Example 4.4-3. Steam enters the condenser of a vapor power plant at 0.1 bar with a quality of 0.95 and condensate exits at 0.1 bar and 45°C. Cooling water enters the condenser in a separate stream as a liquid at 20°C and exits as a liquid at 35°C with no change in pressure. Heat transfer from the outside of the condenser and changes in the kinetic and potential energies of the flowing streams can be ignored. For steady-state operation, determine

(a) the ratio of the mass flow rate of cooling water to the mass flow rate of the condensing steam.

(b) the rate of energy transfer from the condensing steam to the cooling water, in kJ per kg of steam passing through the condenser.

Solution

(a) Determine the ratio of the mass flow rate of cooling water to the mass flow rate of the condensing steam.

The rate of heat transfer from the condensing steam to the cooling water is given by

\[ \dot{Q} = \dot{m}_h (h_1 - h_2) = \dot{m}_c (h_4 - h_3) \]

In this equation, \( \dot{m}_h \) is the mass flow rate of the hot stream (or condensing steam) and \( \dot{m}_c \) is the mass flow rate of the cold stream (or cooling water). The specific enthalpies of the inlet and exit streams are listed in Table E4.4-3. For stream (2), (3), and (4) the enthalpies are taken as saturated liquid at the listed temperature so that \( h_2 = h_l(T_2) \), \( h_3 = h_l(T_3) \), and \( h_4 = h_l(T_4) \).

---

Table E4.4-3 Steam properties from CATT2

<table>
<thead>
<tr>
<th></th>
<th>Temp (°C)</th>
<th>Pressure (MPa)</th>
<th>Enthalpy (kJ/kg)</th>
<th>Quality</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>45.81</td>
<td>0.01</td>
<td>2465</td>
<td>0.95</td>
<td>Liquid Vapor Mixture</td>
</tr>
<tr>
<td>2</td>
<td>45</td>
<td>0.009593</td>
<td>188.4</td>
<td>0</td>
<td>Saturated Liquid</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>0.002339</td>
<td>83.94</td>
<td>0</td>
<td>Saturated Liquid</td>
</tr>
<tr>
<td>4</td>
<td>35</td>
<td>0.005628</td>
<td>146.7</td>
<td>0</td>
<td>Saturated Liquid</td>
</tr>
</tbody>
</table>

Solving for the ratio of the mass flow rate gives

\[
\frac{\dot{m}_1}{\dot{m}_2} = \frac{h_1 - h_2}{h_4 - h_3} = \frac{2465 - 188.4}{146.7 - 83.94} = 36.3
\]

(b) Determine the rate of energy transfer from the condensing steam to the cooling water, in kJ per kg of steam passing through the condenser.

\[
\dot{Q} = \dot{m}_h (h_1 - h_2) \Rightarrow \frac{\dot{Q}}{\dot{m}_h} = h_1 - h_2
\]

\[
\frac{\dot{Q}}{\dot{m}_h} = 2465 - 188.4 = 2276.6 \text{ kJ/kg}
\]

Example 4.4-4
---
A supply line carries a two-phase liquid-vapor mixture of steam at 300 lbf/in$^2$. A small fraction of the flow in the line is diverted through a throttling calorimeter and exhausted to the atmosphere at 14.7 lbf/in$^2$. The temperature of the exhaust steam is measured as 250°F. Determine the quality of the steam in the supply line.

Solution
---
A throttling calorimeter is a device used to reduce the pressure of a gas or liquid stream. This can be simply done by means of a partially opened valve or a porous plug as shown in Figure E4.4-4a.

![Figure E4.4-4a Examples of throttling devices.](image_url)

---
Throttling may be used as a means of controlling the flow rate (valves and flow regulators), maintaining a constant downstream pressure (pressure regulator), or measuring the flow rate (flow metering orifices). For throttling devices we usually make the following assumptions:

* Steady state steady flow (SSSF)
* No work or heat transfer
* Potential and kinetic energy are negligible relative to other energy terms

<table>
<thead>
<tr>
<th>Table E4.4-4 Steam properties from CATT2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Temp</strong></td>
</tr>
<tr>
<td>F</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>

Making energy balance between (1) and (2) we obtain

\[
\frac{h_2 - h_1 + g(z_2 - z_1)}{m} + \frac{1}{2} (V_2^2 - V_1^2) = \frac{\dot{Q}}{m} - \frac{\dot{W}}{m}
\]

For \( g(z_2 - z_1) = 0 \), \( \frac{1}{2} (V_2^2 - V_1^2) = 0 \), \( \frac{\dot{Q}}{m} = 0 \), and \( \frac{\dot{W}}{m} = 0 \), we have

\[
h_2 = h_1 = (1 - x_1)h_{f_1} + x_1h_{g_1} = h_{f_1} + x_1(h_{g_1} - h_{f_1})
\]

The quality of the steam in the supply line is then

\[
x_1 = \frac{h_2 - h_{f_1}}{h_{g_1} - h_{f_1}} = \frac{1169 - 394.1}{1204 - 394.1} = 0.957
\]
4.5 Energy balance on Integrated or Transient System

In real life applications, we usually encounter integrated systems consisting of many components discussed in previous sections.

Example 4.5-1

An industrial process discharges $2 \times 10^5 \text{ ft}^3/\text{min}$ of gaseous combustion products at $400^\circ\text{F}$, 1 atm. As shown in Figure E4.5-1, a proposed system for utilizing the combustion products combined a heat-recovery steam generator with a turbine. At steady state, combustion products exit the steam generator at $260^\circ\text{F}$, 1 atm and a separate stream of water enters at 40 psia, $102^\circ\text{F}$ with a mass flow rate of 275 lb/min. At the exit of the turbine, the pressure is 1 psia and the quality is 93%. Heat transfer from the outer surfaces of the steam generator and turbine can be ignored, as can the changes in kinetic and potential energies of the flowing streams. There is no significant pressure drop for the water flowing through the steam generator. The combustion products can be modeled as air as an ideal gas.

(a) Determine the power developed by the turbine, in Btu/min.
(b) Determine the turbine inlet temperature, in $^\circ\text{F}$.

Solution

\[
\dot{m}_2 h_2 - \dot{m}_1 h_1 + \dot{m}_3 h_5 - \dot{m}_3 h_3 = \dot{Q} - \dot{W}_s
\]

---

Since the gas and water streams do not mix and heat transfer is negligible, we have

\[ \dot{m}_1 = \dot{m}_2, \quad \dot{m}_3 = \dot{m}_5, \text{ and} \]

\[ \dot{W}_s = \dot{m}_1 (h_1 - h_2) + \dot{m}_3 (h_3 - h_5) \]

The mass flow rate \( \dot{m}_1 \) is given by

\[ \dot{m}_1 = \frac{(AV)}{v_1}, \text{ where the specific volume } v_1 \text{ can be obtained from ideal gas law} \]

\[ v_1 = \frac{RT_1}{p_1} = \frac{(\bar{R} / M)T_1}{p_1} \]

\[ v_1 = \left( \frac{1545 \text{ ft} \cdot \text{lb} \cdot \text{F}}{28.97 \text{ lb} \cdot \text{in} \cdot \text{R}} \right) \left( \frac{860^\circ \text{R}}{14.7 \text{ psia}} \right) = 21.667 \text{ lb/ft}^3 \]

The mass flow rate is then

\[ \dot{m}_1 = \frac{2 \times 10^5 \text{ ft}^3 / \text{min}}{21.667 \text{ lb/ft}^3} = 9230.6 \text{ lb/min} \]

The air properties are listed in Table E4.5-1a and steam properties are listed in Table E4.5-1b.

**Table E4.5-1a** Air properties from CATT2

<table>
<thead>
<tr>
<th>Temp F</th>
<th>Pressure psia</th>
<th>Enthalpy (Mass) Btu/lbm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>400</td>
<td>14.7</td>
</tr>
<tr>
<td>2</td>
<td>260</td>
<td>14.7</td>
</tr>
</tbody>
</table>

**Table E4.5-1b** Steam properties from CATT2

<table>
<thead>
<tr>
<th>Temp F</th>
<th>Pressure psia</th>
<th>Enthalpy Btu/lbm</th>
<th>Quality</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>102</td>
<td>40</td>
<td>70.14</td>
<td>Compressed Liquid</td>
</tr>
<tr>
<td>5</td>
<td>101.7</td>
<td>1</td>
<td>1033</td>
<td>Liquid Vapor Mixture</td>
</tr>
</tbody>
</table>

The power developed by the turbine is
\[ \dot{W}_s = \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_5) \]

\[ \dot{W}_s = (9230.6 \text{ lb/min})(206.5 - 172.4) \text{ Btu/lb} + (275 \text{ lb/min})(70.14 - 1033) \text{ Btu/lb} \]

\[ \dot{W}_s = 49,980 \text{ Btu/min} \]

(a) Determine the turbine inlet temperature, in °F.

We need to know 2 properties at (4). Since pressure drop is negligible, \( p_4 = 40 \text{ psia} \), the enthalpy at (4) can be determined from the energy balance around the steam generator:

\[ 0 = \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_4) \]

\[ h_4 = h_3 + \frac{\dot{m}_1}{\dot{m}_3}(h_1 - h_2) \]

\[ h_4 = 70.14 \text{ Btu/lb} + \frac{9230.6}{275}(206.5 - 172.4) \text{ Btu/lb} = 1214.7 \text{ Btu/lb} \]

The inlet temperature at the turbine inlet is \( 355.8^\circ \text{F} \) from Table E4.5-1c.

<table>
<thead>
<tr>
<th>Table E4.5-1c Steam properties from CATT2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp</td>
</tr>
<tr>
<td>F</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>
Example 4.5-2

A tank having a volume of 0.85 m$^3$ initially contains water as a two-phase liquid-vapor mixture at 260°C and a quality of 0.7. Saturated water vapor at 260°C is slowly withdrawn through a pressure-regulating valve at the top of the tank as energy is transferred by heat to maintain the pressure constant in the tank. This continues until the tank is filled with saturated vapor at 260°C. Determine the amount of heat transfer, in kJ. Neglect all kinetic and potential energy effects.

Solution

The mass balance for the content, $m_{cv}$, in the tank is given by

$$\frac{dm_{cv}}{dt} = -\dot{m}_e$$

The energy balance for the tank can be written as

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e = \dot{Q}_{cv} + h_e \frac{dm_{cv}}{dt}$$

Integrating the energy equation with $h_e = \text{constant}$

$$\int dU_{cv} = \int \dot{Q}_{cv} dt + \int h_e dm_{cv}$$

We obtain

$$\Delta U_{cv} = Q_{cv} + h_e \Delta m_{cv}$$

Solving for the heat transfer gives

$$Q_{cv} = \Delta U_{cv} - h_e \Delta m_{cv} = m_2 u_2 - m_1 u_1 - h_e (m_2 - m_1)$$

---

In this equation, \( m_1 \) and \( m_2 \) are the initial and final amounts of mass within the tank, respectively. The steam properties are listed in Table 4.5-2.

### Table E4.5-2 Steam properties from CATT2

<table>
<thead>
<tr>
<th>Temp C</th>
<th>Pressure MPa</th>
<th>Specific Internal Volume m(^3/kg)</th>
<th>Specific Energy kJ/kg</th>
<th>Specific Enthalpy kJ/kg</th>
<th>Quality</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>260</td>
<td>4.689</td>
<td>0.02993</td>
<td>2158</td>
<td>2298</td>
<td>0.7</td>
</tr>
<tr>
<td>2</td>
<td>260</td>
<td>4.689</td>
<td>0.0422</td>
<td>2599</td>
<td>2797</td>
<td>1</td>
</tr>
</tbody>
</table>

The mass initially contained in the tank is

\[
m_1 = \frac{V}{v_1} = \frac{0.85 \text{ m}^3}{0.02993 \text{ m}^3/\text{kg}} = 28.4 \text{ kg}
\]

The mass finally contained in the tank is

\[
m_2 = \frac{V}{v_2} = \frac{0.85 \text{ m}^3}{0.0422 \text{ m}^3/\text{kg}} = 20.14 \text{ kg}
\]

The heat transfer is then

\[
Q_{cv} = m_2u_2 - m_1u_1 - h_e(m_2 - m_1)
\]

\[
Q_{cv} = (20.14)(2599) - (28.4)(2158) - (2797)(20.14 - 28.4) = 14,160 \text{ kJ}
\]