MODULE 6.

POINT CLOUD REGISTRATION AND GEOREFERENCING

Learning Outcomes:
This module discusses point cloud registration and georeferencing. Students should be able to explain the basic concepts behind point cloud registration and georeferencing and perform cloud registration and georeferencing.

6.1 Introduction

In RealWorks Survey, registration refers to aligning the stations properly so that the scans of the same area from different stations overlap each other. Georeferencing is to transform the data from a project or station to another coordinate system. In both cases, coordinate transformation is needed. In registration, the coordinates of one or more stations are transformed to the reference station within the project while in georeferencing, the project or stations can be transformed to an external coordinate system. In 2D transformation, a minimum of two points are required and in 3D, a minimum of three points are needed. Intuitively, in a 2D problem, if only one point is known, a rotation around that point is possible and hence there is no unique transformation. A second known point would fix the transformation. Similarly in a 3D situation, one known point allows free rotation about that point, two known points allow rotation about the axis defined by the two points and three known points fix the transformation. Here is a brief discussion of transformation theories.

6.2 2D Coordinate Transformation

Figure 6.1 shows two coordinate systems XOY and X’O’Y’ and a point A with coordinates \((x_A, y_A)\) in the XOY system. From analytical geometry, the coordinates of the point under X’O’Y’ system can be found with the following formula if the rotation angle \(\alpha\), the translation displacements \(c\) and \(d\) are known:

\[
\begin{align*}
x'_A &= x_A \cos \alpha + y_A \sin \alpha + c \\
y'_A &= -x_A \sin \alpha + y_A \cos \alpha + d
\end{align*}
\]

This is a transformation with four parameters \(\cos \alpha\), \(\sin \alpha\), \(c\) and \(d\). In matrix format:

\[
\begin{bmatrix}
x'_A \\
y'_A \\
1
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & c \\
a_{21} & a_{22} & d \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_A \\
y_A \\
1
\end{bmatrix}
\]

Where \(a_{11}^2 + a_{12}^2 = 1\) and \(a_{21}^2 + a_{22}^2 = 1\).
In point cloud registration and georeferencing, the relationship between the two coordinates systems are usually not explicitly given in terms of rotation angle and translational displacements. Instead, the coordinates of the same set of points under each system are known. In this case, if the coordinates of two points under the two systems are known, four equations can be obtained to solve for the four parameters:

\[
\begin{align*}
    x'_A &= x_A \cos \alpha + y_A \sin \alpha + c \\
    y'_A &= -x_A \sin \alpha + y_A \cos \alpha + d \\
    x'_B &= x_B \cos \alpha + y_B \sin \alpha + c \\
    y'_B &= -x_B \sin \alpha + y_B \cos \alpha + d
\end{align*}
\]

(6.3)

Equation (6.1) can then be used for transformation of all points from XOY to X’O’Y’. If more than two points are known, the transformation can be obtained with the method of least square.

Example 6.1

The coordinates of Points 1 and 2 are given for two systems. The coordinates of Point 3 is given in the first system as show in the table below:

<table>
<thead>
<tr>
<th>Points</th>
<th>XOY system</th>
<th>X’O’Y’ system</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td>1</td>
<td>3425.23</td>
<td>864.52</td>
</tr>
<tr>
<td>2</td>
<td>3058.10</td>
<td>358.75</td>
</tr>
<tr>
<td>3</td>
<td>2532.40</td>
<td>914.30</td>
</tr>
</tbody>
</table>

Find the coordinates of Point 3 under the X’O’Y’ system.
Solution:

Substituting the known coordinates for Points 1 and 2 in the XOY system to the right sides of Equation (6.3) and their coordinates in the X’O’Y’ system to the left side the equations, we have the following system of linear equations:

\[
\begin{align*}
1648.15 &= 3425.23 \cos \alpha + 864.52 \sin \alpha + c \\
100.47 &= -3425.23 \sin \alpha + 864.52 \cos \alpha + d \\
1023.40 &= 3058.10 \cos \alpha + 358.75 \sin \alpha + c \\
104.46 &= -3058.10 \sin \alpha + 358.75 \cos \alpha + d
\end{align*}
\]

Solving the equations, we have:

\[
\begin{align*}
\sin \alpha &= 0.81274 \\
\cos \alpha &= 0.58205 \\
c &= -1048.15 \\
d &= 2381.099
\end{align*}
\]

Substituting \( \cos \alpha \), \( \sin \alpha \), \( c \) and \( d \) into Equation (6.1) to find \( x' \) and \( y' \):

\[
\begin{align*}
  x' &= 0.58205 x + 0.81274 y - 1048.15 \\
      &= 0.58205(2532.40) + 0.81274(914.30) - 1048.15 \\
      &= 1168.94 \\
y' &= -0.81274 x + 0.58205 y + 2381.099 \\
      &= -0.81274(2532.40) + 0.58205(914.30) + 2381.099 \\
      &= 855.08
\end{align*}
\]

6.3 3D Coordinate Transformation

3D coordinate transformation as shown in Figure 6.2 is similar to 2D with an added dimension, the objective is to find the coordinates of Point A in the X’Y’Z’ system if the coordinates of that point in the XYZ system is known. Again from analytical geometry:

\[
\begin{align*}
x' &= x \cos(x', x) + y \cos(x', y) + z \cos(x', z) + c \\
y' &= x \cos(y', x) + y \cos(y', y) + z \cos(y', z) + d \\
z' &= x \cos(z', x) + y \cos(z', y) + z \cos(z', z) + e
\end{align*}
\]
Figure 6.2 3D coordinate transformation.

or in matrix format:

\[
\begin{bmatrix}
x' \\
y' \\
z' \\
1
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & c \\
a_{21} & a_{22} & a_{23} & d \\
a_{31} & a_{32} & a_{33} & e \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
\]  \hspace{1cm} (6.5)

Where \(a_{11} = \cos(x',x), a_{12} = \cos(x',y), \ldots\) are the direction cosines of the unit vectors of the XYZ system in the X'Y'Z' system. c, d and e are the translational displacements in the X', Y' and Z' direction, respectively. This transformation has nine independent parameters since

\[
a_{11}^2 + a_{12}^2 + a_{13}^2 = 1 \\
a_{21}^2 + a_{22}^2 + a_{23}^2 = 1 \\
a_{31}^2 + a_{32}^2 + a_{33}^2 = 1
\]  \hspace{1cm} (6.6)

In the registration or georeferencing process, the software needs to determine the nine parameters first before transforming the coordinates. The concept is similar to 2D transformation except that at least three known points are required. Let Points 1, 2 and 3 be the known points and their coordinates in the XYZ system are \((x_1, y_1, z_1), (x_2, y_2, z_2),\) and \((x_3, y_3, z_3)\) and in the X'Y'Z' system are \((x'_1, y'_1, z'_1), (x'_2, y'_2, z'_2),\) and \((x'_3, y'_3, z'_3)\). Substituting these coordinate into Equation (6.5), nine equations are obtained:

\[
\begin{align*}
x'_1 &= x_1a_{11} + y_1a_{12} + z_1a_{13} + c \\
y'_1 &= x_1a_{21} + y_1a_{22} + z_1a_{23} + d \\
z'_1 &= x_1a_{31} + y_1a_{32} + z_1a_{33} + e \\
x'_2 &= x_2a_{11} + y_2a_{12} + z_2a_{13} + c \\
y'_2 &= x_2a_{21} + y_2a_{22} + z_2a_{23} + d \\
z'_2 &= x_2a_{31} + y_2a_{32} + z_2a_{33} + e
\end{align*}
\]
\[ x_3' = x_3a_{11} + y_2a_{12} + z_3a_{13} + c \]
\[ y_3' = x_3a_{21} + y_3a_{22} + z_3a_{23} + d \]
\[ z_3' = x_3a_{31} + y_3a_{32} + z_3a_{33} + e \]

These nine equations plus the three in Equation (6.6) will give all the \( a \) values as well as \( c, d \) and \( e \) in Equation (6.5).

If more than three points are known, the transformation can be achieved through the method of least squares.

### 6.4 Georeferencing in RealWorks Surveying

Georeferencing in RWS allows transformation of a project or station to an external coordinate system if the coordinates of three or more points are known. Here are the steps for georeferencing RWS:

1. Open a project file as discussed before.
2. In the Module Drop-down menu, select Registration.
3. In the *List* window, select a station to be georeferenced (or a whole project in the *Workspace* window if georeferencing a project is desired).

4. In the *Registration* menu, select *Georeferencing Tool*. 
5. A Tools tab is added to the Workspace window and the Georeferencing tool is open. In the Step 1 Select Station box, the selected station and the number of targets for that station are shown. Click on Target_1 and the By Target button becomes available. Click on the By Target button.

6. The Assign Known Coordinates to Target dialog box appears. Enter the reference X, Y, and Z coordinates for this target in the boxes provided and click OK.
Target_1 is listed in the Step 2 Designated Targets box, removed from the Step 1 Select Station box and highlighted in the 3D View window.

7. Repeat Steps 5-6 for Targe_2 and Target_3.

8. Click on the Apply button in the Georeferencing tool dialog box after the reference coordinates for all three targets are entered.
9. The georeferencing is done and click **Close** in the **Georeferencing** tool dialog box to close the **Georeferencing** tool.

**Questions:**
1. What is the difference between Registration and Georeferencing?
2. Why three known points are needed in Registration or Georeferencing for 3D point clouds?
3. The coordinates of the same two points in two 2D systems are known as shown in the table. The coordinates of a third point is known in the first system (XOY). Find the coordinates of the third points in the second system (X’O’Y’).

<table>
<thead>
<tr>
<th>Points</th>
<th>XOY system</th>
<th>X'OY’ system</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td>1</td>
<td>3572.46</td>
<td>1760.09</td>
</tr>
<tr>
<td>2</td>
<td>2730.20</td>
<td>-1046.85</td>
</tr>
<tr>
<td>3</td>
<td>2507.54</td>
<td>1214.80</td>
</tr>
</tbody>
</table>

4. Explain all the items in the **Georeferencing** tool dialog box.
5. Explain all the items in the **Assign Known Coordinates to Target** dialog box.