Title: On a problem by Don Knuth: Lattice paths of slope 2/5

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Abstract:
We analyze some enumerative and asymptotic properties of Dyck paths under a line of slope 2/5.
This answers to Knuth's problem \#4 from his ``Flajolet lecture'' during the conference ``Analysis of Algorithms'' (AofA'2014) in Paris in June 2014.
Our approach relies on the work of Banderier and Flajolet for asymptotics and enumeration of directed lattice paths.

A key ingredient in the proof is the generalization of an old trick of Knuth himself (for enumerating permutations sortable by a stack), promoted by Flajolet and others as the ``kernel method''.
All the corresponding generating functions are algebraic, and they offer some new combinatorial identities, which can be also tackled in the {\em A=B} spirit of Wilf--Zeilberger--Petkov{\v s}ek.

We show how to obtain similar results for other slopes than 2/5, an interesting case being e.g. Dyck paths below the slope 2/3, which corresponds to the so called Duchon's club model.