Algebra Practice Problems for MDPT Pre Calculus

1. Simplify \(\sqrt[3]{2x^4 \sqrt[4]{64x^2}}\)
   a. \(\sqrt[3]{128x^{10}}\)  b. \(\sqrt[3]{128x^7}\)  c. \(2x^2 \sqrt[4]{32x}\)  d. \(8x^{1/2} \sqrt{2x^3}\)

2. Solve \(-\frac{1}{2} (2x - 3) + \frac{x}{3} = \frac{3}{4} (x - 1) + \frac{1}{9}\)
   a. \(\frac{77}{51}\)  b. \(-\frac{23}{17}\)  c. \(\frac{23}{17}\)  d. \(\frac{23}{51}\)

3. Simplify \((x^{a-b})^{a+b} (x^a)^{-a} (x^{-b})^{a-b}\)
   a. \(x^{2a-ab-2b}\)  b. \(x^{-ab}\)  c. \(3x^{2a-2b}\)  d. \(x^{-3ab}\)

4. Simplify \(\frac{(-3x^2 y^6)}{(6x^3 y^2)^4}\)
   a. \(\frac{1}{16 x^4 y^4}\)  b. \(\frac{9}{8xy}\)  c. \(\frac{x}{2 y^2}\)  d. \(\frac{9}{16 y^2}\)

5. Solve \(2 + \frac{3}{x^2} = \frac{1}{x}\)
   a. -1 or 3/2  b. 2/3 or 1  c. -3/2  d. -1

6. Factor \(x^4 - 81\)
   a. \((x - 3)^2 (x + 3)^2\)  b. \((x - 3) (x + 3) (x^2 + 9)\)
   c. \((x^2 - 9) (x^2 - 9)\)  d. \((x + 3)^2 (x - 3) (x + 3)\)

7. Simplify \((5 - 4t) (3 + 5t)\)
   a. \(-5 + 5t - 3t^2\)  b. \(15 - 13t + 20t^2\)
   c. \(35 + 13t^2\)  d. \(15 + 13t - 20t^2\)

8. Solve \(\frac{8}{a^2 - 16} + \frac{1}{4 - a} = \frac{a}{a + 4}\)
   a. \(\frac{5 + \sqrt{73}}{2}\)  b. -1  c. \(\frac{2}{3}\) and 1  d. 4 and -1
9. Find the x-intercepts of the parabola \( y = x^2 - 3x + 4 \)
   a. \( x = -1 \) or 4
   b. \( x = -4 \) or 1
   c. \( x = \frac{3 \pm i\sqrt{7}}{2} \)
   d. none

10. Solve \( 4^{2x+1} + 8 = 40 \) for \( x \).
    a. \( \frac{3}{2} \)
    b. \( \frac{7}{4} \)
    c. \( \frac{2}{3} \)
    d. 3

11. Simplify the imaginary equation \( \frac{3-2i}{4-3i} \)
    a. \( \frac{16+9i}{5} \)
    b. \( \frac{5}{16+9i} \)
    c. \( -\frac{25}{18+i} \)
    d. \( \frac{18+i}{25} \)

12. Solve \( |2x + 4| \geq 6 \)
    a. \( x \geq -5 \) and \( x \geq 1 \)
    b. \( x \leq -5 \) or \( x \leq 1 \)
    c. \( x \leq -5 \) or \( x \geq 1 \)
    d. \( x \geq -5 \) and \( x \leq 1 \)

13. Find \( f(2p - 1) \) if \( f(x) = 3x^2 - 5x + 1 \)
    a. \( 12p^2 - 22p + 9 \)
    b. \( 12p^2 - 10p + 3 \)
    c. \( 12p^2 - 10p - 7 \)
    d. \( 12p^2 - 22p - 3 \)

14. Find \( p \) if \( f(x) = 12x^2 - 3px \) and 2 is a root of \( f(x) \).
    a. 6
    b. 42
    c. 4
    d. 8

15. Solve \( x(x - 3) = 2 \)
    a. \( 2 \pm \sqrt{22} \)
    b. \( \frac{3 \pm \sqrt{17}}{2} \)
    c. \( \frac{3 \pm \sqrt{17}}{3} \)
    d. \( \frac{3 \pm \sqrt{21}}{2} \)

16. Find the center and radius of the circle \( x^2 + 4x + y^2 - 6y = 3 \)
    a. \( c = (2,-3), r=16 \)
    b. \( c = (-2,3), r=4 \)
    c. \( c = (4,-6), r=3 \)
    d. \( c = (-4,6), r=9 \)

17. Solve \( \frac{\sqrt{2}}{3 - \sqrt{2}} \)
    a. \( \frac{3\sqrt{2} - 3}{7} \)
    b. \( \frac{3\sqrt{2} + 2}{7} \)
    c. \( \frac{7}{3 + \sqrt{2}} \)
    d. \( \frac{3\sqrt{2} - 7}{2} \)
18. If $f(x) = x^2 - 1$ and $g(x) = 2x + 7$, find $\frac{g}{f}(-1)$
   a. 0  b. $-\frac{5}{2}$  c. undefined  d. -1

19. Find the domain of the function $h(x) = \sqrt{3x - 9}$?
   a. $x \geq 3$  b. $x \leq 3$  c. $x \geq 9$  d. $x \leq 9$

20. Solve $\sqrt{x + 1} = x - 1$
   a. 1 and -1  b. 0 and 3  c. 3  d. -1

21. Solve $\sqrt{5x - 1} + \sqrt{x + 3} = 4$
   a. 13  b. no solution  c. 1 and 13  d. 1

22. Solve the system:
   \[
   \begin{align*}
   \frac{x}{3} + \frac{y}{6} &= \frac{2}{3} \\
   \frac{2}{5}x + \frac{y}{4} &= \frac{1}{5}
   \end{align*}
   \]
   a. (8,-12)  b. (3,-4)  c. (1,2)  d. (-3,4)

23. Solve $x^2 - 6x \geq 7$
   a. $-1 \leq x \leq 7$  b. $x \leq -1$ or $x \geq 7$
   c. $x \leq -7$ or $x \geq 1$  d. $x \leq -1$ and $x \geq 7$

24. Solve $\frac{x}{x - 2} \geq 2$
   a. $2 \leq x \leq 4$  b. $x \leq 2$ or $x \geq 4$
   c. $-2 < x < 2$  d. $2 < x \leq 4$

25. A boat can travel 48 miles downstream in the same time that it takes to travel 20 miles upstream. If the current is 14 mph, what is the boat's speed in calm water?
   a. 18 mph  b. 54 mph  c. 34 mph  d. 5.7 mph

26. Refer to the graph of $f(x) = \sqrt{x}$ below to determine which answer below is true
   a. $f(3) > f(9)$  b. $f(3) = f(-3)$  c. $f(3) = f(9)$  d. $f(9) > f(3)$
27. Solve \( \log_7 x + \log_7 (x + 6) = 1 \)
   a. 1  b. -7  c. 1 and -7  d. -1 and 7

28. Which graph is the solution of \( 5x - 2y \leq 8 \)?
   a.  
   b.  
   c.  
   d.  

29. A professional painter can paint a room twice as fast as an amateur. The two working together can paint a particular room in 2 hours. How long would it take the professional to paint the room alone?
   a. 2.5 hrs  b. 6 hrs  c. 4 hrs  d. 3 hrs

30. How many liters of a 6% acid solution must be added to 12 liters of a 10% acid solution to make a 9% acid solution?
   a. 3  b. 6  c. 4  d. 5

31. Solve \( \log_b \sqrt{\frac{63}{21}} \) assuming \( \log_b 3 = .62 \)
   a. 1.86  b. .31  c. \( \sqrt{1.86} \)  d. .206

32. Which is the graph of \( f(x) = x^2 + 4x - 2 \)?
   a.  
   b.  
   c.  
   d.  

33. Which is the graph of \( y = 4^x \)?
   a.  
   b.  
   c.  
   d.  

34. Find \( f(0) \) for the graph below.
   a. -3  
   b. 0  
   c. 2  
   d. can't tell

35. Simplify \( \frac{6x^3 - 13x^2 + 18x - 1}{3x - 2} \)
   a. \( 2x^2 - 3x + 8 + \frac{15}{3x - 2} \)  
   b. \( 2x^2 - 3x - 4 - \frac{7}{3x - 2} \)  
   c. \( 2x^2 - 3x - 8 - \frac{15}{3x - 2} \)  
   d. \( 2x^2 - 3x + 4 + \frac{7}{3x - 2} \)

Answers
   28. a  29. d  30. c  31. b  32. b  33. a  34. c  35. d
Solutions

1. \( \frac{2^{\frac{3}{2}} x^{\frac{1}{2}}}{64^{\frac{1}{4}} x^{\frac{3}{4}}} \) convert to exponent form. Use 64 = 2^6:

\( 2^{\frac{3}{2}} (2^6)^{\frac{1}{4}} x^{\frac{1}{2}} x^{\frac{3}{4}} \) simplify and add exponents:

\( 2^{\frac{3}{2} + \frac{3}{4}} x^{\frac{1}{2} + \frac{3}{4}} = 2^{\frac{11}{6}} x^{\frac{5}{6}} \) convert back to radical form and simplify:

\( \sqrt[6]{2^{11} x^{13}} = 2x^{\frac{5}{6}} \sqrt{2^5} x = 2x^{\frac{6}{2}} \sqrt{32} x \)

2. \( 36 \left[ \frac{1}{2} (2x - 3) + \frac{x}{3} \right] = 36 \left[ \frac{3}{4} (x - 1) + \frac{1}{9} \right] \) multiply through by 36:

\(-18(2x - 3) + 12x = 27(x - 1) + 4\)

\(-36x + 54 + 12x = 27x - 27 + 4\)

\(51x = 77\)

\(x = \frac{77}{51}\)

3. \( x^{a-b^2} x^{a} x^{-ab+b^2} \) using the rule: \( (x^a)^b = x^{ab} \)

\( x^{a-b^2-a^2-ab+b^2} = x^{-ab} \) using the rule: \( x^a x^b = x^{a+b} \)

4. \( \frac{(-3x^2 y)^6}{(6x^3 y^2)^4} = \frac{(-3)^6 x^{12} y^6}{6^4 x^{12} y^8} = \frac{3^6}{2^4 3^4 y^2} = \frac{9}{16 y^2} \)

5. \( 2x^2 + 3 = x \) multiply through by \( x^2 \)

\( 2x^2 - x + 3 = 0 \) set to 0 and factor

\( (2x - 3)(x + 1) = 0 \Rightarrow \) possible solutions are: \( x = 3/2 \) or \( x = -1 \)

Check possible solutions by plugging into original equation:

\( \frac{2 + \frac{3}{(3/2)^2}}{(3/2)^2} = \frac{1}{3} \Rightarrow \frac{10}{3} \neq \frac{2}{3} \Rightarrow x = 3/2 \) is NOT a solution

\( x = -1 \) is ok, so the answer is \( x = -1 \)

6. \( x^4 - 81 = (x^2)^2 - 9^2 \) use the formula: \( a^2 - b^2 = (a + b)(a - b) \)

\( (x^2 - 9)(x^2 + 9) = (x - 3)(x + 3)(x^2 + 9) \)

7. \( (5 - 4t)(3 + 5t) = 15 + 25t - 12t - 20t^2 = -20t^2 + 13t + 15 \)
8. \[
\frac{8}{(a-4)(a+4)} + \frac{-1}{a-4} = \frac{a}{a+4}, \quad \text{factor denominators to find LCM}
\]

\[
8 - (a+4) = a(a-4) \quad \text{multiply through by LCM}
\]

\[
a^2 - 3a - 4 = 0 \quad \text{expand, set equal to 0, and factor}
\]

\[
(a + 1)(a - 4) = 0 \quad \Rightarrow \quad \text{possible solutions are: } x = -1 \text{ or } x = 4
\]

\[
x = -1 \text{ works in the original equation, but } x = 4 \text{ gives a 0 denominator}
\]

Therefore, \( x = -1 \) is the answer.

9. \( y = 0 \) for the \( x \) intercepts:

\[
0 = x^2 - 3x + 4 \quad \text{since this won't factor, use the quadratic formula}
\]

\[
\frac{-(-3) \pm \sqrt{(-3)^2 - (4)(1)(4)}}{2(1)} = \frac{3 \pm \sqrt{-7}}{2} \quad \text{note a negative in the radical}
\]

So, no real solutions \( \Rightarrow \) NO intercepts.

10. \( 4^{2x-1} = 32 \) isolate the term with the exponent

use \( 4 = 2^2 \) and \( 32 = 2^5 \) to get a common base:

\[
(2^2)^{2x-1} = 2^5 \quad \Rightarrow \quad 2^{4x-2} = 2^5
\]

\[
4x - 2 = 5 \quad \text{since bases are equal the exponents must be equal}
\]

\[
x = \frac{7}{4}
\]

11. \[
\frac{(3-2i)(4+3i)}{(4-3i)(4+3i)} = \frac{12 + 9i - 8i - 6i^2}{16 - 9i^2} = \frac{12 + i + 6}{16 + 9} = \frac{18 + i}{25}
\]

12. \(-6 \geq 2x + 4 \geq 6\), remove absolute bars and include \( \pm 6 \)

-10 \( \geq 2x \geq 2\), subtract 2 from all sides

-5 \( \geq x \geq 1\), divide thru by 2

graphs don't overlap, use OR

\[
x \leq -5 \quad \text{OR} \quad x \geq 1
\]

13. Replace \( x \) with \( 2p - 1 \):

\[
f(2p - 1) = 3(2p - 1)^2 - 5(2p - 1) + 1 \quad \text{expand and simplify:}
\]

\[
f(2p - 1) = 12p^2 - 22p + 9
\]
14. If 2 is a root, then \( f(2) = 0 \)

\[ 12(2)^2 - 3(2)p = 0 \text{ plug in 2 and solve for p} \]

\[ p = 8 \]

15. \( x^2 - 3x - 2 = 0 \), multiply and set equal to 0

\[ x = \frac{3 \pm \sqrt{9 - 4(1)(-2)}}{2(1)} \text{ use the quadratic formula} \]

\[ x = \frac{3 \pm \sqrt{17}}{2} \]

16. \( x^2 + 4x + 4 + y^2 - 6y + 9 = 3 + 4 + 9 \), complete the square for \( x \) and \( y \)

\( (x + 2)^2 + (y - 3)^2 = 4^2 \)

\( (x - a)^2 + (y - b)^2 = r^2 \) is a circle with \( c = (a,b) \) and radius \( r \)

\( c = (-2,3), r=4 \) where \( c \) is the center and \( r \) is the radius

17. \[ \frac{\sqrt{2}(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \frac{3\sqrt{2} + 2}{9 - 2} = \frac{3\sqrt{2} + 2}{7} \]

18. \[ \frac{g(x)}{f(x)}(-1) = \frac{g(-1)}{f(-1)} = \frac{2(-1) + 7}{(-1)^2 - 1} = \frac{5}{0} \Rightarrow \text{undefined} \]

19. \( 3x - 9 \geq 0 \) because the number under the radical can't be negative

\[ 3x \geq 9 \]

\[ x \geq 3 \]

20. \( x + 1 = x^2 - 2x + 1 \), square both sides

\[ 0 = x^2 - 3x, \text{ set equal to 0} \]

\[ 0 = x(x - 3), \text{ factor} \]

\[ 0 \text{ and } 3 \]
21. \( \sqrt{5x - 1} = 4 - \sqrt{x + 3} \) isolate a radical term. Square both sides:

\[ 5x - 1 = 16 - 8\sqrt{x + 3} + x + 3 \]
simplify and isolate the radical term

\[ x - 5 = -2\sqrt{x + 3} \] square both sides:

\[ x^2 - 10x + 25 = 4x + 12 \]
set equal to 0 and factor

\[ x^2 - 14x + 13 = 0 \Rightarrow (x - 13)(x - 1) = 0 \]

Check answers \( x = 13 \) and \( x = 1 \) in the original equation:

\[ \sqrt{65 - 1} + \sqrt{13 + 3} = 4 \Rightarrow \sqrt{64} + \sqrt{16} = 4 \Rightarrow 8 + 4 = 4 \text{ 13 doesn't work} \]

\[ \sqrt{5 - 1} + \sqrt{1 + 3} = 4 \Rightarrow \sqrt{4} + \sqrt{4} = 4 \Rightarrow 2 + 2 = 4 \text{ 1 is OK} \]

22. \( 2x + y = 4 \)

\( 8x + 5y = 4 \) multiply both by their LCMs

\( 8x + 4y = 16 \) multiply the first equation by 4

\( 8x + 5y = 4 \) subtract the two equations:

\[ -y = 12 \Rightarrow y = 12 \]

\[ 2x + 12 = 4 \] plug 12 into one of the equations and solve for \( x \)

\( x = 8 \Rightarrow (8, -12) \) is the answer

23. \( x^2 - 6x - 7 \geq 0 \) set to 0

Solve \( x^2 - 6x - 7 = 0 \) for critical points

\( x = -1 \) and \( x = 7 \) are the critical points

Choose test points in the 3 regions around the critical points:

\(-2, 0, \text{ and } 8 \text{ will be used (see graph above)}\)

\[ (-2)^2 - 6(-2) - 7 \geq 0 \Rightarrow 9 \geq 0 \text{ True } \Rightarrow x \leq -1 \text{ is a solution} \]

\[ (0)^2 - 6(0) - 7 \geq 0 \Rightarrow -7 \geq 0 \text{ False } \Rightarrow \text{ mid region not a solution} \]

\[ (8)^2 - 6(8) - 7 \geq 0 \Rightarrow 9 \geq 0 \Rightarrow x \geq 7 \text{ is a solution} \]

Since the regions are separate use OR:

\( x \leq -1 \text{ or } x \geq 7 \)
24. \[ \frac{x}{x-2} - 2 \geq 0 \Rightarrow \frac{-x + 4}{x-2} \geq 0 \] set to 0 and simplify

critical points are \( x = 4 \) or \( x = 2 \) (note: \( x \) cannot equal 2)

Test points 0, 3, and 5 will be used

Test points 0 and 5 make the original equation false but 3 makes the equation true so the mid region is the solution \( 2 < x \leq 4 \)

25. use the equation: distance = (rate)(time) or \( d = rt \)

Downstream, \( r = \) speed of boat + speed of current
\[ 48 = (x + 14)t \] where \( x \) represents the speed of the boat

Upstream, \( r = \) speed of boat - speed of current
\[ 20 = (x - 14)t \]
\[ t = \frac{20}{x - 14} \] solve for \( t \) and plug into first equation

\[ 48 = (x + 14) \frac{20}{x - 14} \] solve for \( x \)
\[ x = 34 \]

26. \( f(9) = \sqrt{9} = 3 \) and \( f(3) = \sqrt{3} \approx 1.4 \) so \( f(9) > f(3) \)

27. \( \log_7{x(x + 6)} = 1 \) use \( \log a + \log b = \log ab \) to write as one log
\[ 7^1 = x(x + 6) \] convert from log to exponent form
\[ x^2 + 6x - 7 = 0 \] set to 0 and solve by factoring
\[ (x + 7)(x - 1) = 0 \] possible answers are \( x = -7 \) or \( 1 \)
For \( \log_b a \) a cannot be negative, so \( x \) cannot be -7
\[ x = 1 \] only

28. \( 5x - 2y = 8 \) change \( \leq \) to = , set equal to \( y \), and graph:
\[ y = \frac{5}{2}x - 4 \] slope is \( 5/2 \) , y intercept is -4

only answers a and c are possible. Plug on (0,0) in the original. If true shade the side with (0,0). If false shade the other:
\[ 5(0) - 2(0) \leq 8 \Rightarrow 0 \leq 8 \] True, so shade the side with (0,0)
answer is a
29. If a pro takes x hrs then an amatuer takes 2x hrs (twice as long).
Pro's rate = 1/x and Amatuer's rate = \( \frac{1}{2x} \)
Use: 1job = time( pro rate + amatuer rate)
\[ 1 = 2(\frac{1}{x} + \frac{1}{2x}) \]
solve for x
\[ x = 3 \]

30. acid1 + acid2 = mixture
\[
\begin{array}{ccc}
\text{acid}1 & \text{acid}2 & \text{mixture} \\
\text{x liters} & \text{12 liters} & (x+12) \text{ liters} \\
.06 & .10 & .09 \\
\end{array}
\]
set up a table
\[
.06x + .10(12) = .09(x + 12)
\]
set up equation from table
\[ 6x + 10(12) = 9(x + 12) \]
multiply by 100 to clear decimals
\[ x = 4 \]

31. \( \log_b \sqrt{3} \) reduce the fraction under the radical sign
\( \log_b 3^{1/2} \) change from radical to exponent form
\[ (1/2) \log_b 3 \]
rewrite using: \( \log_b a^c = c \log_b a \)
\[ (1/2) .62 = .31 \]
sub in .62 for \( \log_b 3 \) and simplify

32. This is the equation of a parabola that opens up, since all answers are possible try looking at x and y intercepts.
y intercept: plug in 0 for x ⇒ y = f(0) = -2 only b & c are possible
x intercepts: plug in 0 for y and solve for x
\[ 0 = x^2 + 4x - 2 \]
use the quadratic formula to find x values
\[ x = -2 \pm \sqrt{6} \approx -2 \pm 2.45 \Rightarrow x \approx -4.45 \text{ or } .45 \]
b is the answer

33. \( 4^x \) can never equal 0 so the graph cannot cross the x axis
a & c are the only possible answers. To find the correct one, check the y intercept (when x = 0).
y = 4^0 = 1 ⇒ graph crosses the y axis at 1

34. \( f(0) \) is the point where x = 0. But x = 0 is the y axis, so the answer is c since the graph crosses the y axis at 2
35. use long division:

\[
\begin{array}{c|ccccc}
\multicolumn{2}{c|}{2x^2} & & & & \\
\hline
3x - 2 & 6x^3 - 13x^2 + 18x - 1 & \text{use } 2x^2 \text{ since } (3x)(2x^2) = 6x^3 \\
\multicolumn{2}{c|}{6x^3 - 4x^2} & \text{multiply } 2x^2 \text{ and } 3x - 2 \\
\multicolumn{2}{c|}{} & \text{change signs and add} \\
\multicolumn{2}{c|}{-9x^2 + 18x} & \text{bring down next term (18x)} \\
\end{array}
\]

\[
\begin{array}{c|ccccc}
\multicolumn{2}{c|}{2x^2 - 3x} & & & & \\
\hline
3x - 2 & 6x^3 - 13x^2 + 18x - 1 & \text{use } -3x \text{ since } (3x)(-3x) = -9x^2 \\
\multicolumn{2}{c|}{6x^3 - 4x^2} & \text{multiply } -3x \text{ and } 3x - 2 \\
\multicolumn{2}{c|}{} & \text{change signs and add} \\
\multicolumn{2}{c|}{-9x^2 + 6x} & \text{bring down next term (-1)} \\
\multicolumn{2}{c|}{+ -} & & & & \\
\multicolumn{2}{c|}{12x - 1} & & & & \\
\end{array}
\]

\[
\begin{array}{c|ccccc}
\multicolumn{2}{c|}{2x^2 - 3x + 4} & & & & \\
\hline
3x - 2 & 6x^3 - 13x^2 + 18x - 1 & \text{use } 4 \text{ since } (3x)(4) = 12x \\
\multicolumn{2}{c|}{6x^3 - 4x^2} & \text{multiply } 4 \text{ and } 3x - 2 \\
\multicolumn{2}{c|}{} & \text{change signs and add} \\
\multicolumn{2}{c|}{-9x^2 + 6x} & & & & \\
\multicolumn{2}{c|}{12x - 1} & & & & \\
\multicolumn{2}{c|}{+ -} & & & & \\
\multicolumn{2}{c|}{12x - 8} & & & & \\
\multicolumn{2}{c|}{7} & & & & \\
\end{array}
\]

answer: \(2x^2 - 3x + 4 + \frac{7}{3x - 2}\)
Geometry Practice Problems

1. \( \triangle ABC \) is an isosceles triangle with base BC. \( L1 \) and \( L2 \) are parallel. \( \angle 1=80^\circ \). Find \( \angle 4 \).

   a. 80° b. 50° c. 45° d. 60°

2. In the figure, the measure of arc \( ABC \) is \( 7\pi / 4 \) and \( O \) is the center. Find \( \angle 1 \).

   a. 30° b. 50° c. 40° d. 45°

3. Find the area of an equilateral triangle with a side of length 12.

   a. \( 72\sqrt{2} \) b. 72 c. \( 36\sqrt{3} \) d. 36

4. Find the area of a circle inscribed in a square with sides of length 8 cm.

   a. \( 4\pi \) cm\(^2\) b. \( 2\pi \) cm\(^2\) c. \( 16\pi \) cm\(^2\) d. \( 8\pi \) cm\(^2\)

5. In the figure, \( \angle 1=40^\circ \). Find Arc \( AB \).

   a. 60° b. 40° c. 20° d. 80°

6. Find the length of one of the equal sides of an isosceles triangle with a perimeter 105 if the base is one-third the length of one of the equal sides.

   a. 45 b. 15 c. 35 d. 25

7. An 8 ft by 10 ft garden is surrounded by a 2 ft walkway. Find the area of the walkway?

   a. 160 ft\(^2\) b. 82 ft\(^2\) c. 158 ft\(^2\) d. 88 ft\(^2\)
8. In the figure, AB=12, DE=9, and BE=4. Find EC.
   a. 12   b. 16   c. 13   d. 21

9. Find the volume of a tent with length 9 ft, height 7 ft, and a 6 foot base.
   a. 567 ft³   b. 189 ft³   c. 94.5 ft³   d. 162 ft³

10. The circle in the figure has a circumference of 10π inches. What is the area of the square circumscribed about the circle?
    a. 25 in²   b. 50 in²   c. 100 in²   d. 25π in²

11. For the right triangle in the figure, find x.
    a. 5   b. √5   c. √3   d. 3

12. In the figure L1 and L2 are parallel. Find x.
    a. 18°   b. 10°   c. 9°   d. 20°

13. 2π/3 radians is how many degrees?
    a. 60°   b. 90°   c. 120°   d. 180°

14. In the figure, BC is a diagonal of rectangle ABCD. EF is perpendicular to AB and ∠DCB=35°. Find ∠EFB.
    a. 35°   b. 45°   c. 55°   d. 65°
15. The object in the figure has a square base with sides of length 4 and a semicircular top. Find the perimeter of the object.
   a. $16+2\pi$  b. $12+2\pi$
   c. $16+4\pi$  d. 32

16. For the parallelogram in the figure $\angle A=50^\circ$. Find $\angle B$.
   a. $180^\circ$  b. $50^\circ$
   c. $130^\circ$  d. $40^\circ$

17. Express $45^\circ$ in radians.
   a. $\pi/8$  b. $\pi/4$  c. $\pi/2$  d. $\pi$

18. In the figure, parallelgram ABCD has an area of 48. If BC = 5 and BE = 4, what is the perimeter of parallelogram ABCD?
   a. 26  b. 32
   c. 34  d. 36

19. The volume of a right cylinder is 300 in$^3$. If the radius is doubled, what will the new volume be?
   a. 900 in$^3$  b. 2400 in$^3$  c. 600 in$^3$  d. 1200 in$^3$

20. Find the area of trapezoid ABCD in the figure below.
   a. 90  b. $11\sqrt{2}+2$  c. $72+6\sqrt{3}$  d. 84
21. Find the volume of the tank in the diagram below.
   where the domes are hemispheres with diameter 6 inches.
   a. $63\pi$ in$^3$  
   b. $396\pi$ in$^3$
   c. $360\pi$ in$^3$  
   d. $1544\pi$ in$^3$

22. Find the arclength of arc AB if O is the center, the measure of arc ACB is 300°, and the radius of the circle is 10.
   a. 60  
   b. $60\pi$
   c. $10\pi/3$  
   d. $50\pi/3$

23. If $\triangle ABC$ and $\triangle abc$ are similar triangles with AC=20, BC=16, bc=12, $\angle B=70^\circ$, and $\angle c=60^\circ$, find $\angle A$ and ac.
   a. $\angle A=50^\circ$, ac=10  
   b. $\angle A=60^\circ$, ac=16
   c. $\angle A=70^\circ$, ac=18  
   d. $\angle A=50^\circ$, ac=15

24. What is the volume of material left if a hole with a radius of 1 is drilled through a solid cube with sides of 4? (see drawing)
   a. $64 - 16\pi$  
   b. $12 - 16\pi$
   c. $16 - 4\pi$  
   d. $64 - 4\pi$

25. In the figure below, the length of arc AB is $\pi$ inches, the radius of the circle is 3 inches, and O is the center. Find the measure of $\angle 1$.
   a. $30^\circ$  
   b. $20^\circ$
   c. $40^\circ$  
   d. $60^\circ$
**Answers**

1. b 2. d 3. c 4. c 5. d 6. a 7. d 8. a 9. b

**Solutions**

1. \( \angle 2 + \angle 3 = 180^\circ - \angle 1 = 180^\circ - 80^\circ = 100^\circ \)
   
   \( \Delta ABC \) isosceles, so \( \angle 2 = \angle 3 \), therefore \( \angle 2 = 50^\circ \)
   
   Since \( L1 \) and \( L2 \) are parallel, \( \angle 2 = \angle 4 \)
   
   \( \angle 4 = 50^\circ \)

2. \( \text{arc AC} = 2\pi - \frac{7\pi}{4} = \frac{\pi}{4} = 180^\circ/4 = 45^\circ \)
   
   Since \( \angle 1 \) is a central angle, \( \angle 1 = \text{arc AC} \)
   
   \( \angle 1 = 45^\circ \)

3. An altitude from a vertex to an opposite side bisects the side.
   
   Use the Pythagorean formula to find the height
   
   \( h^2 + 6^2 = 12^2 \)
   
   \( h = 6\sqrt{3} \)
   
   Area = \( \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(12)(6\sqrt{3}) = 36\sqrt{3} \)

4. \( d = \text{side of the square} = 8 \text{ cm} \), but \( d \) is also the diameter of the circle
   
   \( r = (1/2) d = 4 \text{ cm} \), where \( r \) is the radius of the circle
   
   Area of the circle = \( \pi r^2 = \pi (4 \text{ cm})^2 = 16\pi \text{ cm}^2 \)

5. \( \angle 1 \) is an inscribed angle.
   
   Arc length = twice the measure of the inscribed angle.
   
   Arc AB = 2 \( \angle 1 \); Arc AB = 2 \( 40^\circ \) = 80°

6. If the base = \( x \) then the equal sides = 3\( x \)
   
   Perimeter = 105
   
   105 = \( x + 3x + 3x = 7x \)
   
   \( x = 15 \), so equal sides = (3)(15) = 45

7. Area of garden = (8 ft)(10 ft) = 80 ft²
   
   The garden + walkway together have width 8 ft + 2 ft + 2 ft = 12 ft
   
   The garden + walkway together have length 10 ft + 2 ft + 2 ft = 14 ft
   
   Area of garden + walkway together = (12 ft)(14 ft) = 168 ft²
   
   Area of walkway = 168 ft² – 80 ft² = 88 ft²
8. $\triangle ABC$ and $\triangle DEC$ are similar, so the ratios of corresponding sides are equal
\[
\frac{AB}{DE} = \frac{BC}{EC}
\]
Let $CE = x$, then
\[
\frac{12}{9} = \frac{4 + x}{x}
\]
Cross multiply and solve for $x$: $x = 12$

9. Volume = (area of base) (height)
The base is a triangle with base 6 ft and height 7 ft
Area of base = $(1/2)(7 \text{ ft})(6 \text{ ft}) = 21 \text{ ft}^2$
Volume = $(21 \text{ ft}^2)(9 \text{ ft}) = 189 \text{ ft}^3$

10. Circumference = $\pi d$, where $d$ is the diameter
$10\pi \text{ in} = \pi d$
d = 10 in, this is also the length of the side of the square
Area of a square = $\text{side}^2 = (10 \text{ in})^2 = 100 \text{ in}^2$

11. For right triangles $a^2 + b^2 = c^2$, where $c$ is the hypotenuse
\[
x^2 + 1^2 = 2^2
\]
\[
x^2 = 4 - 1
\]
\[
x = \sqrt{3}
\]

12. $L_1$ and $L_2$ are parallel so the angles are supplemental.
\[
4x + 5x = 180^\circ
\]
x = $20^\circ$

13. Use the fact that $\pi$ radians equals $180^\circ$:
\[
2(180^\circ)/3 = 120^\circ
\]

14. $\overline{AC}$ and $\overline{BD}$ are parallel so $\angle DCB = \angle EBF$
$\angle DCB = 35^\circ$ so $\angle EBF = 35^\circ$
$\angle EFB = 180^\circ - 90^\circ - 35^\circ = 55^\circ$

15. Perimeter of base = $4 + 4 + 4 = 12$
Perimeter of Semicircle: $C = (1/2) \pi d$
d = 4 because of the base.
$C = (1/2) \pi 4 = 2 \pi$
Total perimeter = $12 + 2\pi$
16. Adjacent angles in a parallelogram are supplemental
\[ \angle B + \angle A = 180^\circ \]
\[ \angle B + 50^\circ = 180^\circ \]
\[ \angle B = 130^\circ \]

17. Use the fact that \( 180^\circ \) equals \( \pi \) radians:
\[ \frac{180^\circ}{\pi} = \frac{45^\circ}{x} \]
\[ x = \frac{45^\circ \pi}{180^\circ} \]
\[ x = \frac{\pi}{4} \]

18. Area parallelogram = (base) (height)
\[ 48 = 4 \text{(base)} \]
\[ 12 = \text{base} \]
Perimeter = \( 12 + 5 + 12 + 5 = 34 \)

19. \( V = \pi r^2 h \), where \( V \) is volume of a right cylinder, \( r \) is radius, and \( h \) is the height.

When \( r \) is doubled, \( r^2 \) is replaced by \( (2r)^2 = 4r^2 \)

This increases the original volume by \( 4 \)

\( (300 \text{ in}^3) (4) = 1200 \text{ in}^3 \)

20. \( \triangle CDE \) is a \( 45^\circ-45^\circ-90^\circ \) so
\[ ED = \frac{6\sqrt{2}}{\sqrt{2}} = 6 \]
CE = ED, so \( CE = 6 \)
\( \triangle ABF \) is a \( 30^\circ-60^\circ-90^\circ \) so
\[ AF = \frac{4\sqrt{3}}{2} = 2\sqrt{3} \]

Area = \( \frac{1}{2} (CE)(BC + AD) \)
\[ \text{Area} = \frac{1}{2} (6)(9 + 2\sqrt{3} + 9 + 6) = 72 + 6\sqrt{3} \]

21. \( V_{\text{tank}} = V_{\text{domes}} + V_{\text{cylinder}} \), but two domes = sphere, so
\[ V_{\text{tank}} = V_{\text{sphere}} + V_{\text{cylinder}} \]
\[ V_{\text{tank}} = \frac{4}{3} \pi r^3 + \pi r^2 h \] where \( r \) is 3 inches and \( h \) is 36 inches
\[ V_{\text{tank}} = \frac{4}{3} \pi \times 3^3 + \pi 3^3 \times 36 = 360\pi \]
22. arc measure $AB = 360° - $ arc measure $ACB = 360° - 300° = 60°$

$\angle AOB = 60°$, central angle equals the measure of the intercepted arc

$\text{arclen} = \frac{\angle AOB}{360°} \times 2\pi r = \frac{60}{360°} \times 2\pi \times 10 = \frac{10}{3}\pi$

23. Since the triangles are similar, all corresponding angles are equal and corresponding sides are proportional. So,

$\angle C = \angle c = 60°$

$\angle A = 180° - \angle B - \angle C = 180° - 70° - 60° = 50°$

$\frac{AC}{ac} = \frac{BC}{bc} \Rightarrow \frac{20}{ac} = \frac{16}{12} \Rightarrow ac = 15$

24. $V = V_{\text{cube}} - V_{\text{cylinder}}$

$V = \text{side}^3 - \pi r^2 h = 4^3 - \pi \times 1^2 \times 4 = 64 - 4\pi$

25. $\text{arclen}AB = \frac{\angle AOB}{360°} \times 2\pi r \Rightarrow \pi = \frac{\angle AOB}{360°} \times 2\pi 3$, solve for $\angle AOB$

$\angle AOB = 360° / 6 = 60°$

$\angle 1 = \frac{1}{2} \angle AOB = \frac{1}{2} \times 60° = 30°$
Trigonometry Practice Problems

1. \( \cos (x - \pi/2) = ? \)
   a. -\( \cos x \)  b. \( \sin x \)  c. \( \cos x \)  d. -\( \sin x \)

2. If \( \sin b = \frac{3}{5} \) and \( \frac{\pi}{2} \leq b \leq \pi \), then find \( \tan b \).
   a. \( \frac{4}{3} \)  b. \( \frac{3}{4} \)  c. -\( \frac{3}{4} \)  d. \( \frac{4}{5} \)

3. Find \( \cos A \) in the figure below.
   ![Diagram of a right triangle with sides labeled \( \sqrt{45} \) and \( 2x \).]
   a. \( \frac{2\sqrt{5}}{5} \)  b. \( \frac{1}{2} \)  c. \( \frac{3\sqrt{5}}{2} \)  d. \( 2 \)

4. If \( \sin b = -\frac{5}{13} \) and \( \pi \leq b \leq \frac{3\pi}{2} \), then \( \cos b \).
   a. \( \frac{5}{11} \)  b. \( \frac{12}{13} \)  c. -\( \frac{11}{13} \)  d. -\( \frac{12}{13} \)

5. \( \pi / 6 \) is equal to how many degrees?
   a. 30°  b. 60°  c. 90°  d. 52°

6. Which graph best represents the function: \( y = -(1/2) \sin x \)?
   ![Graphs a, b, c, and d representing different functions with y-values at 2, -1/2, 1/2, and -2, respectively.]
   a.  b.  c.  d.
7. Which graph best represents the function: \( y = \sin 2x \)?
   a. \[
   \begin{array}{cc}
   \text{graph 1} & \text{graph 2} \\
   \end{array}
   \]
   b. \[
   \begin{array}{cc}
   \text{graph 3} & \text{graph 4} \\
   \end{array}
   \]
   
8. Which graph best represents the function: \( y = \sin (x - \frac{\pi}{4}) \)?
   a. \[
   \begin{array}{cc}
   \text{graph 5} & \text{graph 6} \\
   \end{array}
   \]
   b. \[
   \begin{array}{cc}
   \text{graph 7} & \text{graph 8} \\
   \end{array}
   \]

9. On the unit circle (0 to 2\(\pi\) radians), at what radian(s) does \( \sin = -1 \)?
   a. \(\pi\) only
   b. \(\frac{\pi}{2}\) and \(3\pi/2\)
   c. \(3\pi/2\) only
   d. \(\pi/4\) and \(3\pi/4\)

10. At 100 feet from the base of a tower, a wire runs at a 55° angle to the top of the tower (see figure). If the sin 55° \(\approx\) .8192, cos 55° \(\approx\) .5736, and tan 55° \(\approx\)1.4281, find the height of the tower. Round answer to the nearest tenth of a foot.
   a. tower \(\approx\) 81.9 ft
   b. tower \(\approx\) 57.4 ft
   c. tower \(\approx\) 142.8 ft
   d. tower \(\approx\) 174.3 ft

11. If cos b = .5 and b = 60°, then arc cos .5 = ?
   a. 60°
   b. 30°
   c. 5°
   d. 120°
12. On the unit circle (0° to 360°) if tan x = 1, then x = ?
   a. 45° only    b. 45° and 225°
   c. 135° only   d. 135° and 315°

13. Find the distance between points a and b on the rectangular coordinate system shown below.
   a. 7   b. √13
   c. 6   d. √85

14. For x = 0° to x = 360°, inclusive, how many times does sin x = 0?
   a. 1  b. 2  c. 3  d. 4

15. Use the fact that \( \sin 60° = \frac{\sqrt{3}}{2} \) and \( \cos 60° = \frac{1}{2} \) to find the value of sin 120°.
   a. \( \sqrt{3} \)  b. \( \frac{3}{4} \)  c. \( \frac{\sqrt{3}}{2} \)  d. \( \frac{\sqrt{3}}{4} \)

16. 270° is how many radians?
   a. \( \frac{3\pi}{2} \)  b. \( \frac{\pi}{270} \)  c. 270π  d. \( \frac{4\pi}{3} \)

17. What is the supplement of 78°21'? 
   a. 11°39'  b. 12°21'  c. 102°21'  d. 101°39'

**Answers**
1. b  2. c  3. a  4. d  5. a  6. c  7. b  8. a  9. c  
Solutions

1. use the identity:  \( \cos(x-a) = \cos x \cos a + \sin x \sin a \)
   \( \cos(x - \pi/2) = \cos x \cos(\pi/2) + \sin x \sin(\pi/2) \)
   since \( \cos(\pi/2) = 0 \) and \( \sin(\pi/2) = 1 \), \( \cos(x - \pi/2) = \sin x \)

2. \begin{align*}
   \sin b &= \text{(opposite side)} / \text{hypotenuse} = 3/5 \\
   \text{use Pythagorean formula to find } x \\
   x &= -4 \text{ since this is Quadrant II} \\
   \tan b &= \text{(opposite side) / (adjacent side)} = 3 / (-4)
\end{align*}

3. Use the Pythagorean formula to find the missing sides:
   \( \sqrt{45}^2 = (2x)^2 + x^2 \)
   \( 45 = 4x^2 + x^2 = 5x^2 \)
   \( x = 3 \)
   \( \cos A = \text{(adjacent side) / hypotenuse} \)
   \( \cos A = \frac{2x}{\sqrt{45}} = \frac{6\sqrt{5}}{45} = \frac{18\sqrt{5}}{45} = \frac{2\sqrt{5}}{5} \)

4. \begin{align*}
   \sin b &= \text{(opposite side) / hypotenuse} = -5/13 \\
   \text{use Pythagorean formula to find } x \\
   x &= -12 \text{ since this is Quadrant II} \\
   \cos b &= \text{(adjacent side) / hypotenuse} = -12/13
\end{align*}

5. Since \( \pi = 180^\circ, \pi / 6 = 180^\circ / 6 = 30^\circ \)

6. -1/2 is the amplitude. Amplitude affects the range of y values. For this function, y can go from -1/2 to +1/2. Also, the negative sign flips the graph about the x axis. Since the graph of sin is positive from 0 to \( \pi \), this graph will be negative between values.

7. The sin ranges from +1 to -1. Since the amplitude was not changed (see problem 6 above), answers c and d are out. 2 is the period. Period inversely affects when the pattern of the graph repeats. Since the sin function normally repeats at \( 2\pi \), this function will repeat at \( (2\pi) / 2 = \pi \), so b is the correct graph
8. The function $\sin a$ has a period from $a = 0$ to $a = 2\pi$. Since this $\sin$ function has $(x - \pi/4)$ for $a$, it will have a period from $x - \pi/4 = 0$ to $x - \pi/4 = 2\pi$, or, $x = \pi/4$ to $x = 9\pi/4$. This means that the graph is the normal $\sin$ function shifted to the right $\pi/4$ units on the $x$ axis.

9. $\sin \frac{3\pi}{2} = -1$

10. For the tower use: $\tan 55^\circ = \text{tower} / 100$

\[
(100)(\tan 55^\circ) = \text{tower}
\]

\[
(100)(1.4281) \approx \text{tower}
\]

142.8 $\approx$ tower

11. If $\cos b = .5$ then $\arccos .5 = b$

Since $b = 60^\circ$ then $\arccos .5 = 60^\circ$

12. $\tan = \pm 1$ at $45^\circ$ angles and $\tan$ is positive in quadrants I and III, so $x = 45^\circ$ (quad I) or $45 + 180^\circ = 225^\circ$ (quad III)

13. Use the formula for the distance between two points:

\[
d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}, \text{ for points } (x_1, y_1), (x_2, y_2), \text{ and distance } d
\]

\[
d = \sqrt{(5 - 2)^2 + (3 - 3)^2} = \sqrt{7^2 + 6^2} = \sqrt{49 + 36} = \sqrt{85}
\]

14. $\sin 0^\circ = 0$, $\sin 180^\circ = 0$, and $\sin 360^\circ = 0$

15. Use the fact that $\sin 120^\circ = \sin 2(60^\circ)$ and the identity:

\[
\sin 2a = 2 \sin a \cos a
\]

\[
\sin 2(60^\circ) = 2 \sin 60^\circ \cos 60^\circ = 2(\sqrt{3}/2)(1/2) = \sqrt{3}/2
\]

16. Use the fact that $180^\circ = \pi$ radians and set up a proportion:

\[
\frac{270^\circ}{x} = \frac{180^\circ}{\pi} \Rightarrow x = \frac{270\pi}{180} \Rightarrow x = \frac{3\pi}{2}
\]

17. Supplements add to $180^\circ$, so:

$180^\circ = 78^\circ21' + x$

$180^\circ - 78^\circ21' = x$

To subtract, borrow 1 degree which is equal to 60 minutes('):

$179^\circ 60'$

-78° 21'

$101^\circ39'$