In this talk, we consider the $M_t/M_t/1$ queue, the time-varying periodic multi-server queue, queues with jumps of size one and two, and more general ergodic quasi-birth-death processes (QBDs) with time-varying periodic transition rates of period one. We develop asymptotic estimates for the probabilities in the queue length distribution for level independent quasi-birth-death processes (QBD) with time varying periodic rates. The estimates are asymptotic in the level of the process (the length of the queue). These asymptotic estimates highlight the connections between the asymptotic periodic distribution of a stable queue with time-varying rates and the same type of queue with constant rates. The estimates can also be used to approximate other performance measures such as the waiting time distribution. We illustrate the method with several examples.

The basic approach is to solve for the generating function of the QBD using the assumption that if the process is in its asymptotic periodic distribution at some time $t$, then the generating function at time $t$ will equal the generating function at time $t+1$. This will yield a meromorphic function for the generating function in terms of an integral equation. We find the poles of this function to create our asymptotic estimates.

For the single server queue, these estimates take a particularly simple form. Let $\bar{\lambda}$ be the average arrival rate in a time period and $\bar{\mu}$ be the average departure rate. Then an asymptotic estimate for the probability that there are $n$ in the queue at time $t$ is given by $\pi_n(t) \approx f(t) \left(\frac{\bar{\lambda}}{\bar{\mu}}\right)^n$. For constant rates, the formula is exact and $f(t) = \pi_0 = 1 - \frac{\bar{\lambda}}{\bar{\mu}}$. In general, $f(t)$ depends on $\pi_0(t)$. Given $\pi_0(t)$, $f(t)$ can be easily computed for any stable periodic $M_t/M_t/1$ queue. Similar formulas can be developed using this approach for more complex QBDs.