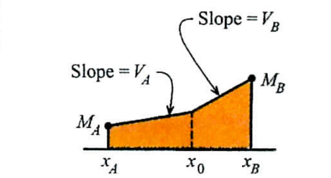
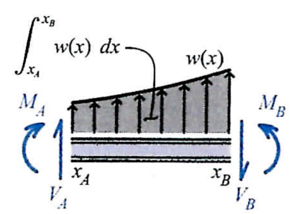
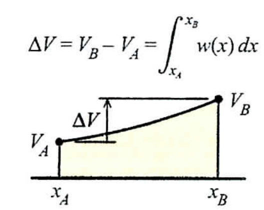
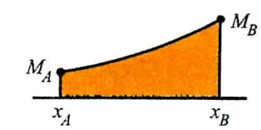
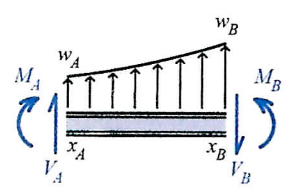
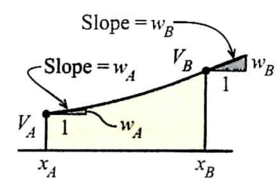
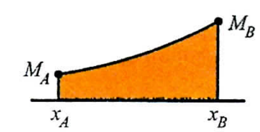
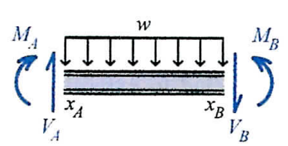
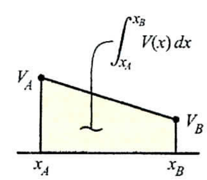
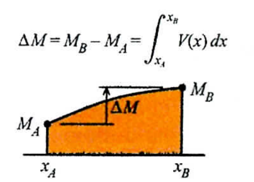
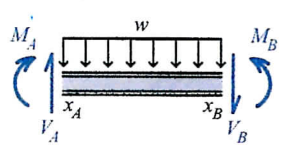
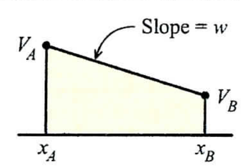
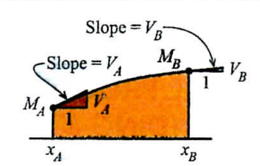
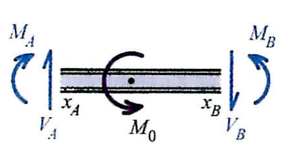
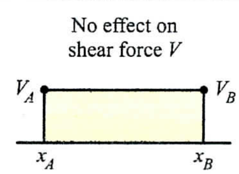
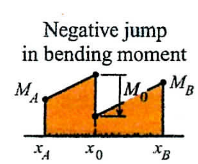
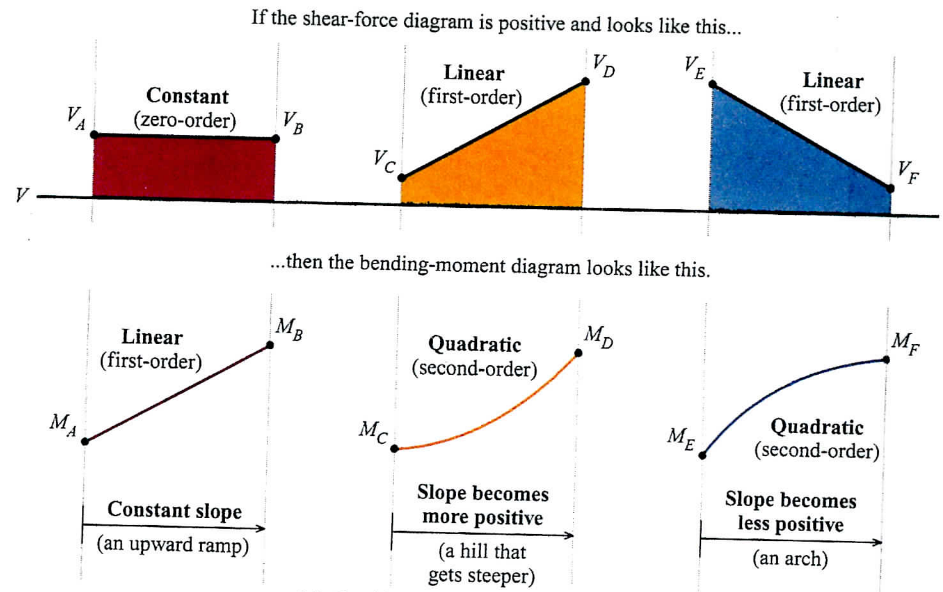
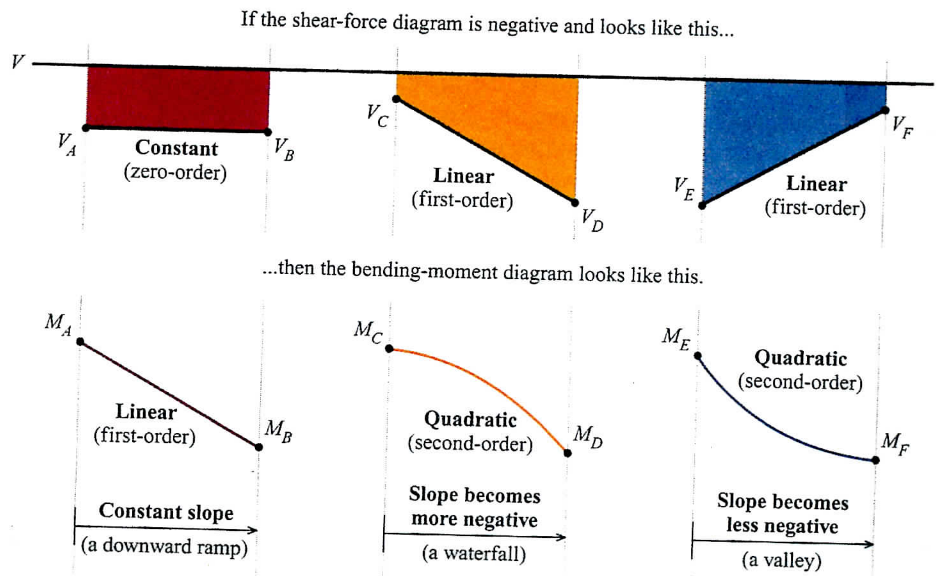


Table 7.1 Construction Rules for Shear-Force and Bending-Moment Diagrams

Equation	Load Diagram w	Shear-Force Diagram V	Bending-Moment Diagram M
Rule 1: Concentrated loads create discontinuities in the shear-force diagram. [Eq. (7.5)]			
$\Delta V = P_0$			
Rule 2: The change in shear force is equal to the area under the distributed-load curve. [Eq. (7.3)]			
$V_B - V_A = \int_{x_A}^{x_B} w(x)\,dx$			
Rule 3: The slope of the V diagram is equal to the intensity of the distributed load w. [Eq. (7.1)]			
$\dfrac{dV}{dx} = w(x)$			
Rule 4: The change in bending moment is equal to the area under the shear-force diagram. [Eq. (7.4)]			
$M_B - M_A = \int_{x_A}^{x_B} V\,dx$			
Rule 5: The slope of the M diagram is equal to the intensity of the shear force V. [Eq. (7.2)]			
$\dfrac{dM}{dx} = V$			
Rule 6: Concentrated moments create discontinuities in the bending-moment diagram. [Eq. (7.6)]			
$\Delta M = -M_0$			

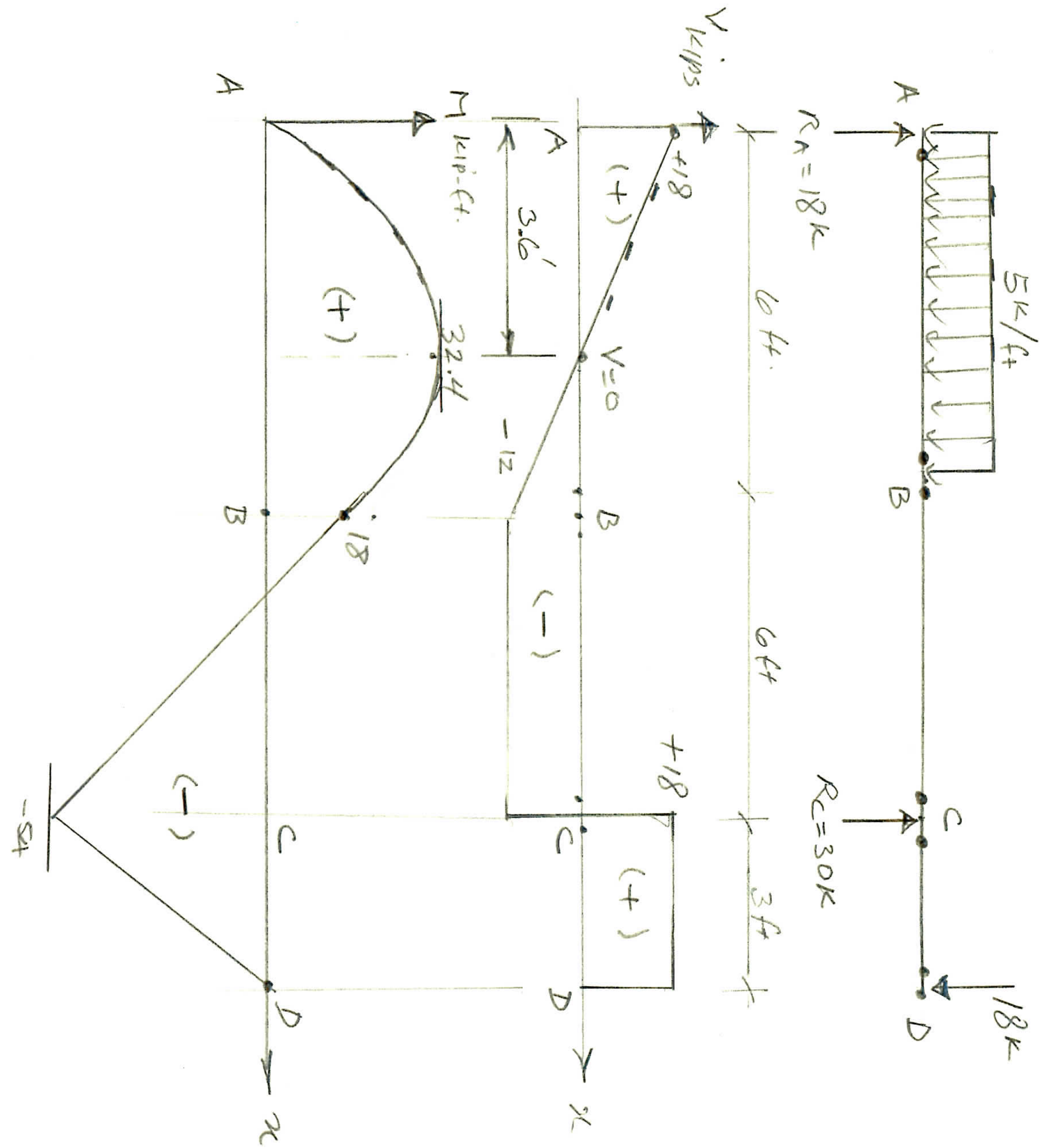


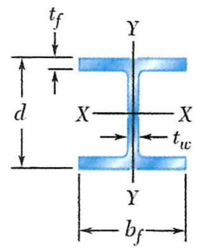
(*a*) Positive shear-force diagrams



(*b*) Negative shear-force diagrams

Relationships between *V* and *M* diagram shapes.


A
B
C
D
5k/ft
$R_A = 18k$
$R_C = 30k$
18k
6 ft
6 ft
3 ft
V: kips
+18
(+)
V=0
-12
(−)
+18
(+)
x
M kip-ft
3.6'
32.4
(+)
18
(−)
-54
x



Appendix C Properties of Rolled-Steel Shapes

(U.S. Customary Units)

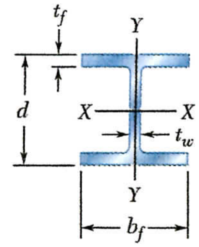
Continued from page A18

W Shapes

(Wide-Flange Shapes)

			Flange		Web	Axis X-X			Axis Y-Y		
Designation†	Area A, in^2	Depth d, in.	Width b_f, in.	Thickness t_f, in.	Thickness t_w, in.	I_x, in^4	S_x, in^3	r_x, in.	I_y, in^4	S_y, in^3	r_y, in.
W12 × 96	28.2	12.7	12.2	0.900	0.550	833	131	5.44	270	44.4	3.09
72	21.1	12.3	12.0	0.670	0.430	597	97.4	5.31	195	32.4	3.04
50	14.6	12.2	8.08	0.640	0.370	391	64.2	5.18	56.3	13.9	1.96
40	11.7	11.9	8.01	0.515	0.295	307	51.5	5.13	44.1	11.0	1.94
35	10.3	12.5	6.56	0.520	0.300	285	45.6	5.25	24.5	7.47	1.54
30	8.79	12.3	6.52	0.440	0.260	238	38.6	5.21	20.3	6.24	1.52
26	7.65	12.2	6.49	0.380	0.230	204	33.4	5.17	17.3	5.34	1.51
22	6.48	12.3	4.03	0.425	0.260	156	25.4	4.91	4.66	2.31	0.848
16	4.71	12.0	3.99	0.265	0.220	103	17.1	4.67	2.82	1.41	0.773
W10 × 112	32.9	11.4	10.4	1.25	0.755	716	126	4.66	236	45.3	2.68
68	20.0	10.4	10.1	0.770	0.470	394	75.7	4.44	134	26.4	2.59
54	15.8	10.1	10.0	0.615	0.370	303	60.0	4.37	103	20.6	2.56
45	13.3	10.1	8.02	0.620	0.350	248	49.1	4.32	53.4	13.3	2.01
39	11.5	9.92	7.99	0.530	0.315	209	42.1	4.27	45.0	11.3	1.98
33	9.71	9.73	7.96	0.435	0.290	171	35.0	4.19	36.6	9.20	1.94
30	8.84	10.5	5.81	0.510	0.300	170	32.4	4.38	16.7	5.75	1.37
22	6.49	10.2	5.75	0.360	0.240	118	23.2	4.27	11.4	3.97	1.33
19	5.62	10.2	4.02	0.395	0.250	96.3	18.8	4.14	4.29	2.14	0.874
15	4.41	10.0	4.00	0.270	0.230	68.9	13.8	3.95	2.89	1.45	0.810
W8 × 58	17.1	8.75	8.22	0.810	0.510	228	52.0	3.65	75.1	18.3	2.10
48	14.1	8.50	8.11	0.685	0.400	184	43.2	3.61	60.9	15.0	2.08
40	11.7	8.25	8.07	0.560	0.360	146	35.5	3.53	49.1	12.2	2.04
35	10.3	8.12	8.02	0.495	0.310	127	31.2	3.51	42.6	10.6	2.03
31	9.12	8.00	8.00	0.435	0.285	110	27.5	3.47	37.1	9.27	2.02
28	8.24	8.06	6.54	0.465	0.285	98.0	24.3	3.45	21.7	6.63	1.62
24	7.08	7.93	6.50	0.400	0.245	82.7	20.9	3.42	18.3	5.63	1.61
21	6.16	8.28	5.27	0.400	0.250	75.3	18.2	3.49	9.77	3.71	1.26
18	5.26	8.14	5.25	0.330	0.230	61.9	15.2	3.43	7.97	3.04	1.23
15	4.44	8.11	4.01	0.315	0.245	48.0	11.8	3.29	3.41	1.70	0.876
13	3.84	7.99	4.00	0.255	0.230	39.6	9.91	3.21	2.73	1.37	0.843
W6 × 25	7.34	6.38	6.08	0.455	0.320	53.4	16.7	2.70	17.1	5.61	1.52
20	5.87	6.20	6.02	0.365	0.260	41.4	13.4	2.66	13.3	4.41	1.50
16	4.74	6.28	4.03	0.405	0.260	32.1	10.2	2.60	4.43	2.20	0.967
12	3.55	6.03	4.00	0.280	0.230	22.1	7.31	2.49	2.99	1.50	0.918
9	2.68	5.90	3.94	0.215	0.170	16.4	5.56	2.47	2.20	1.11	0.905
W5 × 19	5.56	5.15	5.03	0.430	0.270	26.3	10.2	2.17	9.13	3.63	1.28
16	4.71	5.01	5.00	0.360	0.240	21.4	8.55	2.13	7.51	3.00	1.26
W4 × 13	3.83	4.16	4.06	0.345	0.280	11.3	5.46	1.72	3.86	1.90	1.00

†A wide-flange shape is designated by the letter W followed by the nominal depth in inches and the weight in pounds per foot.



Appendix C Properties of Rolled-Steel Shapes

(SI Units)

Continued from page A19

W Shapes

(Wide-Flange Shapes)

Designation[†]	Area A, mm²	Depth d, mm	Flange: Width b_f, mm	Flange: Thickness t_f, mm	Web Thickness t_w, mm	Axis X-X: I_x 10^6 mm⁴	Axis X-X: S_x 10^3 mm³	Axis X-X: r_x mm	Axis Y-Y: I_y 10^6 mm⁴	Axis Y-Y: S_y 10^3 mm³	Axis Y-Y: r_y mm
W310 × 143	18200	323	310	22.9	14.0	347	2150	138	112	728	78.5
107	13600	312	305	17.0	10.9	248	1600	135	81.2	531	77.2
74	9420	310	205	16.3	9.40	163	1050	132	23.4	228	49.8
60	7550	302	203	13.1	7.49	128	844	130	18.4	180	49.3
52	6650	318	167	13.2	7.62	119	747	133	10.2	122	39.1
44.5	5670	312	166	11.2	6.60	99.1	633	132	8.45	102	38.6
38.7	4940	310	165	9.65	5.84	84.9	547	131	7.20	87.5	38.4
32.7	4180	312	102	10.8	6.60	64.9	416	125	1.94	37.9	21.5
23.8	3040	305	101	6.73	5.59	42.9	280	119	1.17	23.1	19.6
W250 × 167	21200	290	264	31.8	19.2	298	2060	118	98.2	742	68.1
101	12900	264	257	19.6	11.9	164	1240	113	55.8	433	65.8
80	10200	257	254	15.6	9.4	126	983	111	42.9	338	65.0
67	8580	257	204	15.7	8.89	103	805	110	22.2	218	51.1
58	7420	252	203	13.5	8.00	87.0	690	108	18.7	185	50.3
49.1	6260	247	202	11.0	7.37	71.2	574	106	15.2	151	49.3
44.8	5700	267	148	13.0	7.62	70.8	531	111	6.95	94.2	34.8
32.7	4190	259	146	9.14	6.10	49.1	380	108	4.75	65.1	33.8
28.4	3630	259	102	10.0	6.35	40.1	308	105	1.79	35.1	22.2
22.3	2850	254	102	6.86	5.84	28.7	226	100	1.20	23.8	20.6
W200 × 86	11000	222	209	20.6	13.0	94.9	852	92.7	31.3	300	53.3
71	9100	216	206	17.4	10.2	76.6	708	91.7	25.3	246	52.8
59	7550	210	205	14.2	9.14	60.8	582	89.7	20.4	200	51.8
52	6650	206	204	12.6	7.87	52.9	511	89.2	17.7	174	51.6
46.1	5880	203	203	11.0	7.24	45.8	451	88.1	15.4	152	51.3
41.7	5320	205	166	11.8	7.24	40.8	398	87.6	9.03	109	41.1
35.9	4570	201	165	10.2	6.22	34.4	342	86.9	7.62	92.3	40.9
31.3	3970	210	134	10.2	6.35	31.3	298	88.6	4.07	60.8	32.0
26.6	3390	207	133	8.38	5.84	25.8	249	87.1	3.32	49.8	31.2
22.5	2860	206	102	8.00	6.22	20.0	193	83.6	1.42	27.9	22.3
19.3	2480	203	102	6.48	5.84	16.5	162	81.5	1.14	22.5	21.4
W150 × 37.1	4740	162	154	11.6	8.13	22.2	274	68.6	7.12	91.9	38.6
29.8	3790	157	153	9.27	6.60	17.2	220	67.6	5.54	72.3	38.1
24	3060	160	102	10.3	6.60	13.4	167	66.0	1.84	36.1	24.6
18	2290	153	102	7.11	5.84	9.20	120	63.2	1.24	24.6	23.3
13.5	1730	150	100	5.46	4.32	6.83	91.1	62.7	0.916	18.2	23.0
W130 × 28.1	3590	131	128	10.9	6.86	10.9	167	55.1	3.80	59.5	32.5
23.8	3040	127	127	9.14	6.10	8.91	140	54.1	3.13	49.2	32.0
W100 × 19.3	2470	106	103	8.76	7.11	4.70	89.5	43.7	1.61	31.1	25.4

[†]A wide-flange shape is designated by the letter W followed by the nominal depth in millimeters and the mass in kilograms per meter.