

The 8th International Conference on Lattice Path Combinatorics and Applications
(August 17 – 20, 2015, California Polytechnic State University Pomona, CA)

Title: *The concept of pseudo-dual of a system of differential equations*

Presented by: Alan Krinik and Gerardo Rubino*

ABSTRACT: Classical performance evaluation of sharing resources systems using queues is usually done assuming a stable model considered in equilibrium. The reason is that in many situations, time scales are such that we can assume that all transients disappear in practice, for most of the reasonable applications we typically have in mind. But there are situations where we are interested in the transient life of the model. In this case, the most basic metrics are built around the state distribution of the model at an arbitrary point in time.

In previous works, we developed an approach to derive the state transient distribution of some Markovian models, that is built around two tools: Jensen's method, also called Uniformization, sometimes Randomization, to transform the problem into a discrete time one, thus changing differential equations into difference equations (allowing then to use lattice path combinatorics), and the concept of dual stochastic processes of William Anderson, as it appears in his known book "Continuous-Time Markov Chains". The latter allows us to work with absorbing chains instead of the irreducible ones associated with queues, and this provides significant simplifications in the analysis.

Recently, we discovered that this dual concept was not dependent on the Markovian properties, but was actually more general, and that it can be seen as a transformation between systems of differential equations. We call it pseudo-dual from this more general point of view. This allows to apply the same general approach previously mentioned to linear systems of differential equations, even to infinite ones (denumerable number of variables and equations), without any contact with the stochastic world.

We will describe the general approach as it works for the analysis of queuing models, with some illustrations. Then, we will show how all this extends to systems of differential equations in general, and we will illustrate the global technique with examples.

.....