Most curve problems are calculated from field measurements ($\Delta$ and chainage), and from the design parameter, radius of curve ($R$). $R$ is dependent on the design speed and $\Delta$. All other curve components can be computed.
GEOMETRY OF CURVE

In triangle BC, O, PI

\[
\frac{T}{R} = \tan \left( \frac{\Delta}{2} \right)
\]

**Tangent** \( T = R \tan \left( \frac{\Delta}{2} \right) \)

In triangle BC, O, B

\[
\frac{1}{2}C/R = \sin \left( \frac{\Delta}{2} \right)
\]

**Chord** \( C = 2R \sin(\Delta/2) \)

\[
\frac{OB}{R} = \cos \left( \frac{\Delta}{2} \right)
\]

\[
OB = R \cos \left( \frac{\Delta}{2} \right)
\]

\[
OB = R - M
\]
R - M = R\cos(\Delta/2)

Mid ordinate (M) \[ M = R\{1 - \cos(\Delta/2)\} \]

In triangle BC, O, PI

Distance O to PI = R + E

\[
\frac{R}{R+E} = \cos(\Delta/2)
\]

E = R\left\{\frac{1}{\cos(\Delta/2)}\right\} - 1

External Distance E = R\{\sec(\Delta/2) - 1\}

**LENGTH OF CURVE**
\[
\frac{L}{2\pi R} = \frac{\Delta}{360}
\]

**Length of Curve** \( L = 2\pi R \left( \frac{\Delta}{360} \right) \)

**DEGREE OF CURVE (D)**

*Highway Definition* The Central Angle subtended by a 100' arc

*Railroad Definition* The Central Angle subtended by a 100' chord

Consider the figure above.

\[
\frac{D}{360} = \frac{100}{2\pi R}
\]

\[
D = \frac{5729.58}{R}
\]

And

\[
\frac{L}{100} = \frac{\Delta}{D}
\]

\[
L = 100\frac{\Delta}{D}
\]

**STATIONING**

The distance along a route in highway surveying is represented by stationing. Stationing is expressed in **units of 100 feet, OR units of 1000 feet**. For example, a point 626.57 feet along the route is expressed as:

\[
6 + 26.57 \text{ in the 100 foot system}
\]

and \( 0+626.57 \) in the thousand foot system
Example

\[ \Delta = 16^\circ 38' \]

\[ R = 1000' \]

PI at 6+26.57

Calculate the stationing of BC, and EC, and find M (mid-ordinate), C (chord) and E (external)

\[ T = R \tan \left( \frac{\Delta}{2} \right) \]

\[ = 1000 \tan 8^\circ 19' \]

\[ = 146.18 \text{ ft} \]

\[ L = 2\pi R \frac{\Delta}{360} \]

\[ = (2\pi)1000(16.6333/360) \]
The stationing around a circular curve is computed as follows:

- Compute the tangent length T
- Subtract T from the station value of PI
- Compute the length of curve L
- Add L to station value of BC to get the EC value

The chainage is calculated as follows:

\[
\begin{align*}
\text{PI at } 6 + 26.57 & \quad - T = 1 + 46.18 \\
\text{BC} & = 4 + 80.39 \\
+L & = 2 + 90.31 \\
\text{EC} & = 7 + 70.70
\end{align*}
\]

\[
\begin{align*}
C & = 2R \sin \left( \frac{\Delta}{2} \right) \\
& = 2 \times 1000 \times \sin 8^\circ 19' \\
& = 289.29 \text{ ft}
\end{align*}
\]

\[
\begin{align*}
M & = R \{ 1 - \cos \left( \frac{\Delta}{2} \right) \} \\
& = 1000(1 - \cos 8^\circ 19') \\
& = 10.52'
\end{align*}
\]

\[
\begin{align*}
E & = R \{ \sec \left( \frac{\Delta}{2} \right) - 1 \} \\
& = 1000(\sec 8^\circ 19')
\end{align*}
\]
A common mistake made by students first studying circular curves to determine the station of the EC by adding the T distance to the PI. Even though EC is physically a distance of T from the PI, the stationing (chainage) must reflect the fact that the route no longer goes through the PI. The route takes the shorter distance (L) from the BC to the EC.

**Example with Degree of Curve**

Given

\[ A = 11^\circ 21' 35'' \]

PI at 14 + 87.33

\[ D = 6^\circ \]

Calculate the station of the BC and EC.

\[ R = \frac{5729.58}{D} = 954.93 \text{ ft} \]

\[ T = R \tan \left( \frac{\Delta}{2} \right) \]

\[ = 954.93 \tan 5.679861^\circ \]
\[ L = 100(\Delta/D) \]
\[ = 100(11.359722/6) \]
\[ = 189.33 \text{ ft} \]

PI at 14 + 87.33

\[ -T \quad 0 + 94.98 \]

BC = 13 + 92.35

+L \quad 1 + 89.33

EC = 15 + 81.68

**DEFLECTION ANGLES**

The most common method of locating a curve in the field is by deflection angles. Typically, the theodolite is set up at the BC, and the deflection angles are turned from the tangent line.
If the layout is to proceed at 20m intervals, the procedure would be as follows. First, compute the deflection angles for the three required arc distances by the following formula:

\[
\text{deflection angle} = \frac{\text{arc}}{L} \left( \frac{\Delta}{2} \right)
\]

BC to first even station (0 + 200):

\[
\frac{6.4250}{89.710} \times 3.262 = 0^\circ 14' 01"
\]

Even station interval

\[
\frac{6.4250}{89.710} \times 20 = 1^\circ 25' 57"
\]

Last even station (0 + 280) to EC:

\[
\frac{6.4250}{89.710} \times 6.448 = 0^\circ 27' 42"
\]

Second, prepare a list of appropriate stations together with cumulative deflection angles.

<table>
<thead>
<tr>
<th>BC</th>
<th>0 + 196.738</th>
<th>0 o 14' 01&quot;</th>
<th>+ 1 o 25' 57&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 + 200</td>
<td>0 + 200</td>
<td>0 o 14' 01&quot;</td>
<td>+ 1 o 25' 57&quot;</td>
</tr>
<tr>
<td>0 + 220</td>
<td>0 + 220</td>
<td>1 o 39' 58&quot;</td>
<td>+ 1 o 25' 57&quot;</td>
</tr>
<tr>
<td>0 + 240</td>
<td>0 + 240</td>
<td>3 o 05' 55&quot;</td>
<td>+ 1 o 25' 57&quot;</td>
</tr>
<tr>
<td>0 + 260</td>
<td>0 + 260</td>
<td>4 o 31' 52&quot;</td>
<td>+ 1 o 25' 57&quot;</td>
</tr>
<tr>
<td>0 + 280</td>
<td>0 + 280</td>
<td>5 o 57' 47&quot;</td>
<td>+ 0 o 27' 42&quot;</td>
</tr>
<tr>
<td>EC</td>
<td>0 + 286.448</td>
<td>6 o 25' 31&quot;</td>
<td>= \Delta/2 approx.</td>
</tr>
</tbody>
</table>

For most engineering layouts, the deflection angles are rounded to the closest minute or half-minute.
CHORD CALCULATIONS

In the previous example, it was determined that the deflection angle for station 0 + 200 was 0° 14' 01"; it follows that 0 + 200 could be located by placing a stake on the transit line at 0'14' and at a distance of 3.262 m (200 - 196.738) from the BC.

Station 0 + 220 could be located by placing a stake on the transit line at 1° 39' 58" and at a distance of 20 m from the stake locating 0 + 200. The remaining stations could be located in a similar manner. However, it must be noted that the distances measured with a steel tape are not arc distances; they are straight lines known as subchords.

To calculate the subchord, \( C = 2R \sin \left( \frac{\Delta}{2} \right) \) may be used. This equation is the special case of the long chord and the total deflection angle. The general case can be stated as follows:

\[
C = 2R \sin \text{deflection angle}
\]

Any subchord can be computed if its deflection angle is known.

First chord: \( C = 2 \times 400 \times \sin 0^\circ 14'01'' = 3.2618 \) m

= 3.262 m (at three decimals, chord = arc)

Even station chord: \( C = 2 \times 400 \times \sin 1025'57''\)
On chords that are short, the difference between the chord and the arc is small.

**FIELD PROCEDURES**

1. Locate PI. Δ angle measured in the field. The radius(R) or degree of curve (D) chosen consistent with the design speed.

2. The surveyor then goes back out to the field and measures off the tangent (T) distance from the PI to locate the BC and EC on the appropriate tangent lines.

3. The transit is then set up at the BC and zeroed and sighted in on the PI. The Δ/2 angle (6° 25' 30" in above example) is then turned off in the direction of the EC. If the computations for T and the field measurements of T have been performed correctly, the line of sight at the Δ/2 angle will fall on EC.

4. The Δ/2 line of sight to EC will contain some error. The surveyor will decide if the resultant alignment is acceptable. (0.10 ft (30 mm) in a ditched highway survey, urban freeway 0.03 ft or (10 mm)).

5. Curve stakes are set by turning off the deflection angle and measuring the chord distance for the appropriate stations. The theodolite is left at the BC for the entire curve stakeout if possible, whereas the distance measuring moves continually forward from station to station.

6. Final verification of the work is after the last even station has been set. The chord distance from the last even station to the EC stake is measured and compared with the theoretical value. The measured value and the theoretical value must check.