Chapter 3

3.5 Energy Balance With Property Evaluation

Example 3.5-1

A piston-cylinder assembly contains 2 lb of air at a temperature of 540°F and a pressure of 1 atm. The air is compressed to a state where the temperature is 840°F and the pressure is 6 atm. During the compression, there is a heat transfer from the air to the surroundings equal to 20 Btu. Using the ideal gas model for air, determine the work during the process, in Btu\(^15\).

Solution

The energy balance on the air is written as

\[ \Delta E = E_2 - E_1 = \Delta KE + \Delta PE + \Delta U = Q + W \]

Neglecting \( \Delta KE \) and \( \Delta PE \), we have

\[ W = \Delta U - Q = m(u_2 - u_1) - Q; \quad (u_2 - u_1) = 51.94 \text{ Btu/lb} \]

| Ideal Gas Properties of Air |
|-----------------------------|-----------------|-----------------|
| \( T \)  | \( h \)  | \( u \)  |
| 540  | 129.06 | 92.04 |
| 840  | 201.56 | 143.98 |

\[ W = (2 \text{ lb})(143.98 - 92.04) \text{ Btu/lb} - (-20 \text{ Btu}) = 123.9 \text{ Btu} \]

**Note:** \( u_2 - u_1 = c_p(T_2 - T_1) = (0.173 \text{ Btu/lb} \cdot \text{°R})(840 - 540) \text{ °R} = 51.9 \text{ Btu/lb} \)

Example 3.5-2

A valve connects two tanks containing carbon monoxide. One tank contains 2 kg of CO gas at 77°C and 0.7 bar. The other tank holds 8 kg of CO gas at 27°C and 1.2 bar. The valve is opened and the gases are allowed to mix while receiving energy by heat transfer from the surroundings. The final equilibrium temperature is 42 °C. Using the ideal gas model with constant $c_v = 0.745 \text{ kJ/kg} \cdot \text{K}$, determine (a) the final equilibrium pressure, in bar (b) the heat transfer for the process, in kJ.

Solution

(a) Determine the final equilibrium pressure

System: gas in both tanks.

The final pressure $p_f$ can be determined from the ideal gas law:

$$p_f = \frac{mRT_f}{V} = \frac{(m_{i1} + m_{i2}) RT_f}{V_1 + V_2}$$

We have $V_1 = \frac{m_{i1}RT_{i1}}{p_{i1}}$ and $V_1 = \frac{m_{i2}RT_{i2}}{p_{i2}}$, therefore

$$p_f = \frac{(m_{i1} + m_{i2}) RT_f}{\frac{m_{i1}RT_{i1}}{p_{i1}} + \frac{m_{i2}RT_{i2}}{p_{i2}}} = \frac{(m_{i1} + m_{i2}) T_f}{\frac{m_{i1}T_{i1}}{p_{i1}} + \frac{m_{i2}T_{i2}}{p_{i2}}}$$

$$p_f = \frac{(10 \text{ kg})(42 + 273) \text{ K}}{(2 \text{ kg})(350 \text{ K}) + (8 \text{ kg})(300 \text{ K})} = 1.05 \text{ bar}$$

(b) Determine the heat transfer for the process, in kJ

The energy balance on the carbon monoxide is written as

$$\Delta E = E_2 - E_1 = \Delta KE + \Delta PE + \Delta U = Q + W$$

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Since $\Delta KE = 0$, $\Delta PE = 0$, and $W = 0$, we have

$$\Delta U = Q \Rightarrow Q = U_t - U_i$$

$$U_i = m_1 c_v T_{1i} + m_2 c_v T_{2i}$$

$$U_t = m_1 c_v T_t + m_2 c_v T_t$$

$$Q = U_t - U_i = m_1 c_v (T_t - T_{1i}) + m_2 c_v (T_t - T_{2i})$$

$$Q = (2 \text{ kg}) \left( \frac{0.745 \text{ kJ}}{\text{kg} \cdot \text{K}} \right) (42 - 77) \degree \text{C} + (8 \text{ kg}) \left( \frac{0.745 \text{ kJ}}{\text{kg} \cdot \text{K}} \right) (42 - 27) \degree \text{C}$$

$$Q = 37.25 \text{ kJ}$$

**Example 3.5-3**

Air undergoes a polytropic ($pV^n$ = constant) compression in a piston–cylinder assembly from $p_1 = 1 \text{ atm}$, $T_1 = 70^\circ \text{F}$ to $p_2 = 5 \text{ atm}$. Employing the ideal gas model with constant specific heat ratio $k$, determine the work and heat transfer per unit mass, in Btu/lb, if (a) $n = 1.3$, (b) $n = k = 1.401$. Evaluate $k$ at $T_1$.

**Solution**

(a) Determine the work and heat transfer per unit mass, in Btu/lb, if $n = 1.3$

$$W = - \int_{V_i}^{V_2} p dV = - \int_{V_i}^{V_2} \frac{k V_2^{1-n} - k V_1^{1-n}}{1-n} dV = - \frac{p_2 V_2^{1-n} - p_1 V_1^{1-n}}{1-n}$$

$$W = - \frac{p_2 V_2 - p_1 V_1}{1-n} = - \frac{m R (T_2 - T_1)}{1-n}$$

Applying the ideal gas law at states (1) and (2) we have

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\[ p_2 V_2 = n \tilde{R} T_2 \]
\[ p_1 V_1 = n \tilde{R} T_1 \]

Taking the ratio of the two equations we have

\[ \frac{p_2}{p_1} = \frac{T_2}{T_1} \frac{V_1}{V_2} \Rightarrow \frac{T_2}{T_1} = \frac{p_2}{p_1} \frac{V_1}{V_2} \]

Since \( p_1 V_1^n = p_2 V_2^n \Rightarrow \frac{V_2}{V_1} = \left( \frac{p_1}{p_2} \right)^{1/n} = \left( \frac{p_2}{p_1} \right)^{-1/n} \)

\[ \frac{T_2}{T_1} = \frac{p_2}{p_1} \left( \frac{p_2}{p_1} \right)^{-(n-1)/n} = \left( \frac{p_2}{p_1} \right)^{(n-1)/n} \]

\[ T_2 = T_1 \left( \frac{p_2}{p_1} \right)^{(n-1)/n} = (460 + 70)^\circ R \left( \frac{5}{1} \right)^{(1.3-1)/1.3} = 768^\circ R \]

The work is then

\[ \frac{W}{m} = - \frac{R (T_2 - T_1)}{1 - n} = \left( \frac{1.986 \text{ Btu}}{28.97 \text{ lb} \cdot \circ R} \right) \frac{(768 - 530)^\circ R}{1.3 - 1} = 54.39 \text{ Btu/lb} \]

From the energy balance

\[ \Delta U = Q + W \Rightarrow \frac{Q}{m} = \frac{\Delta U}{m} - \frac{W}{m} = u_2 - u_1 - 54.39 \text{ Btu/lb} \]

\[ u_2 - u_1 = c_v(T_2 - T_1) \]

\[ c_v = \frac{c_p}{c_v} = \frac{c_p + R}{c_v} \Rightarrow c_v = \frac{R}{k - 1} = \left( \frac{1.986 \text{ Btu}}{28.97 \text{ lb} \cdot \circ R} \right) \frac{1}{1.401 - 1} = 0.171 \frac{\text{Btu}}{\text{lb} \cdot \circ R} \]

\[ u_2 - u_1 = 0.171 \frac{\text{Btu}}{\text{lb} \cdot \circ R} (768 - 530)^\circ R = 40.70 \text{ Btu/lb} \]

\[ \frac{Q}{m} = u_2 - u_1 - 54.39 \text{ Btu/lb} = (40.70 - 54.39) \text{ Btu/lb} = -13.69 \text{ Btu/lb} \]

(b) Determine the heat transfer per unit mass, in Btu/lb, if \( n = k \)

\[ \frac{Q}{m} = u_2 - u_1 - \frac{W}{m} = c_v(T_2 - T_1) + \frac{R(T_2 - T_1)}{k - 1} = \frac{R(T_2 - T_1)}{k - 1} + \frac{R(T_2 - T_1)}{1 - k} = 0 \]

No heat transfer occurs in the polytropic process of an ideal gas for which \( pV^k = \text{constant} \).
The following examples present systems where mass, energy balance, and equation of state must be solved simultaneously to achieve a desired result.

**Example 3.5-4.**

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We have a number of small jets that will be used in attitude control in space. These jets will be powered by low-pressure nitrogen gas heated by an arc at the jet nozzle. Your problem deals with the nitrogen storage system.

The nitrogen is stored in a large well-insulated, 0.4 m³ sphere at 1 bar pressure. At take-off the temperature is 280 K. The mass rate of flow of N₂ will be constant and be equal to 7.0 g/s. Since the pressure inside the sphere must always be kept at 1 bar, a heater will be used inside the sphere.

Under these conditions, what will be the temperature of the N₂ in the sphere, the instantaneous rate of heat flow to the heater, and the total heat required after 10 sec of operation? You can assume nitrogen as an ideal gas with \( c_p = 29.3 \text{ J/mol.}°\text{K} \).

**Solution**

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![Diagram of nitrogen storage system with heater](image)

**Step #1:** Define the system.

System = Nitrogen inside the storage tank at any time. Assume perfect mixing of the air so that the nitrogen temperature is uniform at any time.

**Step #2:** Find equation that contains the temperature of the system.

The temperature of nitrogen inside the tank may be obtained from the energy balance on the system.

**Step #3:** Energy balance

\[
\frac{d}{dt} (mu) = \dot{Q} - \dot{m} \ c_p (T - T_{ref})
\]

In this equation, \( m \) is the mass of nitrogen in the tank at any time, \( \dot{m} \) is the mass flow rate of nitrogen leaving the tank, and \( \dot{Q} \) is the rate of heat supplied to nitrogen to maintain the pressure at 1 bar. Let the reference temperature \( T_{ref} \) be 0°C, the energy equation becomes

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18 Modell and Reid, *Thermodynamics and its Applications*, Prentice Hall
\[
\frac{d}{dt} (mc, T) = \dot{Q} - \dot{m} c_p T
\]

Since there are three unknowns \(m\), \(T\), and \(\dot{Q}\) we need to find other independent relationships to link these variables. The mass balance on the system gives

\[
\frac{dm}{dt} = -\dot{m}
\]

The mass of nitrogen inside the tank can be obtained from the ideal gas law

\[
m = \frac{M_w p V}{R T}
\]

The mass balance becomes

\[
\frac{M_w p V}{R} \frac{d}{dt} \left( \frac{1}{T} \right) = -\dot{m}
\]

The only unknown in the above equation is temperature. Therefore we can solve for temperature just from the mass balance.

**Step #4:** Specify the initial condition for the differential equation

At \(t = 0\), \(T = 280^\circ\text{K}\)

**Step #5:** Solve the resulting equation and verify the solution. Integrating the mass balance

\[
\int_{280}^{T} d\left( \frac{1}{T} \right) = -\frac{\dot{m} R}{M_w p V} \int_{0}^{t} dt
\]

We obtain

\[
\frac{1}{T} - \frac{1}{280} = -\frac{\dot{m} R t}{M_w p V} \Rightarrow \frac{1}{T} = \frac{1}{280} - \frac{\dot{m} R t}{M_w p V}
\]

In this expression, \(R = \text{gas constant} = 8.314 \text{ J/mol} \cdot \text{K} = 8.314 \times 10^{-5} \text{ bar} \cdot \text{m}^3/\text{mol}\)
At $t = 10$ s, we have

$$
\frac{1}{T} = \frac{1}{280} - \frac{7 \times 8.314 \times 10^{-3} \times 10}{28 \times 1 \times 0.4} = 5.196 \times 10^{-4} \text{ K}^{-1} \Rightarrow T = 327.7 \text{K}
$$

This equation can be verified by using it to determine time when there is no nitrogen remaining inside the tank. From the ideal gas law, the temperature can be expressed as a function of $m$, the mass of nitrogen inside the tank:

$$
T = \frac{M_w p V}{\bar{R} m}
$$

When $m$ approaches zero, the temperature must approach infinity to maintain the pressure inside the tank at 1 bar. The time when there is no nitrogen remaining inside the tank can then be solved by setting

$$
\frac{1}{T} = 0
$$

or

$$
\frac{1}{280} - \frac{\dot{m} \bar{R} t}{M_w p V} = 0
$$

Therefore

$$
t = \frac{M_w p V}{280 \bar{R} \dot{m}} = \frac{M_w p V}{\bar{R} T_{t=0} \dot{m}} \quad \text{(Note: } T_{t=0} = 280 \text{ K)}
$$

The time calculated is simply the initial mass of nitrogen in the tank divided by the constant mass flow rate out.

The instantaneous rate of heat flow to the heater, $\dot{Q}$, can be evaluated by substituting the mass $m = \frac{M_w p V}{\bar{R} T}$ into the energy equation

$$
\frac{d}{dt} \left( \frac{M_w p V}{\bar{R} T} c_v T \right) = \dot{Q} - \dot{m} c_p T
$$

$$
\frac{d}{dt} \left( \frac{M_w p V c_v}{\bar{R}} \right) = 0 = \dot{Q} - \dot{m} c_p T \Rightarrow \dot{Q} = \dot{m} c_p T
$$

At $t = 10$ s, $T = 327.7 \text{ K}$, we have

$$
\dot{Q} = \frac{7}{29} \times 29.3 \times 327.7 = 2400 \text{ W}
$$
The total heat required up to 10 sec of operation is obtained by integrating the instantaneous heat flow over this period

\[
Q = \int_0^{10} Q dt = \dot{m} c_p \int_0^{10} T dt
\]

From the relationship for the temperature, \( \frac{1}{T} = \frac{1}{280} - \frac{\dot{m} R t}{M \rho V} \), we have

\[
\frac{1}{T} = \frac{1}{280} - \frac{7 \times 8.314 \times 10^{-5} t}{28 \times 1 \times 0.4} = \frac{1}{280} - 5.196 \times 10^5 t
\]

\[
\frac{1}{T} = 3.5714 \times 10^{-3} - 5.196 \times 10^5 t
\]

\[
T = \frac{1}{3.5714 \times 10^{-3} - 5.196 \times 10^5 t}
\]

Therefore,

\[
Q = \dot{m} c_p \int_0^{10} T dt = \frac{7}{29} \times 29.3 \int_0^{10} \frac{dt}{3.5714 \times 10^{-3} - 5.196 \times 10^5 t}
\]

\[
Q = \frac{7 \times 29.3 \times \ln \frac{3.5714 \times 10^{-3}}{3.5714 \times 10^{-3} - 5.196 \times 10^{-4}}}{5.196 \times 10^{-5}} = 2.22 \times 10^4 \text{ J}
\]