Some applications of the Vector spaces:

1) It is easy to highlight the need for linear algebra for **physicists** - Quantum Mechanics is entirely based on it. Also important for time domain (state space) control theory and stresses in materials using tensors.

2) In **circuit theory**, matrices are used to solve for current or voltage. In electromagnetic field theory which is a fundamental course for communication engineering, conception of divergence, curl are important.
For other fields of engineering, computer memory extensively uses the conception of partition of matrices. If the matrices size gets larger than the space of computer memory it divides the matrices into submatrices and does calculation.

3) Linear operator plays a key role in **computer graphics**. For many CAD software generates drawing using linear operators, And don't forget about cryptography.

4) Matrices can be cleverly used in **cryptography**. Exchanging secret information using matrix is very robust and easy in one sense. How about **MATLAB**? This software is widely used in engineering fields and MATLAB's default data type is matrix. And, of course, Linear Algebra is the underlying theory for all of **linear differential equations**. In electrical engineering filed, vector spaces and matrix algebra come up often.

5) **Least square** estimation has a nice subspace interpretation. Many linear algebra texts show this. This kind of estimation is used a lot in digital filter design, tracking (Kalman filters), control systems, etc.

Note1: The method of **least squares** is a standard approach to the approximate solution of overdetermined systems, i.e., sets of equations in which there are more equations than unknowns. "Least squares" means that the overall solution minimizes the sum of the squares of the errors made in the results of every single equation.

Note2: In mathematics, a system of linear equations is considered **overdetermined** if there are more equations than unknowns. The terminology can be described in terms of the concept of constraint counting. Each unknown can be seen as an available degree of freedom. Each equation introduced into the system can be viewed as a constraint that restricts one degree of freedom.

6) **Fourier Analysis**. The discrete Fourier transform is a nice finite dimensional example, and the FFT algorithm is just fun to learn about. Continuous time is nice too, but then you are in infinite dimensional space of course ...

Note: A **Fast Fourier transform** (**FFT**) is an algorithm to compute the discrete Fourier transform (DFT) and its inverse. There are many different FFT algorithms involving a wide range of mathematics, from **simple complex-number arithmetic** to **group theory** and **number theory**; this article gives an overview of the available techniques and some of their general properties, while the specific algorithms are described in subsidiary articles linked below.

7) Incidence matrices from graphs that represent **circuit topology**. The Kirchhoff voltage and current laws can then be nicely represented in matrix form. Yes, I had an undergrad electrical engineering class that covered this stuff, and included state-space analysis, matrix exponentials, etc.

Note: In mathematics, an **incidence matrix** is a matrix that shows the relationship between two classes of objects. If the first class is X and the second is Y, the matrix has one row for each element of X and one column for each element of Y. The entry in row x and column y is 1 if x and y are related (called **incident** in this context) and 0 if they are not.

8) Orthogonal projections are used all the time. For example, in **adaptive beam forming**, if the interference signals have a very high signal to noise, we essentially project the data orthogonal to the interference subspace in order to maximize the signal to noise of the desired signals. In the limit of infinite interference to noise, you get exactly the subspace projection.

9) Solving Nonhomogeneous PDEs (**Partial Differential Equations**) using Eigen function expansions. This is infinite dimension again, but relating sturm-liouville to symmetric matrices, and solving Ax=c by eigenvector expansions is fun. This kind of problem comes up in **Electrodynamics** (Electrical Engineering), fluids (mechanical/civil/chemical engr.), and quantum mechanics (electrical/materials/chemical engr). Etc.

10) ODEs (Ordinary Differential Equations), of course. Basic signals and systems courses are basically based on the fact that complex exponentials are the eigenfunctions of constant coefficient ODEs. Fourier transform is basically a projection onto this space.

In linear algebra, the **Singular Value Decomposition** (**SVD**) is a factorization of a real or complex matrix, with many useful applications in signal processing and statistics.

Formally, the singular value decomposition of an $m \times n$ real or complex matrix M is a factorization of the form

$$M = U\Sigma V^*,$$

where U is a $m \times n$ real or complex unitary matrix, Σ is an $m \times n$ rectangular diagonal matrix with nonnegative real numbers on the diagonal, and V^* (the conjugate transpose of V) is an $n \times n$ real or complex unitary matrix. The diagonal entries $\Sigma_{i,i}$ of Σ are known as the **singular values** of M. The m columns of U and the n columns of V are called the **left-singular vectors** and **right-singular vectors** of M, respectively.

11) SVD is used everywhere for things like **compressing images**, **decomposing 2-D filters into simple outer products of 1-D filters** (much more efficient to implement). SVD for numeric is also important...

Just a note of interest:

Polynomials have a great use in science, mainly in **approximations using interpolations**. Since the set of polynomials with degree smaller than n is a vector space, we can take an orthonormal basis for it and easily find approximation for any real value function (depending on the inner product of course). Note that the reason we can do this is that the real valued functions are also a vector space

Note1: In the mathematical field of numerical analysis, **interpolation** is a method of constructing new data points within the range of a discrete set of known data points.

Note2: In engineering and science, one often has a number of data points, obtained by sampling or experimentation, which represent the values of a function for a limited number of values of the independent variable. It is often required to **interpolate** (i.e. estimate) the value of that function for an intermediate value of the independent variable. This may be achieved by curve fitting or regression analysis.