Minimization of Energy in Reverse Osmosis Water Desalination Using Constrained Nonlinear Optimization

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This work focuses on the minimization of energy in reverse osmosis water desalination. First, a set of dimensionless parameters were derived to characterize the reverse osmosis desalination process. On the basis of the assumptions of constant pump efficiency and no pressure change in the retentate, the minimization of energy cost per volume of produced permeate or specific energy consumption (SEC) for three different reverse osmosis modules (single-stage, two-stage, and single-stage with an energy recovery device (ERD)) were then formulated and solved as constrained nonlinear optimization problems. Without ERD, the optimal solution to SEC normalized by the feed salinity was solely dependent on a dimensionless parameter γ that is comprised of the membrane area, hydraulic permeability, feed rate, and salinity. In the thermodynamic limit where γ approaches infinity, the minimal SEC approaches 4 and 3.596 times the feed salinity and the fractional recovery approaches 0.5 and 0.574 for single-stage and two-stage reverse osmosis modules, respectively. However, the water yield approaches zero in both cases. With an ERD, the SEC can be further reduced to the feed salinity while the fractional recovery approaches zero. It is also shown that the SEC flattens out quickly as γ increases, and a cutoff of γ (around 0.5−1.5 for one-stage and 1−3 for two-stage membrane modules) can be used to achieve a reasonable water yield as well as a low SEC slightly above the theoretical global minimum.

1. Introduction

Worldwide, about three billion people lack access to clean and safe drinking water. As an important water desalination technology, reverse osmosis membrane separation can potentially help alleviate the water crisis faced by many nations. It is reported that the energy consumption of the pumps to drive the membrane module accounts for a major portion of the total cost of water desalination. In some extreme cases, it reaches 45% of the total production cost. Therefore, research is needed to reduce the energy cost per volume of produced permeate or specific energy consumption (SEC) in order to make this technology more affordable to people.

Considerable efforts have been made to reduce the SEC by enhancing membrane performance. A highly permeable membrane module allows the water to pass through the membrane easier, which will lead to more efficient water production and a lower energy cost. As another approach to reduce the SEC, energy recovery devices (ERDs) have been used to exchange energy between the brine and the feed. For example, the brine can pass through a rotary turbine that drives an auxiliary pump to pressurize the feed. There are four main types of ERDs: a Pelton wheel, Grundfos Pelton wheel, turbo charger, and pressure exchanger, with the last one being the most efficient. It is reported that a direct pressure transmission of the high-pressure brine to the feed can reach an energy recovery efficiency up to 98% and lead to a significant economic effect. The amortization time of the ERD installation is typically about 2−4 years.

Reducing the SEC in the reverse osmosis desalination processes might also be achieved by optimizing the operating conditions and/or membrane module configuration using the empirical Taguchi method and model-based analysis and optimization techniques. For example, it is reported that applying pressure slightly above the concentrate osmotic pressure would result in the lowest energy cost. Multiojective optimization and optimization of overall cost have also been proposed to account for the capital cost, feed intake and pretreatment, and cleaning and maintenance cost.

Recently, for the first time, Zhu and co-workers studied extensively the effect of the thermodynamic restriction on the cost minimization of the reverse osmosis/nanofiltration membrane desalination process. They developed a first of its type rigorous theoretical cost minimization framework to quantify the energy cost, membrane cost, brine management cost, and effect of energy recovery and feed fluctuation on cost minimization. They further examined the energy minimization of the two-pass membrane desalination process and concluded that two-pass membrane desalination is less energy efficient than single pass. Additionally, Zhu et al. discovered for the first time a very important lumped dimensionless cost factor (the Zhu number) to evaluate the relative contribution of the membrane cost over the energy cost in the total water production cost. The Zhu number reflects the impact of feedwater osmotic pressure, salt rejection requirement, membrane permeability, purchase price of electrical energy, and membrane module. As shown in their work, if the Zhu number is less than 0.01 (for example, seawater desalination), the membrane cost is less than about 10% of the energy cost, indicating that for seawater desalination the top priority is to lower the energy consumption, while if the Zhu number is larger than 1 (for example, low salinity brackish water), the membrane cost is on the same order of energy cost and therefore significant effort should be devoted to lower the membrane cost as well.

Despite previous research efforts in the area of energy minimization in reverse osmosis desalination, there is no generic optimization framework that is formulated from the perspective of scalable parameters. Several simple and physically meaningful dimensionless groups are derived in this paper to clearly illustrate the coupled behavior between membrane property (area and permeability), feed conditions (flow rate and salinity), and operating conditions (pressure difference and fractional recov-
The derived equation is then included as a constraint in the optimization problem. It is shown by various examples that an unambiguous understanding of the relationship between the SEC and the process conditions can be obtained, and the conclusions are scalable.

2. Optimization of SEC in Reverse Osmosis Processes

2.1. Single-Stage Reverse Osmosis Module. In a typical reverse osmosis water desalination process, the feed passes through a pump and the pressurized fluid enters the membrane channel, where water is allowed to flow through it while the transport of salts is blocked. As shown in Figure 1, the mass balance on a control volume represented by a membrane area of \( dA \) is given by

\[
-dQ = dA \cdot L_p \cdot (\Delta P - \Delta \pi)
\]  

(1)

where \(-dQ\) is the flow rate of water across the membrane area \( dA \), \( L_p \) is the membrane hydraulic permeability, and \( \Delta P \) and \( \Delta \pi \) are the differences in the system pressure and osmotic pressure across the membrane, respectively.

It is generally acknowledged that the osmotic pressure varies linearly with salt concentration:

\[
\pi = f_{os} C
\]  

(2)

where \( f_{os} \) is the osmotic pressure coefficient and \( C \) is the salt concentration in the solution. As a result, a mass balance of the salt can be written as

\[
Q \cdot \frac{\pi_r}{f_{os}} = (Q + dQ) \cdot \frac{(\pi_r + d\pi_r)}{f_{os}} + (-dQ) \cdot \frac{\pi_p}{f_{os}}
\]  

(3)

where \( \pi_r \) and \( \pi_p \) are the osmotic pressures of the retentate and permeate, respectively. If \( dQ \cdot d\pi_r \) is ignored, an integration of eq 3 from the beginning of the membrane channel to a specific location where the osmotic pressure of the retentate is \( \pi_r \) yields

\[
\int_{Q_0}^{Q} \frac{dQ}{Q} = - \int_{\pi_r}^{\pi_p} \frac{d\pi_r}{\pi_r - \pi_p}
\]  

(4)

where \( Q \) is the feed flow rate and \( \pi_r \) is the osmotic pressure of the feed. If \( \pi_p \ll \pi_r \), holds along the entire membrane channel (this assumption is valid if the rejection rate is high or \( \pi_p \ll \pi_r \)), an analytical solution to eq 4 can be obtained by substituting \( d(\pi_r - \pi_p) \) for \( d\pi_r \) and the result is given as

\[
Q = \frac{\Delta \pi}{Q_f} = \frac{\Delta \pi_r}{\Delta \pi_0}
\]  

(5)

where \( \Delta \pi = \pi_r - \pi_p \) and \( \Delta \pi_0 \) is \( \Delta \pi \) at the entrance of the membrane channel (or \( \pi_0 \)).

Integration of eq 1 from the entrance to the end of the membrane channel using the relationship represented by eq 5 yields

\[
\int_{Q_0}^{Q_f} \frac{dQ}{\Delta P - \Delta \pi_0/Q_f} = \int_{0}^{L_p} L_p \, dA
\]  

(6)

where \( Q_p \) is the permeate flow rate.

On the basis of the assumption of a negligible pressure drop along the flow direction, the solution to eq 6 is derived as

\[
\frac{Q_p}{\Delta P} + \frac{\Delta \pi_0/Q_f}{(\Delta P)^2} \ln \frac{1 - \Delta \pi_0/Q_f}{1 - \Delta \pi_0/\Delta P} = AL_p
\]  

(7)

Therefore, the average driving force \( (\Delta P - \Delta \pi) \) is

\[
\Delta P - \Delta \pi = \frac{Q_f}{AL_p} = \frac{1 - \Delta \pi_0/\Delta P}{1 + \Delta \pi_0/Q_f} \ln \frac{1 - \Delta \pi_0/\Delta P}{1 - \Delta \pi_0/\Delta \pi_0}
\]  

(8)

Equation 8 can be written in a compact dimensionless form as follows

\[
\beta = \alpha \left[ 1 + \frac{\alpha}{\gamma} \ln \frac{1 - \frac{\alpha}{\gamma}}{1 - \frac{1}{\gamma}} \right]
\]  

(9)

or

\[
\gamma = \alpha \left[ Y + \alpha \ln \frac{1 - \frac{1}{\gamma}}{1 - \frac{1}{\gamma}} \right]
\]  

(10)

where \( \alpha = \Delta \pi_0/\Delta P \), \( \beta = (AL_p\Delta \pi_0)/Q_p \), \( Y = Q_f/Q_0 \), and \( \gamma = (AL_p\Delta \pi_0)/Q_f = \beta Y \). \( \alpha \) is the feed salinity to pressure ratio, \( Y \) is the water fractional recovery. The common numerator in \( \beta \) and \( \gamma \) (= \( AL_p\Delta \pi_0 \)) is the water flow rate that would cross the same membrane at an average driving force of \( \Delta \pi_0 \). In this sense, \( \beta \) is inversely proportional to the average driving force normalized by \( \Delta \pi_0 \), which can also be derived using eq 8: \( (\Delta P - \Delta \pi_0)/\Delta \pi_0 = Q_p/(AL_p\Delta \pi_0) = 1/\beta \). The flux commonly used in the literature is directly related to \( \beta \) as \( Q_f/AL_p = (L_p\Delta \pi_0)/\beta \). Moreover, the driving forces normalized by \( \Delta \pi_0 \) at the entrance and outlet of the membrane module can also be expressed by these dimensionless parameters. For example, at the entrance, \( (\Delta P - \Delta \pi_0)/\Delta \pi_0 = 1/\alpha - 1 \). At the exit, \( (\Delta P - \Delta \pi_0)/(\Delta P - \Delta \pi_{exit}) = 1/\alpha - 1/(1 - Y) \). The driving force at the exit of the membrane must be non-negative, implying that \( 1 - Y - \alpha \geq 0 \) or \( \Delta P \geq \Delta \pi_0/(1 - Y) \). At the theoretical thermodynamic limit, \( 1 - Y - \alpha = 0 \) or \( \Delta P = \Delta \pi_0/(1 - Y) \). A schematic of the normalized forces described using dimensionless parameters along the membrane channel is shown in Figure 2. This figure is plotted to scale based on \( \alpha = 0.5 \).

Equation 9 or 10 is the physical constraint on the reverse osmosis process which must be satisfied in the process optimization. It implies that \( Y \) and applied pressure \( \Delta P \) cannot be independent if the membrane and feed conditions are given or the dimensionless parameter \( \gamma \) is specified.

On the basis of eq 10, the contours of \( \alpha \) and \( \gamma/\alpha = AL_p\Delta P/Q_p \) as function of \( \gamma \) and \( Y \) are plotted in Figure 3. These two plots can be used to illustrate the relationship between membrane...
property ($AL_p$), feed conditions ($\Delta \pi_0$ and $Q_f$), and operating conditions ($Y$ and $\Delta P$). For example, when $\gamma$ is fixed, $\alpha$ decreases as $Y$ increases. When $Y$ is fixed, it is seen from Figure 3a that $\alpha$ decreases as $Y$ decreases. These imply that $\Delta P$ should be high when either $Y$ is high or $AL_p$ is low. The relationship between $\Delta \pi_0$ and $\Delta P$ is not obvious in Figure 3a because both $\gamma$ and $Y$ are functions of $\Delta \pi_0$. However, from Figure 3b it is clear that $\Delta P$ should increase in order to maintain the same $Y$ when the feed salinity $\Delta \pi_0$ increases.

The energy cost in the reverse osmosis desalination process is typically described based on the concept of specific energy consumption (SEC) or the electrical energy demand per cubic meter of permeate$^{2,32-34}

$$SEC = \frac{Q_f \Delta P}{\eta_{\text{pump}} Q_f} = \frac{1}{\eta_{\text{pump}}} \frac{\Delta \pi_0}{\alpha Y}$$

where $\eta_{\text{pump}}$ is the pump efficiency, which can be determined from the pump characteristic curve typically provided by the manufacturer.$^{36}$ As a preliminary study, the pump efficiency is assumed to be a constant and a modified SEC defined as $SEC_m = \eta_{\text{pump}} SEC$ is introduced to indicate this fact. In the operation of reverse osmosis desalination, the pump might be chosen so that it is operated near the best efficiency point (BEP). The effect of pump efficiency will be included in future studies. It can be verified that both $SEC_m$ and SEC have the same unit as $\eta_{\text{pump}}$. On the basis of eq 10 or the results in Figure 3, the contour of $SEC_m/\Delta \pi_0$ is plotted in Figure 4. It is clearly seen that there is a minimum of $SEC_m/\Delta \pi_0$ for any given $\gamma$. Moreover, the optimal solution decreases as $\gamma$ increases. In order to find the optimal solutions numerically, the following constrained nonlinear optimization problem is formulated

$$\min_{\alpha, \gamma} SEC_m = \frac{\Delta \pi_0}{\alpha Y}$$

s.t.

$$\gamma = \alpha \left[ Y + \alpha \ln \left( \frac{1 - \alpha}{1 - Y - \alpha} \right) \right]$$

where $\gamma$ is specified based on the feed conditions and membrane properties. To avoid taking logarithms of nonpositive numbers during the iteration procedure in solving eq 12, a new variable $z = \ln((1 - \alpha)/(1 - Y - \alpha))$ is introduced. According to this definition

$$Y = (1 - \alpha)(1 - e^{-z})$$

On the basis of eq 13, eq 9 can be written as

$$\beta = \frac{\alpha(Y + \alpha z)}{Y}$$

Similarly, eq 10 is converted to

$$\gamma = \alpha(Y + \alpha z)$$

Therefore, an optimization problem equivalent to eq 12 is derived as follows

$$\min_{\alpha, z} SEC_m = \frac{\Delta \pi_0}{\alpha Y}$$

s.t.

$$Y = (1 - \alpha)(1 - e^{-z})$$

$$\gamma = \alpha(Y + \alpha z)$$

which can be solved using standard optimization packages. Note that there are only two variables because $Y$ is an explicit function of $\alpha$ and $z$. If global optimization techniques are not used, there is a possibility of finding local minima. However, different initial values have been tried to yield the same results. Figure 4 also indicates there is a unique minimum for each given $\gamma$.

The optimal solutions to $SEC_m/\Delta \pi_0$ and the desired $\alpha$ and $Y$ and at different values of $\gamma$ (from 0.1 to 3) are shown in Figure 5. It is seen that as $\gamma$ increases, the optimal $SEC_m/\Delta \pi_0$ and $Y$ decrease while the optimal $\alpha$ increases. At the theoretical limit where $\gamma \rightarrow \infty$, the optimal solution is $SEC_m/\Delta \pi_0 = 4$ and $\alpha = Y = 0.5$. In this extreme case, $1 - \alpha - Y = 0$, which suggests that the system is in thermodynamic equilibrium at the exit of the membrane channel. Because $\gamma \rightarrow \infty$ implies that either $AL_p \rightarrow \infty$ or $Q_f = 0$, it is not desired to operate the reverse osmosis desalination process at this extreme condition. In fact, a long tale is observed in the profile of $SEC_m/\Delta \pi_0$ at optimal conditions. When $\gamma$ is above 1.5, the improvement in $SEC_m/\Delta \pi_0$ is not obvious. Therefore, the reverse osmosis membrane module might be operated at $\gamma = 0.5-1.5$ in order to get a reasonable

*Figure 2*. Schematic of normalized driving forces described by dimensionless parameters along the membrane channel.

*Figure 3*. Contours of $\alpha$ and $\gamma/\alpha$ and as a function of $\gamma$ and $Y$. 

*Figure 4*. Schematic of normalized driving forces described by dimensionless parameters along the membrane channel. 

*Figure 5*. Contours of $\alpha$ and $\gamma/\alpha$ and as a function of $\gamma$ and $Y$. 

*Figure 6*. Schematic of normalized driving forces described by dimensionless parameters along the membrane channel.
water yield and a SEC$_m$/Δπ$_0$ slightly above 4, the optimal SEC$_m$/Δπ$_0$ of the unconstrained optimization problem (i.e., without the equality constraint γ = α(Y + αz)). Therefore, from a practical point of view, the approach taken in Zhu et al.$^2$ is neat and reasonable.

The driving forces in the reverse osmosis processes under optimal conditions are shown in Figure 6a. Clearly, the driving forces at both the entrance and the exit of the membrane channel as well as the average driving force (ΔP - Δπ) decrease as γ increases. As γ approaches infinity, ΔP - Δπ$_0$ and ΔP - Δπ$_\text{exit}$ approach Δπ$_0$ and zero, respectively. ΔP - Δπ also approaches zero, which can be verified using eq 13, or

\[
1 - Y - \alpha = (1 - \alpha)e^{-z} \quad (17)
\]

At the thermodynamic limit where 1 - Y - α = 0, either α → 1 (which implies Y → 0) or z → ∞ (which implies γ → ∞). In either case, $\beta = \gamma / Y \rightarrow \infty$. Therefore

\[
\lim_{1 - Y - \alpha \to 0} \frac{\Delta P - \Delta \pi}{\Delta \pi_0} = \lim_{\rho \to \infty} \frac{1}{\beta} = 0 \quad (18)
\]

Because $Q_p$ is proportional to the average driving force, it will be zero when γ approaches infinity (where SEC$_m$/Δπ$_0$ = 4), as shown in Figure 6b. The fact that the optimal $Q_p$ reduces as the optimal SEC$_m$/Δπ$_0$ decreases suggests that both SEC$_m$/Δπ$_0$ and $\beta$ (note that $1/\beta$ is proportional to $Q_p$) should be included in the optimization problem. This becomes meaningful when feed intake and pretreatment costs are not negligible. There are at least two approaches to account for the effect of recovery. One is to include the recovery in the objective function. The other is to add an inequality constraint $Y \geq Y_{\text{min}}$ in the optimization problem to guarantee a minimum recovery. Due to the limit of space, only the former is discussed in this subsection while the latter is applied in a different example later. In Figure 7, SEC$_m$/Δπ$_0$ + $w\cdot β$ (where $w$ is the weight factor) is plotted as a function of γ for $w = 0.1, 0.5, 1, 2$. In each case a minimum is observed at γ around 0.5 - 1.5. The corresponding SEC$_m$ and $Q_p$ can also be calculated from the same figure based on the optimal γ. One may also solve the following optimization problem

\[
\begin{align*}
\min_{\alpha, \gamma, \beta} J &= \frac{1}{\alpha Y} + w\beta \\
\text{s.t.,} \quad Y &= (1 - \alpha)(1 - e^{-z}) \\
\gamma &= \alpha(Y + \alpha z) \\
\beta &= \gamma / Y
\end{align*} \quad (19)
\]

The optimal solutions based on the same weighting factors are shown in Table 1. The results are consistent with those in Figure 7.

To implement the results obtained from this work, one may first determine the membrane property (AL$_p$) experimentally.
Figure 7. Optimization of $\gamma$ accounting for both SEC, and $Q_f$.

Table 1. Optimization of SEC Accounting for Water Yield

<table>
<thead>
<tr>
<th>$W$</th>
<th>$\gamma$</th>
<th>$Y$</th>
<th>$\alpha$</th>
<th>$1/\beta$</th>
<th>SEC/Δπ₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.2425</td>
<td>0.5334</td>
<td>0.4616</td>
<td>0.4293</td>
<td>4.0616</td>
</tr>
<tr>
<td>0.5</td>
<td>0.7985</td>
<td>0.5874</td>
<td>0.3962</td>
<td>0.7356</td>
<td>4.2970</td>
</tr>
<tr>
<td>1.0</td>
<td>0.6331</td>
<td>0.6214</td>
<td>0.3544</td>
<td>0.9816</td>
<td>4.5403</td>
</tr>
<tr>
<td>2.0</td>
<td>0.4912</td>
<td>0.6606</td>
<td>0.3070</td>
<td>1.3448</td>
<td>4.9316</td>
</tr>
</tbody>
</table>

This can be done by recording the water recoveries at different feed rates and the fixed applied pressure and salinity. A plot $Q_f/\Delta\pi_0$ vs $\alpha[Y + \alpha \ln(1 - \alpha)/(1 - Y - \alpha)]$ should yield a straight line passing through the original point according to eq 10. The slope is just $L_p$. Then, based on $\gamma = AL_p\Delta\pi_0/Q_f$, the operator calculates the range of $Q_f(0.7 - 2 AL_p\Delta\pi_0)$ that is optimal in terms of SEC and water yield. That is, if the membrane property ($AL_p$) and feed salinity ($\Delta\pi_0$) and are fixed, $Q_f$ cannot be randomly chosen for the sake of energy cost. Subsequently, based on the actual $\gamma$ value (e.g., $\gamma = 1$) and Figure 5, the operator determines the optimal applied pressure ($\alpha = 0.45$ or $\Delta P = 2.2 \pi_0$). In this condition, the theoretical water recovery will be 54% and the theoretical permeate flow rate will be $AL_p\Delta\pi_0/Y_f = 0.54 L_p\Delta\pi_0$. If $Q_f$ cannot be varied in some cases, $\gamma$ will be fixed and Figure 5 can still be used to find out the optimal applied pressure. However, in this case the optimal SEC/Δπ₀ may or may not be close to 4, depending on what the actual $\gamma$ is. For example, if $\gamma = 0.4$ (fixed), the smallest SEC/Δπ₀ is 5.3, occurring at an optimal applied pressure of 3.2$AL_p\Delta\pi_0$ (or a recovery of 61%). A lower applied pressure with a recovery of 50% would lead to a SEC/Δπ₀ greater than 5.3 in this case.

2.2. Two-Stage Reverse Osmosis Module. A reduction of energy consumption in the reverse osmosis desalination process can also be achieved by using a multiple-stage configuration.37 A schematic of the two-stage reverse osmosis desalination process is shown in Figure 8. In this process, the retentate from the first stage is pressurized further by passing through a pump and sent to the second reverse osmosis module where additional water is recovered.

With results obtained in the single-stage reverse osmosis process, the optimization of a two-stage process can be formulated with a little additional effort. Following a similar procedure, the definitions of dimensionless parameters $\alpha$, $\beta$, and $\gamma$ are listed in Table 2. In this case, the SEC assuming constant pump efficiency is

$$SEC_m = \frac{Q_f AL_p \Delta P_1 + Q_f (1 - Y_1) \Delta P_2}{Q_f Y_1 + Q_f (1 - Y_1) Y_2}$$

(20)

Therefore, the optimization problem is formulated as

$$\min_{\alpha_1, \alpha_2, z_1, z_2} SEC_m = \frac{\Delta \pi_0 Y_f}{Y_1 + (1 - Y_1) Y_2}$$

s.t.

$$Y_1 = (1 - \alpha_1)(1 - e^{-z_1})$$

$$Y_2 = (1 - \alpha_2)(1 - e^{-z_2})$$

(21)

$$\gamma_1 = \alpha_1(1 - \alpha_1)(1 - e^{-z_1} + \alpha_1 z_1)$$

$$\gamma_2 = \alpha_2(1 - \alpha_2)(1 - e^{-z_2} + \alpha_2 z_2)$$

$$\gamma_{total} = \gamma_1 + (1 - Y_1) \gamma_2$$

$$\alpha_2 \leq \alpha_1/(1 - Y_1)$$

where $\gamma_{total} = AL_p L_p \Delta \pi_0 / Q_f$ is directly related to the total membrane area. The inequality constraint is to guarantee a non-negative $\Delta P$ across the second pump.

The above problem is useful in the design of the reverse osmosis desalination process where the allocation of membrane areas in each stage might be optimized. However, in the operation of reverse osmosis desalination where the areas are fixed, one more constraint should be added. For example, if the membrane areas are equal, the equality constraint $\gamma_1 = \gamma_{total}/2$ should be included in eq 21.

![Figure 8. Schematic of a simple two-stage reverse osmosis process.](image-url)
The optimal $\frac{SEC_m}{\Delta \pi_0}$ and overall fractional recovery $Y_{overall}$ for single-stage and two-stage reverse osmosis with/without the equal membrane area constraint are shown in Figure 9. It is seen that there is not much difference in three cases where $\gamma$ is below 0.3. However, the advantage of using two stages becomes apparent as $\gamma$ becomes greater because $\frac{SEC_m}{\Delta \pi_0}$ is smaller. Another advantage of using two stages is that $Y_{overall}$ is also higher even though it is not included in the objective function. The optimal $\frac{SEC_m}{\Delta \pi_0}$, fractional recoveries, area allocation, pressure rises across the pumps, and average driving forces.
across the membrane module at each stage with/without the equal area constraint are shown in Figures 10–14. It is seen that when γ is below 0.2, the SECs, fraction recoveries, and area allocations in each stage do not lie on a smooth curve formed by other points where γ is above 0.2 without the equal area constraint. This is because the optimal ∆P/∆π (calculated by 1/R(1 − Y1) − 1/R1) is 0 when γ is below 0.2, which implies that the two-stage membrane module is in fact a single stage. Therefore, the allocation of A1 and A2 can be arbitrary, and Y1, Y2, (SECm/∆π0), and (SECm/∆π0)2 will change accordingly. In this case, using different initial guesses would change the converged optimal solution to SECm/∆π0, Y, and A at each stage in Figures 10a, 11a, and 12a. However, the overall SECm/∆π0 and Y as well as ∆P at each stage would remain the same. For the two-stage module, excluding the equal area constraint would lead to a slightly lower SECm/∆π0 (not differentiable from Figure 9a) and a higher overall water recovery for most γs in the range of interest. The Yoverall using equal membrane area allocation is higher if γ is in the range of 0.37–0.83. From Figure 13, it is seen that the pressure rise across the second pump is always lower than the one across the first pump. Similarly, the average driving force in the second stage (Qp/2Lp = ∆π0/β3(1 − Y1)) is consistently lower than the one in the first stage (Qp/1Lp = ∆π0/β3), as shown in Figure 14. Moreover, as γ increases, the optimal average driving force decreases. These are consistent with the fact that SECm/∆π0 is lower in the second stage and reduces as γ increases. Therefore, it can be concluded that operating the reverse osmosis near the thermodynamic limit (where the driving force is smaller) could significantly reduce the specific energy cost. The fact that SECm/∆π0 flattens out quickly implies that a γ = 1–3 should maintain a low SEC and a high water flow yield at the same time. This conclusion is similar to the one drawn in the single-stage reverse osmosis.

Table 3. Optimal Solutions of Two-Stage Reverse Osmosis at the Thermodynamic Limit

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>SECm/∆π0</td>
<td>4.4115</td>
<td>2.3473</td>
<td>3.5963</td>
</tr>
<tr>
<td>∆P/∆π0 (for pump)</td>
<td>1.5321</td>
<td>0.8152</td>
<td>2.3473</td>
</tr>
<tr>
<td>Y</td>
<td>0.3473</td>
<td>0.3473</td>
<td>0.5740</td>
</tr>
<tr>
<td>α</td>
<td>0.6527</td>
<td>0.6527</td>
<td></td>
</tr>
<tr>
<td>γ∞</td>
<td>∞</td>
<td>∞</td>
<td></td>
</tr>
<tr>
<td>∆P − ∆π0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

At the thermodynamic limit where γtotal approaches infinity, it is shown that the optimal solution is the same for the two-stage reverse osmosis processes with/without the equal area constraint. A detailed solution to the fractional water recoveries, pressure rises, and SECm/∆π0, etc., at each stage and in the overall process are shown in Table 3. It is expected that the optimal SECm will be even smaller when three or more stages are used. This conclusion is substantiated by previous research, which shows that the SEC decreases when the number of membrane elements increases.5

Figure 13. Optimal ∆P across the pump for the two-stage reverse osmosis module (a) without and (b) with equal membrane area constraint.

Figure 14. Optimal normalized average driving forces in each stage for the two-stage reverse osmosis module (a) without and (b) with equal membrane area constraint.

Figure 15. Schematic of a simple reverse osmosis process with an energy recovery device.
2.3. Single-Stage Reverse Osmosis Module with Energy Recovery.

Because the brine from the membrane module has a higher pressure than the initial pressure of the feed, the energy consumption in the reverse osmosis desalination can be reduced if the brine and the feed exchanges energy through an ERD. Consider a process shown in Figure 15: if the pressure in the brine at the exit of the ERD is $P_e$, the following equations can be written

$$
\frac{\text{SEC}}{\Delta \pi_0} = a \gamma, \quad \gamma = \frac{\Delta P_{\text{brine}}}{Q_f}
$$

$$
\alpha = b \gamma, \quad \gamma = \frac{\Delta P_{\text{brine}}}{Q_f}
$$

$$
Y_{\text{fr}} = \frac{Q_r}{Q_f}, \quad \gamma = \frac{\Delta P_{\text{brine}}}{Q_f}
$$

$$
\left(\frac{\Delta P - \Delta P_{\text{brine}}}{Q_f} \right) = c \gamma, \quad \gamma = \frac{\Delta P_{\text{brine}}}{Q_f}
$$

---

**Figure 16.** Optimal SEC/Δπ₀ as a function of γ and energy recovery efficiency $\eta_{\text{erd}}$ (a) without and (b) with the minimum recovery constant $Y_{\text{min}} = 0.5$.

**Figure 17.** Optimal $\alpha$ as a function of γ and energy recovery efficiency $\eta_{\text{erd}}$ (a) without and (b) with the minimum recovery constant $Y_{\text{min}} = 0.5$.

**Figure 18.** Optimal fractional recovery as a function of γ and energy recovery efficiency $\eta_{\text{erd}}$ (a) without and (b) with the minimum recovery constant $Y_{\text{min}} = 0.5$.

**Figure 19.** Optimal normalized average driving force as a function of γ and energy recovery efficiency $\eta_{\text{erd}}$ (a) without and (b) with the minimum recovery constant $Y_{\text{min}} = 0.5$. 
\[ Q(P_i - P_0) = Q_b(P_l - P_e) \quad (22) \]

or

\[ Q(P_i - P_l) + Q_b(P_l - P_e) = Q(P_i - P_0) \quad (23) \]

It can be derived that

\[ Q(P_i - P_l) = \Delta P \times \left[ Q_l - \frac{P_l - P_e}{P_l - P_0} Q_b \right] \quad (24) \]

Let \( \eta_{\text{erd}} = (P_l - P_0)(P_l - P_0) \) be the energy recovery efficiency, it is easily verified that \( \eta_{\text{erd}} \leq 1 \) because \( P_e \geq P_0 \).

The optimization problem is then formulated as follows

\[
\min_{\alpha, \gamma} \text{SEC}_m = \frac{1 + \eta_{\text{erd}} Y - \eta_{\text{erd}}}{\alpha Y} \\
\text{s.t.} \quad \gamma = \alpha(Y + \alpha z) \\
Y = (1 - \alpha)(1 - e^{-\gamma}) \\
Y \geq Y_{\text{min}}
\]

where \( Y_{\text{min}} \) is the minimum fractional recovery set by the plant operator. The reason for inclusion of the inequality constraint will become apparent later.

The optimal \( \text{SEC}_m/\Delta \pi_{0} \), \( \alpha \), \( Y \), and \( 1/\beta \) (normalized average driving forces) for different values of \( \eta_{\text{erd}} \) (0–90%) with/without the minimum fractional recovery (\( Y_{\text{min}} = 0.5 \)) are shown in Figures 16–19. At a fixed \( \gamma \), as \( \eta_{\text{erd}} \) increases, the optimal \( \text{SEC}_m/\Delta \pi_{0} \) and \( 1/\beta \) decreases continuously. Moreover, the optimal \( Y \) and \( \Delta P/\Delta \pi_{0} \) also decrease. At \( \eta_{\text{erd}} = 90\% \) and \( \gamma = 3 \), the optimal \( \text{SEC}_m/\Delta \pi_{0} \) is only 1.7365, which is less than the one in both the single-stage and the two-stage membrane modules without ERD. The corresponding \( \alpha = 0.7566 \) and \( Y = 0.2417 \). At the theoretical limit of \( \eta_{\text{erd}} = 100\% \), the optimal solution is \( \text{SEC}_m/\Delta \pi_{0} = 1 \), \( Y = 0 \), \( \alpha = 1 \), and \( \gamma \) can be any value. This is not desired as no water will be recovered under such a condition. However, if the constraint of minimal water recovery (for example, \( Y_{\text{min}} = 0.5 \)) is included in the optimization problem, a slightly higher \( \text{SEC}_m/\Delta \pi_{0} \) with a minimum water recovery of 0.5 can be obtained at the same time. This higher fractional recovery is achieved by using a higher \( \Delta P \) so that the average driving force is also higher. As a comparison, at \( \gamma = 3 \), when \( \eta_{\text{erd}} = 90\% \) and 100%, the optimal \( \text{SEC}_m/\Delta \pi_{0} \) satisfying \( Y_{\text{min}} = 0.5 \) is 2.2000 and 2.0000, respectively. The corresponding \( \alpha = 0.5000 \) and \( Y = 0.5000 \) in both cases. The operation is slightly above the thermodynamic limit.

3. Conclusions

A set of dimensionless parameters is derived to characterize the reverse osmosis process. It reveals the coupled behavior between membrane property (area and permeability), feed conditions (flow rate and salinity), and operating conditions (pressure difference and fractional recovery). With the introduction of dimensionless parameters, the mathematical models of reverse osmosis modules with various configurations can be easily built and the derived optimal conditions for SEC are scalable.

At the thermodynamic limit, both the driving force at the exit of the membrane channel and the average driving force are zero. The conclusion is equivalent to \( \beta = \infty \). If the water fractional recovery is not zero (which is meaningful for a desalination process), it can also be translated to \( \gamma = \infty \).

The optimal \( \text{SEC}_m/\Delta \pi_{0} \) is solely a function of \( \gamma \) for both one-stage and two-stage reverse osmosis modules. Even though the global minimum \( \text{SEC}_m/\Delta \pi_{0} \) occurs at the thermodynamic limit (4 and 3.596 times the feed salinity for single-stage and two-stage reverse osmosis modules, respectively) where \( \gamma = \infty \), the water yield \( Q_0 \) will be zero. However, the fact that \( \text{SEC}_m/\Delta \pi_{0} \) flattens out quickly as \( \gamma \) increases suggests that a cutoff of \( \gamma \) (around 0.5–1.5 for one stage and 1–3 for two-stage) can be used to achieve reasonable water yield and low \( \text{SEC}_m \). In this case, the system is operated close to the thermodynamic limit. The two-stage module is shown to be better than the one-stage one in terms of both \( \text{SEC}_m \) and fractional water recovery. The performance of membrane modules with three or more stages is expected to be even better in terms of \( \text{SEC}_m \).

Using ERDs could significantly reduce \( \text{SEC}_m \). The theoretical global minimum of \( \text{SEC}_m/\Delta \pi_{0} \) is equal to the salinity of the feed, but the water yield is zero. A minimal water recovery can be included in the optimization scheme to obtain a slightly higher \( \text{SEC}_m \) with a much better water recovery.

As a concluding comment, the nonlinear optimization framework developed in this work provides a flexible way to minimize \( \text{SEC}_m \) subject to various equality and inequality constraints. It can be easily adapted to solve other problems raised in a desalination plant.

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Nomenclature

\( \alpha = \Delta \pi_{0}/\Delta P \), dimensionless
\( \beta = AL_{0}/\Delta \pi_{0}Q_{0}, \) dimensionless
\( \Delta = \text{difference across the membrane} \)
\( \Delta \pi_{0} = \text{osmotic pressure difference at the entrance of the membrane module, bar} \)
\( \eta_{\text{erd}} = \text{ERD efficiency, dimensionless} \)
\( \eta_{\text{pump}} = \text{pump efficiency, dimensionless} \)
\( \gamma = AL_{0}/\Delta \pi_{0}Q_{0}, \) dimensionless
\( \pi = \text{osmotic pressure, bar} \)
\( A = \text{membrane area, m}^2 \)
\( C = \text{salt concentration, kg cm}^{-3} \)
\( f_{oa} = \text{osmotic pressure coefficient, bar} \cdot \text{m}^2 \cdot \text{kg}^{-1} \)
\( L_{0} = \text{hydraulic permeability, m s}^{-1} \cdot \text{bar}^{-1} \)
\( P = \text{pressure, bar} \)
\( Q = \text{flow rate, m}^3 \cdot \text{s}^{-1} \)
\( \text{SEC}_m = \text{specific energy consumption times the pump efficiency, J} \cdot \text{m}^{-3} \)
\( Y = Q_{0}/Q_{L}, \) dimensionless
\( z = \ln(1 - \alpha)/(1 - Y - \alpha), \) dimensionless
\( b = \text{brine} \)
\( f = \text{feed} \)
\( p = \text{permeate} \)
\( r = \text{retentate} \)

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