LOZENGE TILINGS OF HALVED HEXAGONS WITH DEFECTS

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MacMahon's boxed plane partition formula from 1911 enumerates the lozenge tilings of a hexagon with side-lengths a, b, c, a, b, c in cyclic order on the triangular lattice. More recently several authors have enumerated the number of tilings for hexagons with different types of defects. Proctor treated the case where a "maximal staircase" is removed, and several others have found formulas for hexagons with triangles removed from the boundary.

Potential lozenges used in the tiling of a region can be given weights. For a given tiling μ, define $\mathrm{wt}(\mu)$ to be the product of the weights of all lozenges used in μ. Rather than counting the number of tilingss of a weighted region R, one tries to determine its *tiling generating function*, $M(R)$ defined by

$$M(R) = \sum_{\mu \in \mathcal{M}} \mathrm{wt}(\mu),$$

where $\mathcal{M}$ is the set of all tilings of R. Evidently, if all weights are 1, $M(R)$ is precisely the number of tilings of R.

We present results for both unweighted and certain weighted hexagonal regions with both a maximal staircase and boundary triangles removed. Treating these regions as halved hexagons allows us, with the help of Ciucu's factorization theorem, to recover known results in a new way.