Number of Points of Schubert Varieties over Finite Fields[1]

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(Joint work with Christian Krattenthaler)

Let V be an n-dimensional vector space over a field $\mathbb{F}$. Consider the Grassmannian $G_{k,n}$ of k-dimensional linear subspaces of V. It is well-known that $G_{k,n}$ can be viewed as a projective algebraic variety defined by the vanishing of certain quadratic homogeneous polynomials in $\binom{n}{k}$ variables with integer coefficients. Morever, $G_{k,n}$ contains an important and interesting class of subvarieties known as *Schubert varieties.*

When $\mathbb{F}$ is the finite field $\mathbb{F}_q$ with q elements, it makes to sense to ask for (a nice formula for) the cardinality of $G_{k,n}(\mathbb{F}_q)$ and more generally, for (the number of $\mathbb{F}_q$-rational points of) each of its Schubert subvarieties. The answer in the case of Grassmannians is simply the Gaussian binomial coefficient ${\begin{bmatrix} n \\ k \end{bmatrix}}_q$. Answers in the case of Schubert varieties (in Grassmannians) are also known and these range from classical ones deduced from the cellular decomposition á la Ehresmann [1] or more complex ones given by Guerra and Vincenti [3, 4] and by Ghorpade and Tsfasman [2]. We will review these formulas and answer a question of Tsfasman concerning direct combinatorial equivalence of these formulas. Furthermore, we will describe yet another formula, arguably the most elegant one, for the number of $\mathbb{F}_q$-rational points of Schubert varieties in Grassmannians.

This is a joint work with C. Krattenthaler.

References

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