Shreeram Abhyankar and his work on Enumerative Combinatorics[1]

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Shreeram Shankar Abhyankar (1930–2012) was an extraordinary mathematician and educator, who has contributed significantly to many parts of mathematics and has left a lasting influence on numerous students and peers. In this talk, we will attempt to give a brief account of major research contributions of Abhyankar, with particular emphasis on the period 1982–1988 during which he worked almost exclusively on enumerative combinatorics and its applications to algebraic geometry. In particular, we will give a short account of Schubert varieties (in Grassmannians and in flag manifolds) and questions concerning them that led Abhyankar to enumerative combinatorics of Young tableaux. We will outline Abhyankar's enumerative proof of the straightening law of Doubilet-Rota-Stein and the remarkable consequences of this approach to an explicit determination of the multiplicity and Hilbert function of a class of determinantal varieties. Connections with lattice path combinatorics will be highlighted and a sketch of an alternative proof of Abhyankar's formula using the counting of families of nonintersecting lattice paths with a given number of turns will be given.

We will also share interesting bijective approaches between the families of nonintersecting paths and certain tableaux and monomials. They extend popular Robinson-Schensted-Knuth correspondences. If time permits, we will touch upon some of the subsequent developments influenced by Abhyankar's work on enumerative combinatorics.

References

[1] S. S. Abhyankar, *Enumerative Combinatorics of Young Tableaux*, Monogr. Textbooks Pure Appl. Math. **115**, Marcel Dekker, New York, 1988.

[2] S. R. Ghorpade, Abhyankar's work on Young tableaux and some recent developments, in: *Algebraic Geometry and Its Applications*, pp. 233–265, Springer-Verlag, New York, 1994.

[3] S. R. Ghorpade, *Young bitableaux, lattice paths and Hilbert functions*, J. Statist. Plann. Inference **54** (1996), 55–66.

[4] S. R. Ghorpade, Remembering Shreeram S. Abhyankar, With appendices by A. Sathaye, B. Singh, S. D. Cutkosky and R. V. Gurjar, *Asia Pac. Math. Newsl.* **3** (2013), no. 1, 22–30.

[5] S. Mulay and A. Sathaye (Coordinating Eds.), Shreeram Abhyankar (July 22, 1930–November 2, 2012), *Notices Amer. Math. Soc.* **61** (2014), no. 10, 1196–1216.

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[1]Abstract of a memorial talk at the 8th International Conference on Lattice Path Combinatorics and Applications, CalPoly, Pomona, California, USA, August 17–20, 2015