

COUNTING COLORED PATHS USING PARTIAL BELL POLYNOMIALS

MICHAEL D. WEINER

ABSTRACT. We consider families of colored Dyck paths and count the number of elements. For this sequence, we prove a recurrence relation and give a representation in terms of partial Bell polynomials that unifies several known results and simplifies the handling of different colorings. We give explicit examples and show how the counting changes for various restrictions and number of colors. In order to illustrate our method, we will show (as an example) that the number of Dyck paths with $4n$ steps and k peaks containing only ascents of even length, colored by triangular numbers, is given $\binom{2n}{k-1} \binom{n+2k-1}{n-k} \frac{1}{k}$.

PENN STATE ALTOONA, 3000 IVYSIDE PARK, ALTOONA, PA 16601