Title: Two Recent Applications of Lattice Path Theory to Other Areas of Combinatorics

Author(s):
Antonio Blanca
Samuel Connolly
Zachary Gabor
Anant Godbole

Speaking Author: Author 4

Email: godbolea@etsu.edu

Abstract:
It is natural to ask, given a permutation with no three-term ascending subsequence, at what index the first ascent occurs. We shall show, using both a recursion and a bijection, that the number of 123-avoiding permutations at which the first ascent occurs at positions $k,k+1$ is given by the k-fold Catalan convolution $C_{n,k}$. For $1\le k\le n$, $C_{n,k}$ is also seen to enumerate the number of 123-avoiding permutations with n being in the kth position. Two interesting discrete probability distributions, related obliquely to the Poisson and geometric random variables, are derived as a result. Critical to the bijective proof are the various combinatorial interpretations of the Catalan convolutions due to Tedford, and the bijections between Dyck paths and avoiding permutations due to Krattenthaler.

Next, let $P_{n,k}$ be the set of the strings over $\Sigma=\{N,S,E,W\}$ corresponding to all lattice paths of length n
that start at the origin and end up at a distance of at most k from it. We show that there exists a universal cycle for $P_{n,k}$, i.e., a cyclic coded listing of length $\vert P_{n,k}\vert$ and containing each appropriate lattice path exactly once.