# The 8th International Conference on Lattice Path Combinatorics & Applications

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Title: The Combinatorics Governing the Periodicity of $p(n, d)$ Modulo $M$.

Abstract: From the generating function, one can quickly see that $p(n, d)$, the number of partitions of $n$ into parts of size at most $d$, is periodic modulo $M$. It is natural to ask if there is a purely combinatorial explanation for this periodicity.

The search for an explanation led us to study the geometry of lattice points in polyhedra, which ultimately inspired a new decomposition of partitions into their "$\ell$-box remainder" and "$\ell$-box quotient." These two new objects bear many similarities to the $\ell$-core and $\ell$-quotient of a partition, which were used in the famous combinatorial proof of the first four Ramanujan congruences by Garvan, Kim, and Stanton. This new $\ell$-box decomposition does lead to a combinatorial proof of the periodicity of $p(n, d)$ modulo $M$, and the proof provides substantial structural information about the behavior of $p(n, d)$.

Some immediate consequences of this work include new proofs of several infinite families of known Ramanujan-type congruences for $p(n, d)$. Furthermore, these methods apply equally well to partitions whose parts come from any fixed finite set $A$, which allows for many new generalizations of the previously known infinite families of congruences.

This talk is joint work with Felix Breuer and Brandt Kronholm.