Chapter 2

Conservation of Energy

2.1 Mechanical Concepts of Energy

2.1-1 Work and Kinetic Energy

Consider a moving body with velocity \( \mathbf{V} \) shown in Figure 2.1-1. The body is acted on by a resultant force \( \mathbf{F} \), which could be a function of \( s \), a distance along the path from point 0. \( \mathbf{F}_s \) is a component of \( \mathbf{F} \) along the path.

![Figure 2.1-1 Force acting on a moving body.](image)

The magnitude of the component \( \mathbf{F}_s \) is related to the change in the magnitude of \( \mathbf{V} \) by Newton’s second law of motion:

\[
F_s = ma = m \frac{dV}{dt}
\]  

(2.1-1)

Using the chain rule, the above expression can be written as

\[
F_s = m \frac{dV}{dt} = m \frac{dV}{ds} \frac{ds}{dt} = mV \frac{dV}{ds}
\]  

(2.1-2)

We have used the definition \( V = \frac{ds}{dt} \). Separating the variables and integrating from \( s_1 \) to \( s_2 \) gives

\[
\int_{V_i}^{V_f} mVdV = \int_{s_1}^{s_2} F_s ds
\]  

(2.1-3)

\[
\frac{1}{2} m \left( V_f^2 - V_i^2 \right) = \int_{s_1}^{s_2} \mathbf{F} \cdot ds
\]  

(2.1-4)
The quantity \( \frac{1}{2} mV^2 \) is the kinetic energy, KE, of the body. The integral \( \int_{s_1}^{s_2} F_s ds \) is the work of the force \( F_s \) acting on the body from \( s_1 \) to \( s_2 \). \( F_s ds \) is the scalar product of the force vector \( \mathbf{F} \) and the displacement vector \( ds \). Both kinetic energy and work are scalar quantities. The work done on the body can be considered a transfer of energy to the body, where it is stored as kinetic energy.

**Example 2.1-1**

A spear of mass, \( m = 0.3 \) kg, is propelled horizontally from a spear gun by a scuba diver. The initial speed of the spear is \( V_0 = 30 \) m/s. The force that resists its motion through the water is given by \( F_D = kV^2 \), where \( k = 0.033 \) N·sec\(^2\)/m\(^2\). The spear is effective against sharks when its speed is above \( V = 10 \) m/s. Estimate the effective range of the spear.

**Solution**

**Step #1:** Define the system.

System: spear.

**Step #2:** Find equation that contains \( s \), the distance the spear travels.

Conservation of momentum: \( \sum \mathbf{F} = ma = m \frac{dV}{dt} \)

In this equation \( s \) is related to \( V \), speed of the spear by \( V = \frac{ds}{dt} \)

**Step #3:** Apply the momentum balance on the spear.

\[
m \frac{dV}{dt} = m \frac{dV}{ds} \frac{ds}{dt} = - F_D
\]

**Step #4:** Specify the boundary conditions for the differential equation.

The effective range of the spear is the distance from \( s = 0 \) where the initial speed of the spear is 30 m/s to the distance \( s \) where the speed of the spear is 10 m/s.

**Step #5:** Solve the resulting equation and verify the solution.

\[
m \frac{dV}{ds} \frac{ds}{dt} = - F_D
\]

\[
mV \frac{dV}{ds} = - kV^2
\]

\[
\frac{m}{k} \int_{v_0}^{v} \frac{dV}{V} = - \int_{s_0}^{s} ds
\]
\[ s = -\frac{m}{k} \ln \left( \frac{V}{V_o} \right) = \frac{m}{k} \ln \left( \frac{V_o}{V} \right) \]

\[ s = \frac{0.3}{0.033} \ln \left( \frac{30}{10} \right) = 10.0 \text{ m} \]

**2.1-2 Potential Energy**

Gravitational work can be defined as the work due to gravitational force field. The gravitational work, \( W_g \), required to raise a body of mass \( m \) vertically from \( z_1 \) to \( z_2 \) is

\[ W_g = \int_{z_1}^{z_2} F_g \, dz = \int_{z_1}^{z_2} mg \, dz = mg \int_{z_1}^{z_2} \, dz = mg(z_2 - z_1) \]

The quantity \( mgz \) is the gravitational energy, \( PE \). The change in gravitational potential energy, \( \Delta PE \), is

\[ \Delta PE = PE_2 - PE_1 = mg(z_2 - z_1) \]

Potential energy is regarded as an *extensive property* of the body. For this course, it is assumed that elevation differences are small enough that the gravitational force can be considered constant.

**Example 2.1-2**

Consider a 3000-lb car cruising steadily on a level road at 65 mile/h. Now the car starts climbing a hill that is sloped 6\(^\circ\) from the horizontal. Determine the additional power delivered by the engine in order for the car to maintain its speed at 65 mile/h.

**Solution**

The additional power required is simply the gravitational work that must be done per unit time to raise the elevation of the car, which is equal to the change in the potential energy of the car per unit time:

\[ \dot{W}_g = mg\Delta z/\Delta t = mgV_{vertical} \]

\[ V_{vertical} = 65 \text{ mile/h} = (65)(5280)(\sin 6^\circ)/3600 = 9.965 \text{ ft/s} \]

\[ \dot{W}_g = mgV_{vertical} = \frac{(3000 \text{ lb})(32.2 \text{ ft/s}^2)(9.965 \text{ ft/s})}{32.2 \text{ lb}_f \text{ ft/s}^2} = 29,895 \text{ lb}_f \text{ ft/s} \]

Since 1 hp = 550 lb\(_f\) ft/s

\[ \dot{W}_g = 29,895/550 = 54.4 \text{ hp} \]
2.2 General Definition of Work

The work \( W \) done by, or on, a system evaluated in terms of forces and displacements is given by

\[
W = \int_{s_i}^{s_f} F \cdot ds
\]

The differential of work, \( dW \), is said to be *inexact* because the integral \( \int_{s_i}^{s_f} dW = W \) can be evaluated without specifying the path or the details of the process. On the other hand, the differential of a property such as volume \( V \) is said to be *exact* since the integral \( \int_{V_i}^{V_f} dV = V_f - V_i \) depends only on the initial and final values \( V_i \) and \( V_f \) respectively.

Work is defined as any other transfer of energy except the energy transfer due to a difference in temperature between the objects. Work is done on a system whenever a piston is pushed, a liquid within a container is stirred, or a current is run through a resistor. In each case, the system’s energy will increase, and usually its temperature too. However the system is not being heated since the flow of energy is not a spontaneous one caused by a difference in temperature. Notice that both heat and work refer to energy in transit. The total energy inside a system can be defined but not heat or work. It is only meaningful to specify how much heat entered a system, or how much work was done on a system.

The usual sign convention for the first law of thermodynamics is as follows:

**The flow of heat into a system is a positive flow, while a flow of work into a system is a negative flow.**

Thus, if 10 J of heat \( \delta Q \) flow into a system, it is regarded as \( \delta Q = +10 \) J, while if 10 J of work \( \delta W \) flow into a system, it is regarded as \( \delta W = -10 \) J.

The sign convention for heat flow stated above has been universally used; unfortunately, the sign convention for work flow has not been universally accepted. In the U. S. the convention for work as stated above has been in widespread use while in Europe the opposite convention for work has been more commonly used\(^1\) [1].

The symbol ‘\( d \)' means an exact differential quantity where \( \int dE = E_f - E_i \). Energy is a state function. Heat and work are path functions and the differentials of heat and work, \( \delta Q \) and \( \delta W \), respectively, are nonexact differentials so that \( \int \delta Q \neq Q_f - Q_i \).

There are various forms of energy that matter may possess, in particular, kinetic energy \( KE \), potential energy \( PE \), internal energy \( U \), electrical energy \( EE \), and magnetic energy \( ME \). Only kinetic, potential, and internal energies will mostly be considered in this

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A system possesses kinetic energy by virtue of its velocity, a system possesses potential energy by virtue of its height above a reference plane, and a system possesses internal energy by virtue of the random thermal motion of the atoms and molecules of which it is composed.

### 2.2-1 Power

The time rate at which energy transfer occurs is important information. **Power** is the rate of energy transfer by work. When a work interaction involves a force, power \( \dot{W} \) is given by

\[
\dot{W} = F \cdot V
\]

The total work during the time interval \( t_1 \) to \( t_2 \) can be obtained by integrating the power over this time interval.

\[
W = \int_{t_1}^{t_2} \dot{W} dt = \int_{t_1}^{t_2} F \cdot V dt
\]

**Example 2.2-1.**

Determine the power required for a bicyclist traveling at 20 miles per hour to overcome the drag force imposed by the surrounding air. This **aerodynamic drag** force is given by

\[
F_d = 0.5 C_d A \rho V^2
\]

In this equation, \( C_d \) is a constant called the **drag coefficient**, \( A \) is the frontal area of the bicycle and rider, and \( \rho \) is the air density. You can use the following data

<table>
<thead>
<tr>
<th>( C_d )</th>
<th>( A )</th>
<th>( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.88</td>
<td>3.9 ft(^2)</td>
<td>0.075 lb/ft(^3)</td>
</tr>
</tbody>
</table>

**Solution**

\[
\dot{W} = F \cdot V = F_d V = (0.5 C_d A \rho V^2) V = 0.5 C_d A \rho V^3
\]

\[
V = 20 \text{ mi/h} = (20)(5280)/3600 = 29.33 \text{ ft/s}
\]

\[
\dot{W} = (0.5)(0.88)(3.9)(0.075)(29.33) = 3.248 \times 10^3 \text{ lb-ft}^2/\text{s}
\]

\[
\dot{W} = (3.248 \times 10^3 \text{ lb-ft}^2/\text{s}) \frac{1 \text{ lbf}}{32.2 \text{ lb-ft/s}^2} = 100.88 \text{ ft-lbf/s}
\]

\[
\dot{W} = 100.88 \text{ lbf-ft/s} \frac{1 \text{ hp}}{550 \text{ ft-lbf/s}} = 0.183 \text{ hp}
\]

---

2.2-2 Expansion or Compression Work

Consider a closed system shown in Figure 2.2-1 consisting of a gas contained in a piston-cylinder assembly as the gas expands. The work done by the gas as the piston is displaced a distance \( dx \) is

\[
\delta W = Fdx = pAdx = pdV
\]  
(E.2.2-1)

For a change in volume from \( V_A \) to \( V_B \), the work is obtained by integrating the above expression

\[
W = \int_{V_A}^{V_B} pdV
\]  
(E.2.2-2)

A relationship between the absolute gas pressure at the piston surface and the gas volume must be known before Eq. (E.2.2-2) can be integrated. If the gas is ideal, pressure \( p \) and volume \( V \) is related by the ideal gas law:

\[
pV = nR_gT
\]  
(E.2.2-3)

In this equation \( n \) is the number of moles, \( R_g \) is the gas constant, and \( T \) is the absolute temperature.

**Example 2.2-2.**

Determine the work done by 0.1 gmole (or mole) gas when the piston slowly moves from A to B. At position A, the gas volume is 1000 cm\(^3\) and the gas temperature is 300 K. At position B, the gas volume is 2000 cm\(^3\). The process is isothermal and ideal gas law is applicable with ideal gas constant \( R_g = 82.057 \text{ cm}^3\text{·atm/gmol·K} \). Determine the initial gas pressure.
Solution

The work performed by the gas is given by

\[ W = \int_{V_A}^{V_B} pdV \]

Substituting \( p = nR g \frac{T}{V} \) and integrating gives

\[ W = nR g \frac{T}{V} V \int_{V_A}^{V_B} dV = nR g T \ln \frac{V_B}{V_A} = (0.1)(82.057)(300) \ln (2000/1000) \]

\[ W = 2.46 \times 10^3 \text{ cm}^3 \cdot \text{atm} \]

Since 1 atm = 1.01325×10^5 Pa = 1.01325×10^5 N/m^2 and cm^3 = 10^-6 m^3

\[ W = (2.46 \times 10^3)(10^{-6})(1.01325 \times 10^5) = 249.4 \text{ N} \cdot \text{m} = 249.4 \text{ J} \]

The initial gas pressure is at position A where \( V = 1000 \text{ cm}^3 \)

\[ p = nR g \frac{T}{V} = (0.1) (82.057)(300)/1000 = 2.46 \text{ atm} \]

2.3 Energy Transfer by Heat

Heat is not a property therefore the amount of energy transfer by heat for a process from state 1 to state 2 is given by

\[ Q = \int_{t_1}^{t_2} \delta Q \]  \hspace{1cm} (2.3-1)

If the net rate of heat transfer \( \dot{Q} \) is given, the amount of energy transfer by heat during a period of time can be obtained by integrating the rate of heat transfer from time \( t_1 \) to time \( t_2 \).

\[ Q = \int_{t_1}^{t_2} \dot{Q} dt \]  \hspace{1cm} (2.3-2)

The heat flux \( \dot{q} \) is defined as the heat transfer rate per unit surface area. The unit of heat flux in SI system is J/s·m^2 or W/ m^2. The net rate of heat transfer, \( \dot{Q} \), can be obtained from the heat flux by the integral

\[ \dot{Q} = \int_{A} \dot{q} dA \]  \hspace{1cm} (2.3-3)

An adiabatic process involves no heat transfer between the system and the surroundings.
2.3-1 Modes of Heat Transfer

Conduction

Conduction refers to energy transfer by molecular interactions. Energy carriers on the molecular level are 'electrons' and 'phonons' where the latter is a quantized lattice vibration. The interaction is a nearest-neighbor process that extends only a few molecular dimensions. Energy transport over a distance is by a staged transfer through molecular distances.

Convection

Convection refers to energy transport over macroscopic distances by bulk movement of matter. Once matter reaches its destination, energy dissipated by conduction. In general, the total heat transfer is a superposition of energy transport by molecular interactions and by the bulk motion of the fluid.

Radiation

Radiation refers to energy transfer by propagation of electromagnetic waves. Energy is absorbed or emitted by electrons changing their energy levels as a result of the temperature of the body. A packet of energy emitted this way is called a 'photon' which has an energy \( E \) given by Planck's Law,

\[
E = h \nu
\]

where \( h = 6.625 \times 10^{-34} \) J-s/(molecule) is 'Planck's constant' and \( \nu \) is the frequency of the electromagnetic wave. Unlike conduction and convection, radiation heat transfer does not require any matter in the region over which the temperature difference exists to promote the transport of heat. The following figure gives an analogy to show the differences between the three modes of heat transfer.

![Figure 2.3-1 Three modes of heat transfer.](image)

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