ME219

Background:

1- Strains due to Normal Stresses:
   ε_x , ε_y, ε_z

2- Strains due to Shear Stresses:
   γ_{xy}, γ_{yz}, γ_{xz}

3- State of Stress: **Plane Stress**

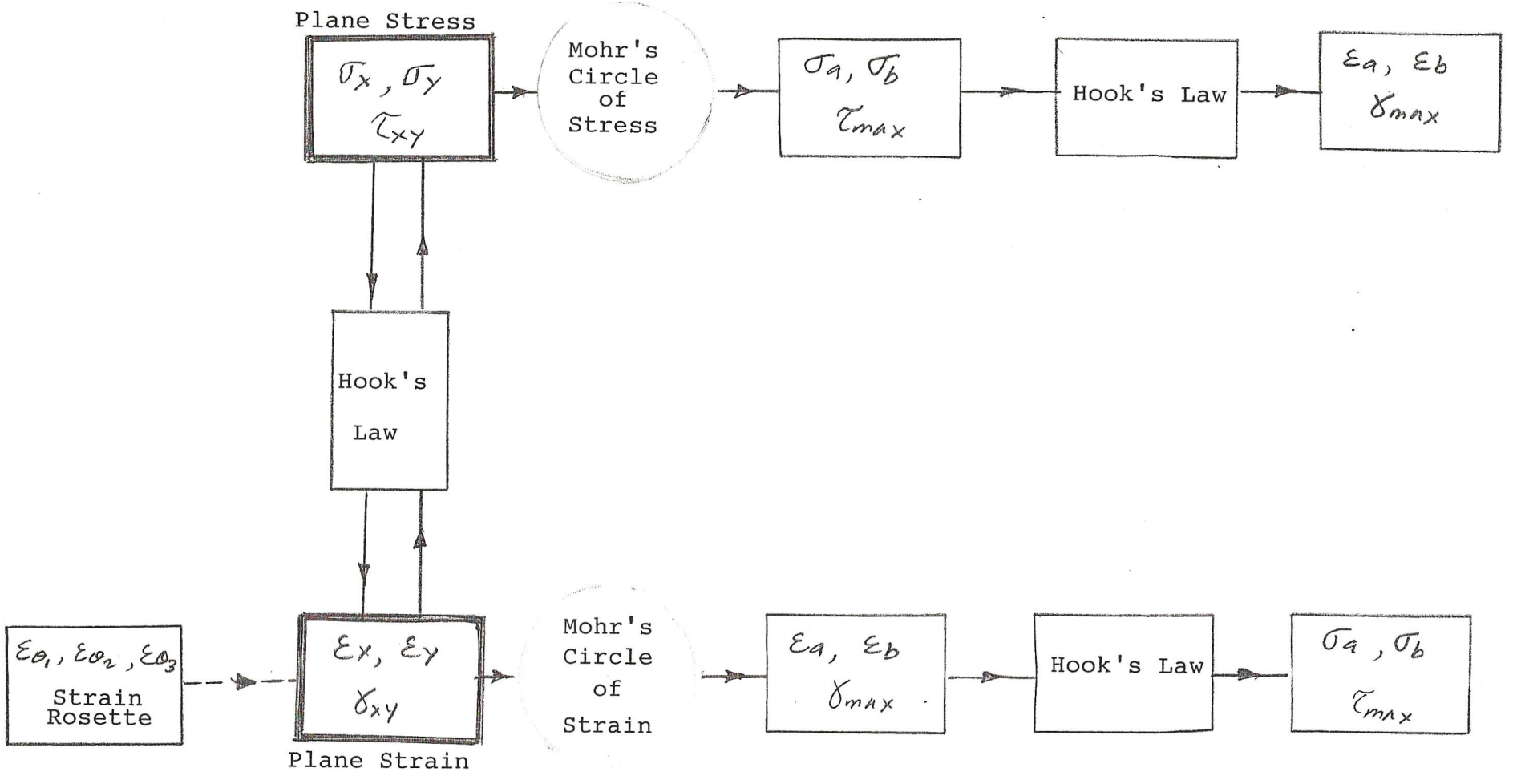
4- Stress Transformation:
   a- Purpose
   b- Procedures: Using Equations or Mohr's Circle

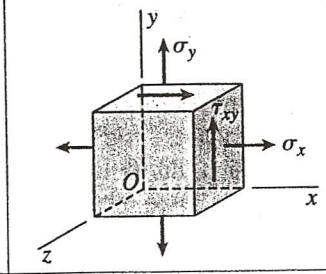
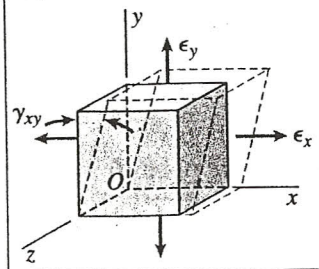
Today's Subject:

5- **Plane Strain**

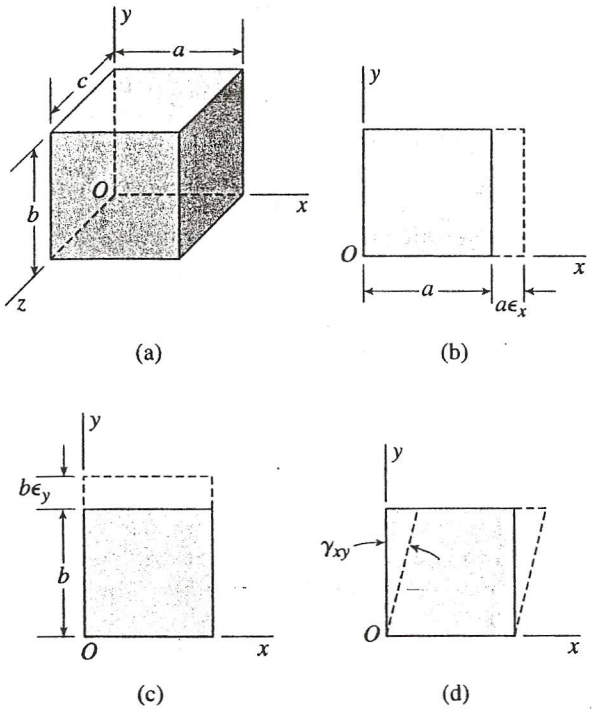
6- **Strain Transformation:**
   a- Purpose
   b- Procedures: Using Equations or Mohr's Circle

7- Examples

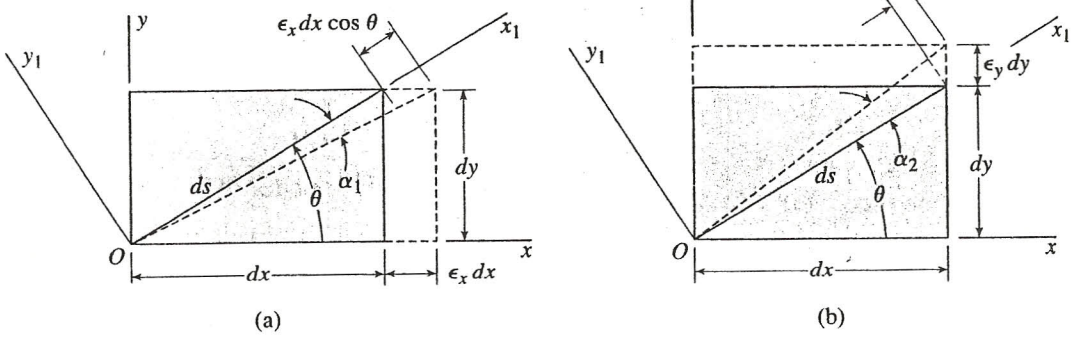

Plane Stress
σ_x, σ_y
τ_{xy}
Mohr's Circle of Stress
σ_a, σ_b
τ_{max}
Hook's Law
$\varepsilon_a, \varepsilon_b$
γ_{max}
Hook's Law
$\varepsilon_{\theta_1}, \varepsilon_{\theta_2}, \varepsilon_{\theta_3}$
Strain Rosette
$\varepsilon_x, \varepsilon_y$
γ_{xy}
Plane Strain
Mohr's Circle of Strain
$\varepsilon_a, \varepsilon_b$
γ_{max}
Hook's Law
σ_a, σ_b
τ_{max}

	Plane stress	Plane strain
		
Stresses	$\sigma_z = 0 \quad \tau_{xz} = 0 \quad \tau_{yz} = 0$ σ_x, σ_y, and τ_{xy} may have nonzero values	$\tau_{xz} = 0 \quad \tau_{yz} = 0$ σ_x, σ_y, σ_z, and τ_{xy} may have nonzero values
Strains	$\gamma_{xz} = 0 \quad \gamma_{yz} = 0$ ϵ_x, ϵ_y, ϵ_z, and γ_{xy} may have nonzero values	$\epsilon_z = 0 \quad \gamma_{xz} = 0 \quad \gamma_{yz} = 0$ ϵ_x, ϵ_y, and γ_{xy} may have nonzero values

Comparison of Plane Stress & Plane Strain



Strain Components ε_x, ε_y and γ_{xy} in xy plane

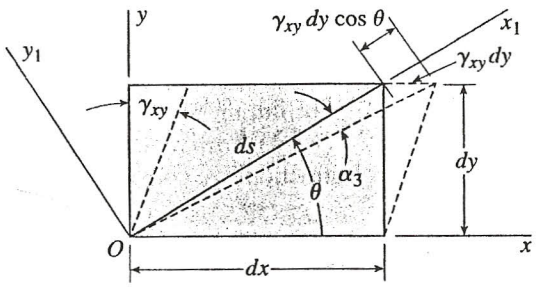


Deformation in Plane-Strain:

a- Normal Strain ε_x

b- Normal Strain ε_y

c- Shear Strain γ_{xy}



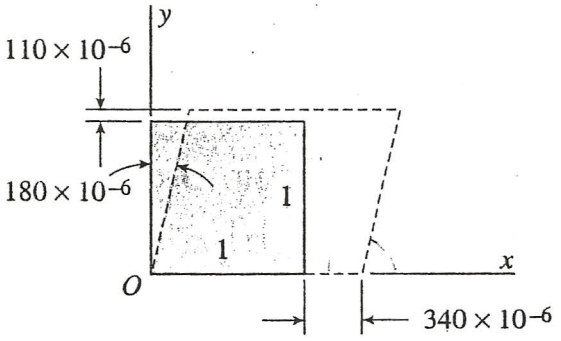
Corresponding variables in Transformation Equations of Plain Stress vs. Plane Strain

Stresses	Strains
σ_x	ϵ_x
σ_y	ϵ_y
τ_{xy}	$\gamma_{xy}/2$
σ_{x_1}	ϵ_{x_1}
$\tau_{x_1 y_1}$	$\gamma_{x_1 y_1}/2$

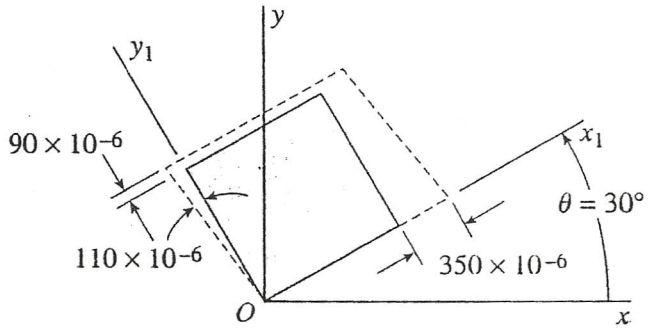
Result of strain transformation of class exercise :

a- Plane Strain Element
b- Strain Element oriented @ $\theta = 30°$
c- Principal Strains Element
d- Maximum Shear Strain Element

For details refer to Mohr's Circle.



(a)



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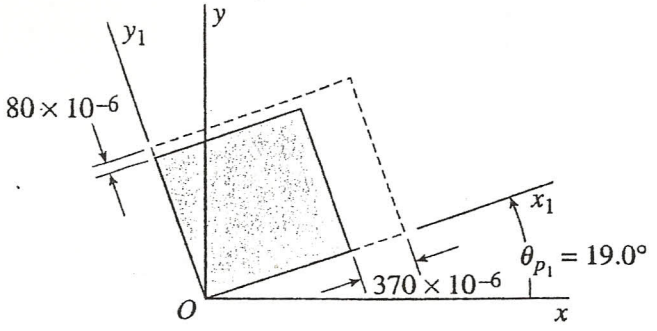
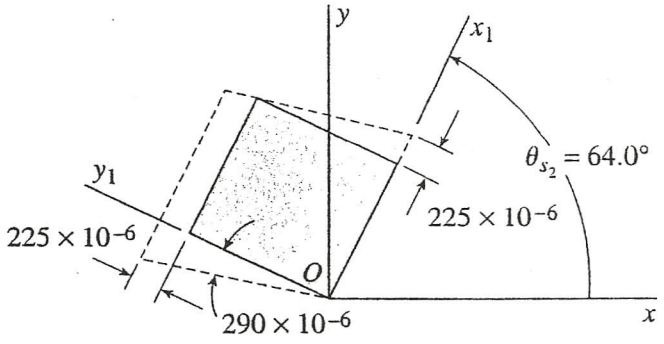
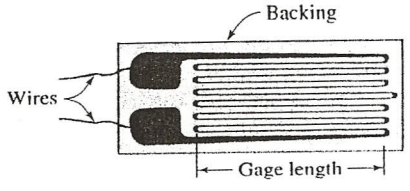


(c)



(d)



Electrical-resistance strain gage (shown greatly enlarged).

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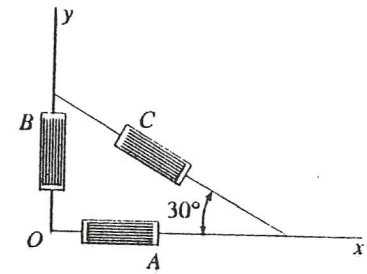
Question 1: (20 Points)

On the surface of a space vehicle the strains are monitored by means of a strain rosette as shown. During certain maneuver the following readings were recorded: $\varepsilon_A = 1{,}100 \times 10^{-6}$ $\varepsilon_B = 200 \times 10^{-6}$ and $\varepsilon_C = 200 \times 10^{-6}$.

The material is special magnesium alloy with: $E = 6{,}000$-ksi , $\upsilon = 0.35$, $G = 2{,}200$-ksi.

a- Use **Mohr's Circle** to determine the principal strains and maximum shear strain γ at this point.

b- Find maximum shear stress τ and state whether it is in-plane or out-of-plane shear.



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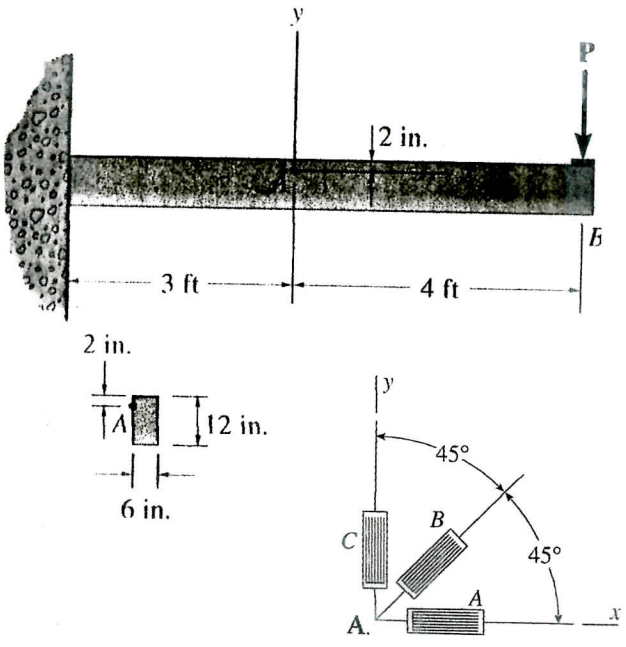
Question 1:
The strains on the surface of an experimental devise are monitored by means of three strain gages A, B and C put at $\theta = 60°, 90°$ and $120°$ respectively. The reading of the gages are $\varepsilon_A = -90 \times 10^{-6}$, $\varepsilon_B = -190 \times 10^{-6}$ and $\varepsilon_C = -240 \times 10^{-6}$.
a- Find the reading of a gage put at $\theta = 0°$, along the x-axis.
b- Use Mohr's Circle to determine the in-plane principal strains and their directions.
c- Determine the absolute maximum shear strain and its plane.

Answers:
$\varepsilon_A = -90 \times 10^{-6}$
$\varepsilon_a = -40 \times 10^{-6}$, $\varepsilon_b = -240 \times 10^{-6}$, $\varepsilon_c = 120 \times 10^{-6}$
$(\gamma_{max})_{xz} = 360 \times 10^{-6}$

Question 2:
A load P = 72-kips is applied to the steel beam shown:
a-Determine the reading of strain gages in 45° rosette at point A.
b-Find principal strains and absolute maximum shear strain using strain transformation.
Assume: E = 29,000-ksi G = 11,500 ksi

Answers:
$\varepsilon_x = 552 \times 10^{-6}$, $\varepsilon_y = -144 \times 10^{-6}$, $\varepsilon_{45} = 168 \times 10^{-6}$
$\varepsilon_a = 554 \times 10^{-6}$, $\varepsilon_b = -146 \times 10^{-6}$, $\varepsilon_c = -144 \times 10^{-6}$
$(\gamma_{max})_{xy} = 700 \times 10^{-6}$



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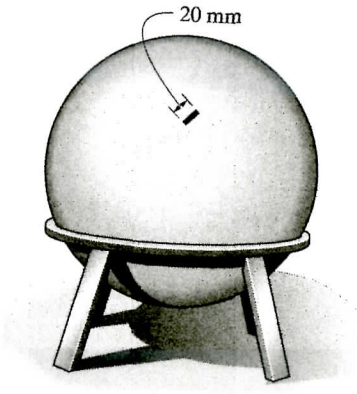
Question 3

The spherical pressure vessel has an inner diameter of 2-m and a thickness of 10-mm. A strain gage having a length of 20-mm is attached to it, and it is observed to increase in length by 0.012-mm when the vessel is pressurized. Determine the pressure causing this deformation, the maximum in-plane shear stress, and the absolute maximum shear stress at a point on the outer surface of the vessel.

Assume: E = 200-GPa $\upsilon = 0.3$



Answers:

p = 3.43 MPa

$\tau_{\text{max in plane}} = 0$ MPa

$\tau_{\text{max out-of plane}} = 85.7$ MPa

Question 4:

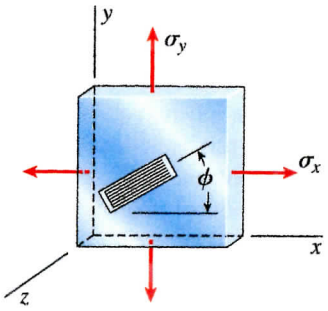
A steel plate with $E = 30 \times 10^6$ psi and $\upsilon = 0.30$ is loaded in biaxial stress with $\sigma_x = 18{,}000$-psi and σ_y as shown. A strain gage put at $\varphi = 30°$ measures the strain $\varepsilon_{30} = 407 \times 10^{-6}$, determine σ_y, the in-plane shear stress, the absolute maximum shear stress, in-plane shear strain and absolute maximum shear strain.

Answers:

$\sigma_y = 2{,}400$-psi

$\tau_{\text{max in plane}} = 7{,}800$-psi, $\tau_{\text{max out-of plane}} = 9{,}000$-psi

$(\gamma_{max})_{xy} = 676 \times 10^{-6}$, $(\gamma_{max})_{xz} = 780 \times 10^{-6}$



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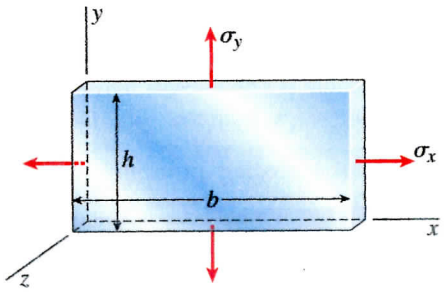
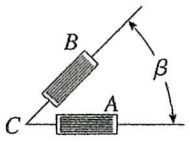
Question 5

A thin rectangular plate in biaxial stress is subjected to σ_x and σ_y as shown. The plate has the width of b =10-in, the height of h = 8-in, and thickness of t = 1.5-in. After plate is loaded the strain gage on the surface of the plate show measurements of ε_A = 600-μ and ε_B = 448-μ at β = 40°. Determine σ_x and σ_y.

Plate is made of brass with E = 15,000-ksi and υ = 0.375





Answers:

σ_x = 12,000-psi

σ_y = 8,000-psi

Question 6

A solid circular bar with a diameter of d = 1.5-in is subjected to an axial force P and a torque T. After rod is loaded the strain gages mounted on the surface of the rod give readings of ε_A = 100-μ and ε_B = - 55-μ. The rod is made of steel having E = 30,000-ksi and υ = 0.29. Determine:

a-The axial load P and the torque T.

b-The absolute maximum shear stress and shear strain in the rod.

Answers:

P = 5,300-lbs , T = 1,390 lb-in

τ_{max} in plane = 2,580-psi , $(\gamma_{max})_{xy}$ = 222-μ