

Some Applications of the Eigenvalues and Eigenvectors of a square matrix

1. Communication systems:

Eigenvalues were used by Claude Shannon to determine the theoretical limit to how much information can be transmitted through a communication medium like your telephone line or through the air. This is done by calculating the eigenvectors and eigenvalues of the communication channel (expressed as a matrix), and then waterfilling on the eigenvalues. The eigenvalues are then, in essence, the gains of the fundamental modes of the channel, which themselves are captured by the eigenvectors.

2. Designing bridges:

The natural frequency of the bridge is the eigenvalue of smallest magnitude of a system that models the bridge. The engineers exploit this knowledge to ensure the stability of their constructions.

[Watch the video on the collapse of the Tacoma Narrow Bridge which was built in 1940]

3. Designing car stereo system:

Eigenvalue analysis is also used in the design of the car stereo systems, where it helps to reproduce the vibration of the car due to the music.

4. Electrical Engineering:

The application of eigenvalues and eigenvectors is useful for decoupling three-phase systems through symmetrical component transformation.

5. Mechanical Engineering:

Eigenvalues and eigenvectors allow us to "reduce" a linear operation to separate, simpler, problems. For example, if a stress is applied to a "plastic" solid, the deformation can be dissected into "principle directions"- those directions in which the deformation is greatest. Vectors in the principle directions are the eigenvectors and the percentage deformation in each principle direction is the corresponding eigenvalue.

Oil companies frequently use eigenvalue analysis to explore land for oil. Oil, dirt, and other substances all give rise to linear systems which have different eigenvalues, so eigenvalue analysis can give a good indication of where oil reserves are located. Oil companies place probes around a site to pick up the waves that result from a huge truck used to vibrate the ground. The waves are changed as they pass through the different substances in the ground. The analysis of these waves directs the oil companies to possible drilling sites.

Eigenvalues are not only used to explain natural occurrences, but also to discover new and better designs for the future. Some of the results are quite surprising. If you were asked to build the strongest column that you could to support the weight of a roof using only a specified amount of material, what shape would that column take? Most of us would build a cylinder like most other columns that we have seen. However, Steve Cox of Rice University and Michael Overton of New York University proved, based on the work of J. Keller and I. Tadjbakhsh, that the column would be stronger if it was largest at the top, middle, and bottom. At the points of the way from either end, the column could be smaller because the column would not naturally buckle there anyway.

Does that surprise you? This new design was discovered through the study of the eigenvalues of the system involving the column and the weight from above. Note that this column would not be the strongest design if any significant pressure came from the side, but when a column supports a roof, the vast majority of the pressure comes directly from above.

Very (very, very) roughly then, the eigenvalues of a linear mapping is a measure of the distortion induced by the transformation and the eigenvectors tell you about how the distortion is oriented. It is precisely this rough picture which makes **PCA** (Principal Component Analysis = A statistical procedure) very useful.