# Department of Mathematios and Statistics 

## Colloquium Series



From Fibonacci numbers to polynomials, integrals, and vector spaces

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Abstract: Many people have heard of the famous sequence of Fibonacci numbers, which starts $0,1,1,2,3,5,8, \ldots$ Each term in the sequence is the sum of the two previous terms. In other words, $F_{n}=F_{n-1}+F_{n-2}$. We can generalize this idea by adding coefficients to the recurrence to consider sequences which satisfy $X_{n}=a X_{n-1}+b X_{n-2}$. The collection of all sequences satisfying this recurrence is denoted by $\mathcal{R}(a, b)$. Viewed this way, the Fibonacci sequence is simply one element among many in the space $\mathcal{R}(1,1)$. It turns out that many of the famous facts about Fibonacci numbers are facts which are true of sequences in $\mathcal{R}(a, b)$. Finally, by letting $a=x$ and $b=1$, we obtain the Fibonacci polynomials $F_{n}(x)$. In joint work stemming from the Park City Math Institute in 2017, we investigated a new sequence obtained by integrating $e_{n}=\int_{0}^{\infty} F_{n}(x) e^{-x} d x$. In this talk, we'll show how one particular Fibonacci fact allows us to investigate the sequence $\left\{e_{n}\right\}$.

Keywords: Fibonacci, combinatorics, sequences

