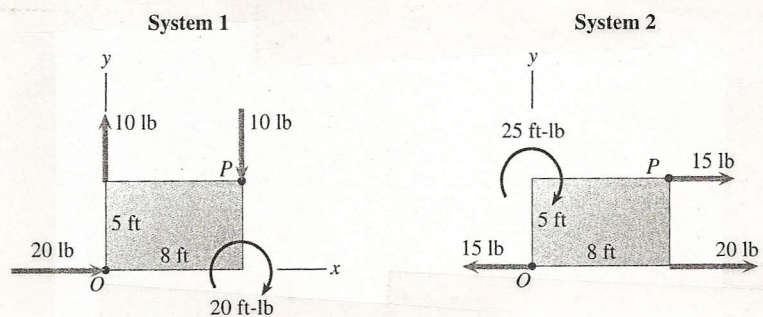


**Question:**

Are these two systems Equivalent?

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Are these two systems Equivalent?

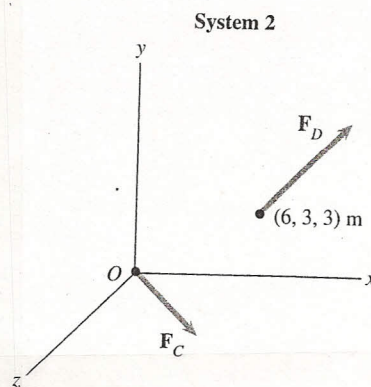
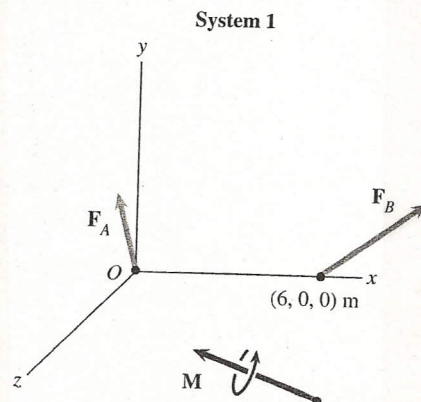
$$F_A = -10\mathbf{i} + 10\mathbf{j} - 15\mathbf{k} \text{ (kN)},$$

$$F_B = 30\mathbf{i} + 5\mathbf{j} + 10\mathbf{k} \text{ (kN)},$$

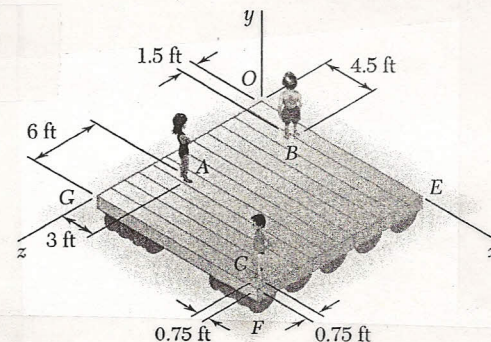
$$M = -90\mathbf{i} + 150\mathbf{j} + 60\mathbf{k} \text{ (kN-m)},$$

$$F_C = 10\mathbf{i} - 5\mathbf{j} + 5\mathbf{k} \text{ (kN)},$$

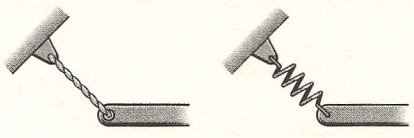
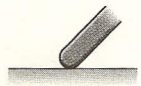
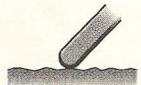
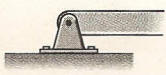
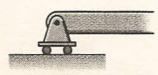
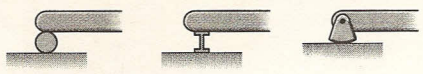
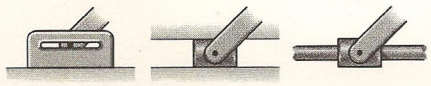

$$F_D = 10\mathbf{i} + 20\mathbf{j} - 10\mathbf{k} \text{ (kN)}.$$

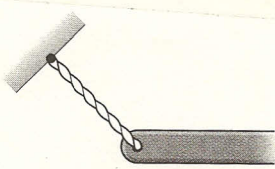
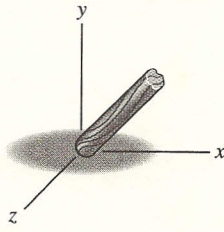
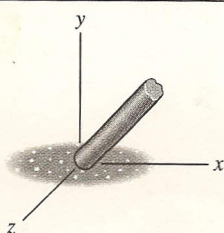
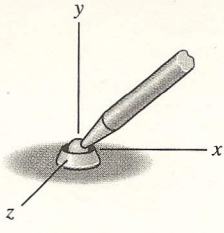
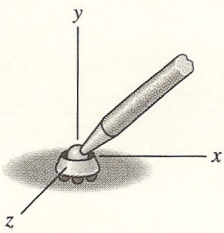
**Question:**

Three children are standing on a 15 × 15-ft raft. If the weights of the children at points A, B, and C are 85 lb, 60 lb, and 90 lb, respectively, determine the magnitude and the point of application of the resultant of the three weights.



**Table 5.1** Supports used in two-dimensional analysis

Supports
 <p>Rope or Cable      Spring</p>
 <p>Contact with a Smooth Surface</p>
 <p>Contact with a Rough Surface</p>
 <p>Pin Support</p>
 <p>Roller Support</p>
 <p>Equivalents</p>
 <p>Constrained Pin or Slider</p>
 <p>Built-in (Fixed) Support</p>

Supports
 <p>Rope or Cable</p>
 <p>Contact with a Smooth Surface</p>
 <p>Contact with a Rough Surface</p>
 <p>Ball and Socket Support</p>
 <p>Roller Support</p>

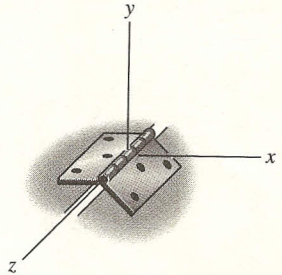
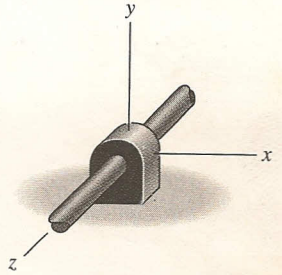
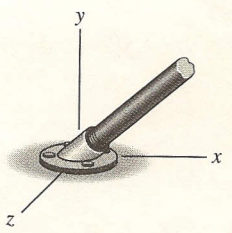
Supports
 <p>Hinge (The <math>z</math> axis is parallel to the hinge axis.)</p>
 <p>Bearing (The <math>z</math> axis is parallel to the axis of the supported shaft.)</p>
 <p>Built-in (Fixed) Support</p>



Table 5.2 Supports used in three-dimensional applications.

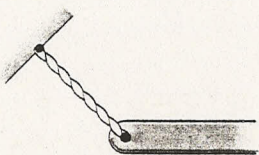
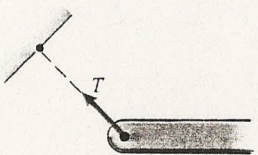
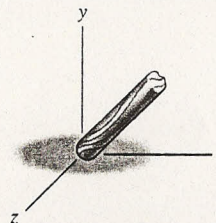
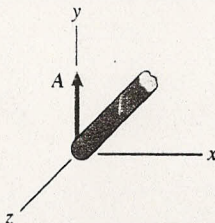
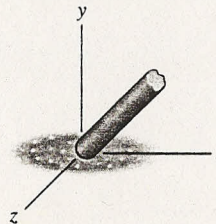
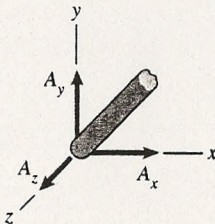
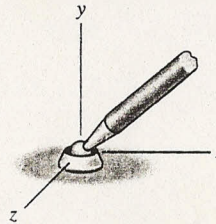
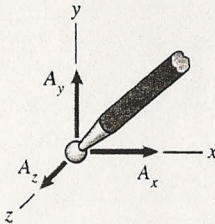
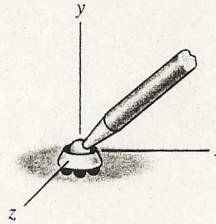
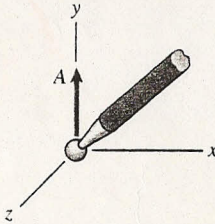
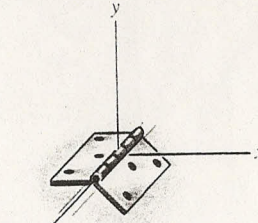
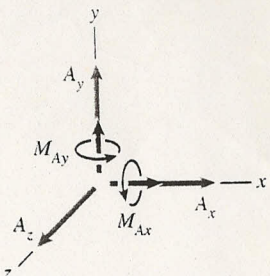
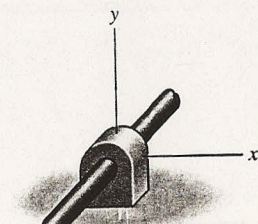
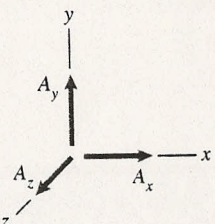
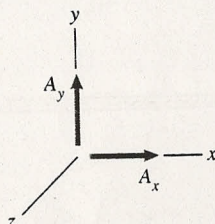
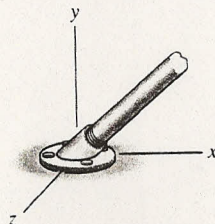
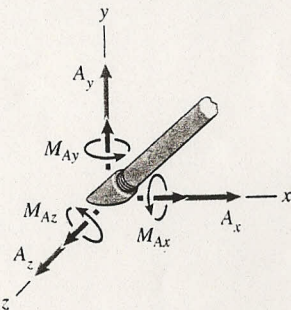
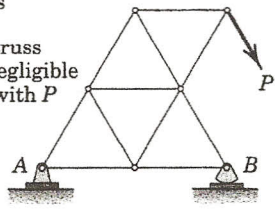
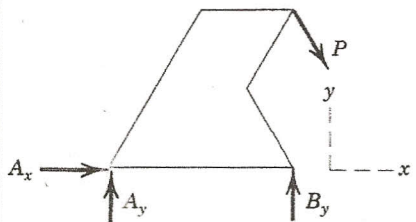
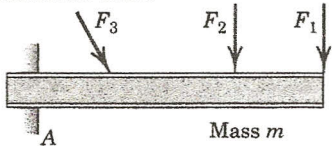
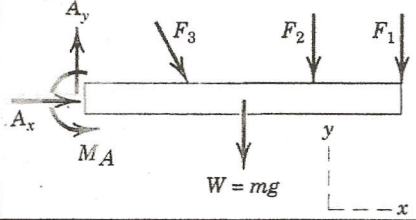
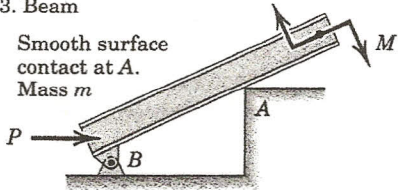
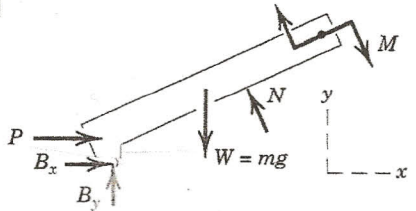
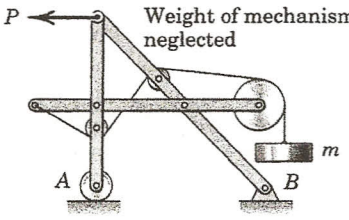
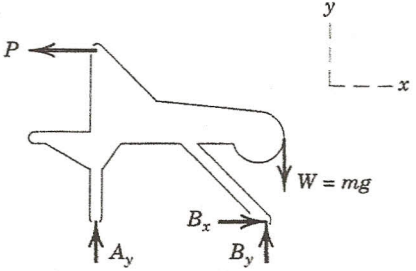
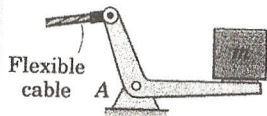
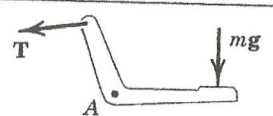

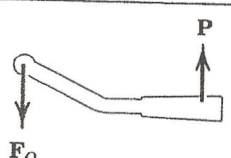
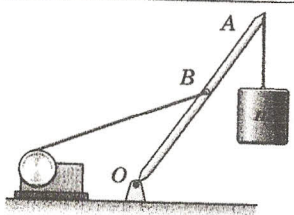
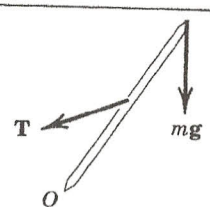
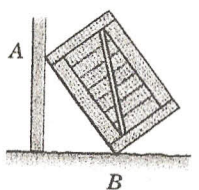
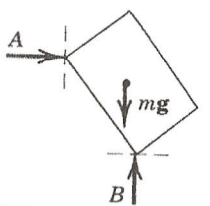
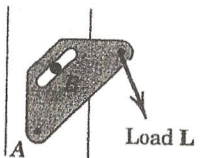
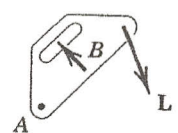
Supports	Reactions
 Rope or Cable	 One Collinear Force
 Contact with a Smooth Surface	 One Normal Force
 Contact with a Rough Surface	 Three Force Components
 Ball and Socket Support	 Three Force Components
 Roller Support	 One Normal Force

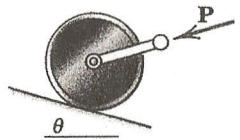
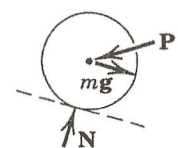
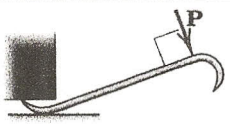
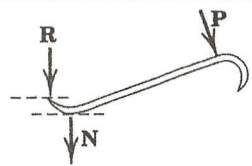
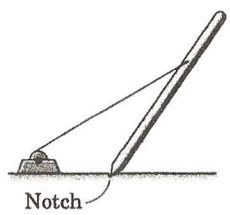
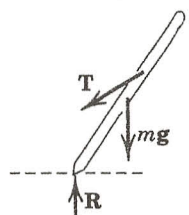
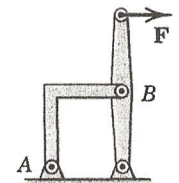
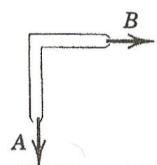
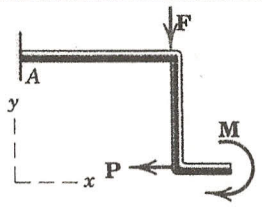
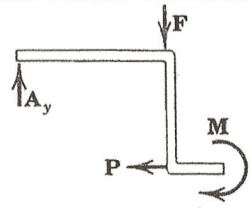
Table 5.2 (continued)

Supports	Reactions
 Hinge (The $z$ axis is parallel to the hinge axis.)	 Three Force Components, Two Couple Components
 Bearing (The $z$ axis is parallel to the axis of the supported shaft.)	 (When no couples are exerted)
	 (When no couples and no axial force are exerted)
 Built-in (Fixed) Support	 Three Force Components, Three Couple Components

SAMPLE FREE-BODY DIAGRAMS	
Mechanical System	Free-Body Diagram of Isolated Body
<p>1. Plane truss</p> <p>Weight of truss assumed negligible compared with <math>P</math></p> 	
<p>2. Cantilever beam</p> 	
<p>3. Beam</p> <p>Smooth surface contact at A.</p> <p>Mass <math>m</math></p> 	
<p>4. Rigid system of interconnected bodies analyzed as a single unit</p> <p>Weight of mechanism neglected</p> 	

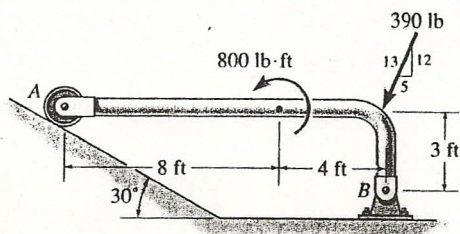


	Body	Incomplete FBD
1. Bell crank supporting mass $m$ with pin support at A.		
2. Control lever applying torque to shaft at O.		
3. Boom OA, of negligible mass compared with mass $m$ . Boom hinged at O and supported by hoisting cable at B.		
4. Uniform crate of mass $m$ leaning against smooth vertical wall and supported on a rough horizontal surface.		
5. Loaded bracket supported by pin connection at A and fixed pin in smooth slot at B.		

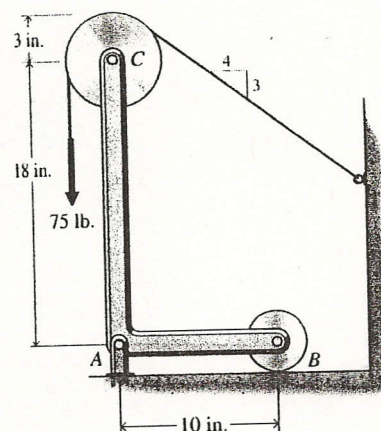
	Body	Wrong or Incomplete FBD
1. Lawn roller of mass $m$ being pushed up incline $\theta$ .		
2. Prybar lifting body A having smooth horizontal surface. Bar rests on horizontal rough surface.		
3. Uniform pole of mass $m$ being hoisted into position by winch. Horizontal supporting surface notched to prevent slipping of pole.		
4. Supporting angle bracket for frame; Pin joints.		
5. Bent rod welded to support at A and subjected to two forces and couple.		

**Question 1 to 4:**

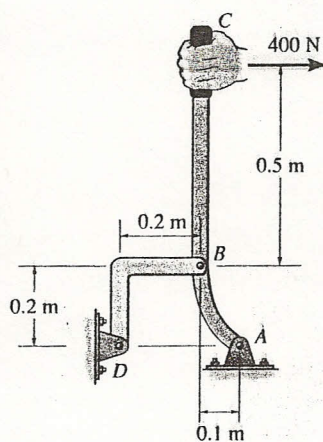
For the assemblies shown find reactions at supports A and B.



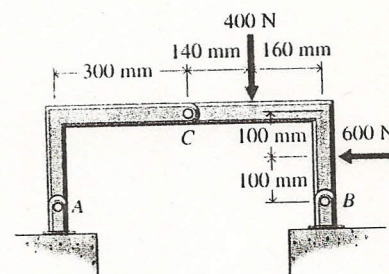
(1)



(2)



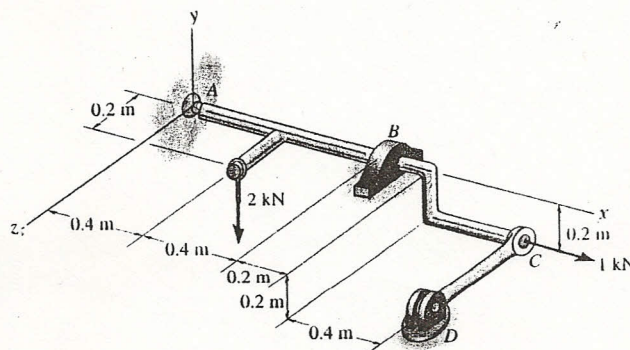
(3)



(4)

**Question 5:**

In the assembly shown find the reactions at the ball-and-socket joint and journal bearing B. Bearing B does not exert any axial thrust and has negligible moment reactions.





NAME: \_\_\_\_\_

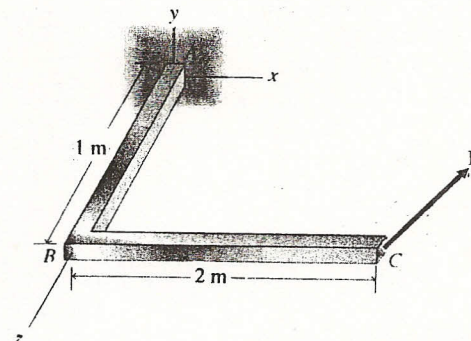
SID # \_\_\_\_\_

Question 1 : (20 Points)

The rod ABC has a constant cross-sectional area with a weight of 10 kN/m. The force  $\mathbf{F} = -4\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$  (kN) is applied at C. Support A is fixed (built-in) at the wall.

a- Draw Free-Body-Diagram of rod ABC.

b- Determine the force and moment reactions at supports A.



NAME: \_\_\_\_\_

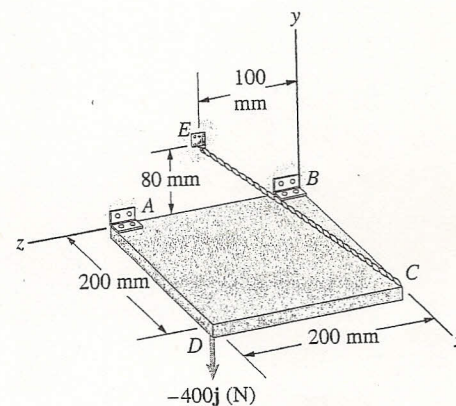
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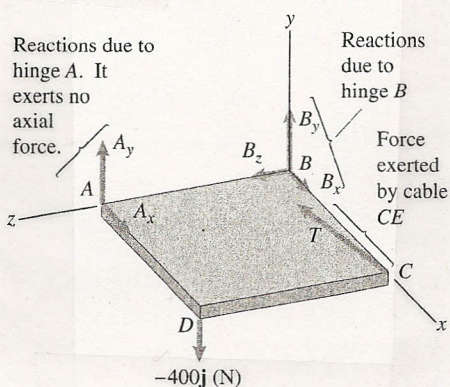
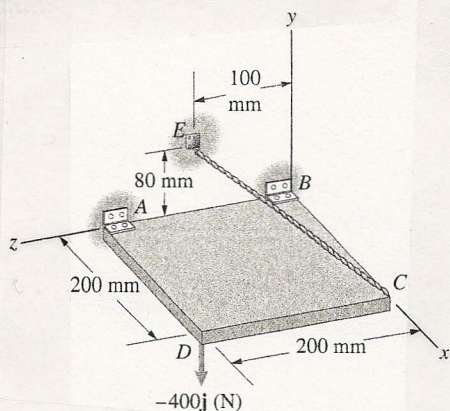
Question 1 : (20 Points)

The plate shown is supported by two hinges A and B, and cable CE. The hinges do not exert couples on the plate, and hinge A does not exert axial thrust on the plate.

a- Draw Free Body Diagram of the plate.

b- Determine the reactions at the hinges and the tension in the cable.





**Draw the Free-Body Diagram**

$$\vec{T} = T(-0.842\mathbf{i} + 0.337\mathbf{j} + 0.421\mathbf{k}). \text{ N}$$

**Apply the Equilibrium Equations**

$$\Sigma F_x = A_x + B_x - 0.842T = 0,$$

$$\Sigma F_y = A_y + B_y + 0.337T - 400 = 0,$$

$$\Sigma F_z = B_z + 0.421T = 0.$$

$$\begin{aligned} \Sigma \mathbf{M}_{(\text{point } B)} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2 & 0 & 0 \\ -0.842T & 0.337T & 0.421T \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.2 \\ A_x & A_y & 0 \end{vmatrix} \\ &+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2 & 0 & 0.2 \\ 0 & -400 & 0 \end{vmatrix} \\ &= (-0.2A_y + 80)\mathbf{i} + (-0.0842T + 0.2A_x)\mathbf{j} \\ &+ (0.0674T - 80)\mathbf{k} = 0. \end{aligned}$$

The scalar equations are

$$\Sigma M_x = -0.2A_y + 80 = 0,$$

$$\Sigma M_y = -0.0842T + 0.2A_x = 0,$$

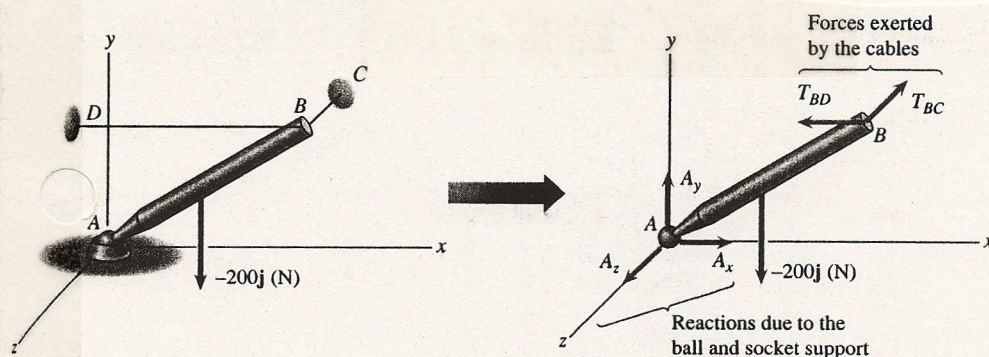
$$\Sigma M_z = 0.0674T - 80 = 0.$$

Solving these equations, we obtain the reactions

$$T = 1187 \text{ N}, \quad A_x = 500 \text{ N}, \quad A_y = 400 \text{ N}.$$

$$B_x = 500 \text{ N}, \quad B_y = -400 \text{ N}, \quad B_z = -500 \text{ N}.$$





(a) Obtaining the free-body diagram of the bar.

**Apply the Equilibrium Equations** The sums of the forces in each coordinate direction equal zero:

$$\begin{aligned}\Sigma F_x &= A_x - T_{BD} = 0, \\ \Sigma F_y &= A_y - 200 = 0, \\ \Sigma F_z &= A_z - T_{BC} = 0.\end{aligned}\tag{5.22}$$

Let  $\mathbf{r}_{AB}$  be the position vector from A to B. The sum of the moments about A is

$$\begin{aligned}\Sigma \mathbf{M}_{(\text{point A})} &= [\mathbf{r}_{AB} \times (-T_{BC} \mathbf{k})] + [\mathbf{r}_{AB} \times (-T_{BD} \mathbf{i})] \\ &\quad + \left[ \frac{1}{2} \mathbf{r}_{AB} \times (-200 \mathbf{j}) \right] \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0.6 & 0.4 \\ 0 & 0 & -T_{BC} \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0.6 & 0.4 \\ -T_{BD} & 0 & 0 \end{vmatrix} \\ &\quad + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.5 & 0.3 & 0.2 \\ 0 & -200 & 0 \end{vmatrix} \\ &= (-0.6T_{BC} + 40) \mathbf{i} + (T_{BC} - 0.4T_{BD}) \mathbf{j} + (0.6T_{BD} - 100) \mathbf{k}.\end{aligned}$$

The components of this vector (the sums of the moments about the three coordinate axes) each equal zero:

$$\begin{aligned}\Sigma M_x &= -0.6T_{BC} + 40 = 0, \\ \Sigma M_y &= T_{BC} - 0.4T_{BD} = 0, \\ \Sigma M_z &= 0.6T_{BD} - 100 = 0.\end{aligned}$$

Solving these equations, we obtain the tensions in the cables:

$$T_{BC} = 66.7 \text{ N}, \quad T_{BD} = 166.7 \text{ N}.$$

(Notice that we needed only two of the three equations to obtain the two tensions. The third equation is redundant.)

Then from Eqs. (5.22) we obtain the reactions at the ball and socket support:

$$A_x = 166.7 \text{ N}, \quad A_y = 200 \text{ N}, \quad A_z = 66.7 \text{ N}.$$

## DISCUSSION

Notice that by summing moments about A we obtained equations in which the unknown reactions  $A_x$ ,  $A_y$ , and  $A_z$  did not appear. You can often simplify your solutions in this way.

**Question 1 :**

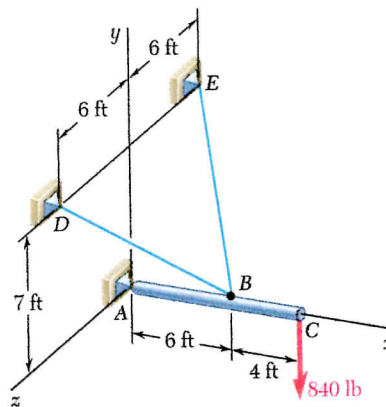
A 10-ft boom is acted upon by the vertical force of 840-lb and is held by two cables and a ball-and-socket support at A. Determine the tension in each cable and the reaction at support A.

Answers:

$$T_{BE} = 1,100\text{-lb}$$

$$T_{BD} = 1,100\text{-lb}$$

$$\mathbf{A} = 1200\mathbf{i} - 560\mathbf{j} + 0\mathbf{k} \text{ lb}$$

**Question 2 :**

A shaft is loaded through a pulley and a lever as shown. The bearings A and B exert only force reactions on the shaft, and bearing B does not have any axial thrust. Draw free-Body-Diagram of the shaft, find force P required for equilibrium and the reaction in each bearing.

Answers:

$$P = 150\text{-lb}$$

$$\mathbf{A} = -200\mathbf{i} - 117\mathbf{j} + 928\mathbf{k} \text{ lb}$$

$$\mathbf{B} = 0\mathbf{i} - 267\mathbf{j} - 278\mathbf{k} \text{ lb}$$

