Chapter 3

Convective Mass Transfer

3.1 Introduction

The mass transfer coefficient for the transport of species A between two locations within a fluid may be defined from the following relations:

(Gases): $\quad N_A = k_c(c_{A1} - c_{A2}) = k_G(p_{A1} - p_{A2}) = k_y(y_{A1} - y_{A2})$

(Liquids): $\quad N_A = k_c(c_{A1} - c_{A2}) = k_L(c_{A1} - c_{A2}) = k_x(x_{A1} - x_{A2})$

In these equations, N_A is the molar flux of species A and the mass transfer coefficient k has different subscript and different units depending on the units of the driving force used in the expression. Since many mass operations involve the transfer of material between two contacting phases, different subscript for the mass transfer coefficient is also used to distinguish between the phases.

The mass transfer coefficients might be obtained from the correlations given in chapter 8 where the Prandtl number ($Pr = v/\alpha$) is replaced by Schmidt number ($Sc = v/D_{AB}$) and the Nusselt number ($Nu = hL/k$) is replaced by Sherwood number ($Sh = k_cL/D_{AB}$). Table 3.1-1 lists some correlations to determine the mass transfer coefficient for external forced convection flow. The expressions for the flat plate are obtained from the solutions of the boundary layer equations. The other formulas are experimental correlations.

--------- **Table 3.1-1** Mass transfer coefficients for external forced convection flow. -------

Correlations	Geometry	Conditions
$Sh_x = 0.332Re_x^{1/2}\,Sc^{1/3}$	Flat plate	Laminar, local, T_f, $0.6 \le Sc \le 50$
$\overline{Sh_x} = 0.664Re_x^{1/2}\,Sc^{1/3}$	Flat plate	Laminar, average, T_f, $0.6 \le Sc \le 50$
$Sh_x = 0.0296Re_x^{4/5}\,Sc^{1/3}$	Flat plate	Turbulent, local, T_f, $Re_x \le 10^8$ $0.6 \le Sc \le 50$
$\overline{Sh_x} = (0.037Re_x^{4/5} - 871)\,Sc^{1/3}$	Flat plate	Mixed, average, T_f, $Re_{x,c} = 5 \times 10^5$ $Re_x \le 10^8$, $0.6 \le Sc \le 50$
$\overline{Sh_D} = 0.3 + [0.62\,Re_D^{1/2}Sc^{1/3}$ $\times[1 + (0.4/Sc)^{2/3}]^{-1/4}]$ $\times[1 + (Re_D/282{,}000)^{5/8}]^{4/5}$	Cylinder	Average, T_f, $Re_DSc > 0.2$ Cross flow
$\overline{Sh_D} = 2 + (0.4Re_D^{1/2} + 0.06Re_D^{2/3})Sc^{0.4}$ $\times(\mu/\mu_s)^{1/4}$	Sphere	Average, T_∞, $3.5 < Re_D < 7.6 \times 10^4$ $0.71 < Sc < 380$, $1.0 < (\mu/\mu_s) < 3.2$

These correlations are valid for equimolar mass transfer or low mass transfer rate where the mole fraction of species A is less than about 0.05. For higher mass transfer rate the coefficients might be corrected by the log mean concentration difference. Instead of using k_c one should use $k_c/(1 - y_A)_{lm}$ where

$$(1 - y_A)_{lm} = \frac{(1 - y_{A1}) - (1 - y_{A2})}{\ln\left(\dfrac{1 - y_{A1}}{1 - y_{A2}}\right)}$$

Example 3.1-1 --
Air at 32°C is humidified by flowing over a 1.2-m-long container filled with water. The interfacial temperature is 20°C. If the initial humidity of the air is 25% and its velocity is 0.15 m/s, calculate (a) the convective mass transfer coefficient, and (b) the amount of water evaporated per unit width of the container.
 (Ref. <u>Fundamentals of Heat Transfer</u> by Incropera and DeWitt, Wiley, 5th Edition, 2002)
Solution --

The film temperature is $T_f = (32 + 20)/2 = 26°C$

Air at 26°C: $v = 1.51 \times 10^{-5}$ m²/s, $D_{AB} = 2.77 \times 10^{-5}$ m²/s.

Water vapor pressure: $p_A{}^{sat}(20°C) = 0.02308$ atm, $p_A{}^{sat}(32°C) = 0.04696$ atm.

Partial pressure of water vapor at the air-water interface is $p_{As} = p_A{}^{sat}(20°C) = 0.02308$ atm. Mole fraction of water vapor at this location is $y_{As} = 0.02308$.

Partial pressure of water vapor in the ambient air is $p_{A\infty} = 0.25 \times p_A{}^{sat}(32°C) = 0.25 \times 0.04696$ atm $= 0.01174$ atm. Mole fraction of water vapor at this location is $y_{A\infty} = 0.01174$.

Since both y_{As} and $y_{A\infty}$ are less than 0.05, we could use k_c without the correction factor $(1 - y_A)_{lm}$.

For parallel flow to a flat plate, laminar flow exists with $Re < 300,000$.

$$Re = \frac{U_\infty L}{v} = \frac{(0.15)(1.2)}{1.51 \times 10^{-5}} = 1.192 \times 10^4$$

The average Sherwood number over the container may be obtained from the following correlation:

$$\overline{Sh_L} = 0.664 Re^{1/2} Sc^{1/3} = 0.664(1.192 \times 10^4)^{1/2}\left(\frac{1.51}{2.77}\right)^{1/3} = 59.22 = \frac{k_c L}{D_{AB}}$$

The convective mass transfer coefficient is then

$$k_c = 59.22\frac{D_{AB}}{L} = 59.22\frac{2.77 \times 10^{-5}}{1.2} = \mathbf{1.367 \times 10^{-3} \ m/s}$$

The molar flux of water is given by

$$N_A = k_c(c_{As} - c_{A\infty})$$

The molar concentrations can be evaluated from the ideal gas law with gas constant $R = 0.08205$ atm·m^3/(kmol·°K):

$$c_{As} = \frac{p_{As}}{RT_s} = \frac{0.02308}{(0.08205)(273+20)} = 9.6\times10^{-4} \text{ kmol/m}^3.$$

$$c_{A\infty} = \frac{p_{A\infty}}{RT_\infty} = \frac{0.01174}{(0.08205)(273+32)} = 4.691\times10^{-4} \text{ kmol/m}^3.$$

$$N_A = k_c(c_{As} - c_{A\infty}) = 1.367\times10^{-3}(9.6\times10^{-4} - 4.691\times10^{-4}) = 6.711\times10^{-7} \text{ kmol/m}^2\cdot\text{s}$$

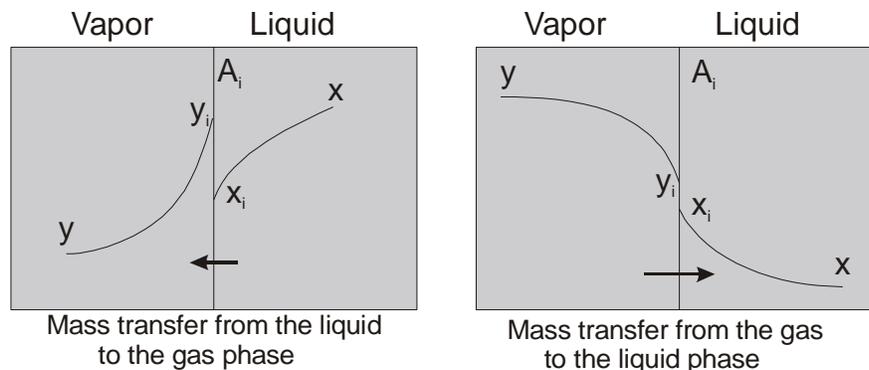The amount of water evaporated per m width of the container is

$$W_A = (1)(1.2)(18)\, N_A = \mathbf{1.449\times10^{-5} \text{ kg/s}}$$

Example 3.1-2 ---
In a wetted-wall tower, an air-H$_2$S mixture is flowing by a film of water which is flowing as a thin film down a vertical plate. The H$_2$S is being absorbed from the air to the water at a total pressure of 1.50 atm abs and 30°C. The value of k_c of 9.567×10^{-4} m/s has been predicted for the gas-phase mass-transfer coefficient. At a given point the mole fraction of H$_2$S in the liquid at the liquid-gas interface is 2.0×10^{-5} and p_A of H$_2$S in the gas is 0.05 atm. The Henry's law equilibrium relation is $p_A(\text{atm}) = 609 x_A$ (mole fraction in liquid). Calculate the rate of absorption of H$_2$S.
 (Ref: Transport Processes and Separation Process by C.J. Geankoplis, Prentice Hall, 4th Edition, 2003)
Solution --

| Vapor | Liquid | | Vapor | Liquid |

Mass transfer from the liquid
to the gas phase

Mass transfer from the gas
to the liquid phase

The rate of absorption of H$_2$S per unit area of the thin film is given by

$$N_A = \frac{k_c}{(1-y_A)_{lm}}(c_A - c_{Ai}) = \frac{k_y}{(1-y_A)_{lm}}(y_A - y_{Ai}) = \frac{k_c c}{(1-y_A)_{lm}}(y_A - y_{Ai})$$

$$N_A = \frac{k_c P}{(1-y_A)_{lm} RT}(y_A - y_{Ai})$$

The mole fraction of H_2S in the gas phase is given by

$$y_A = \frac{p_A}{p} = \frac{0.05}{1.5} = 0.0333$$

The partial pressure of H_2S in the gas phase at the interface is determined from Henry's law and the mole fraction of H_2S in the liquid at the liquid-gas interface.

$$p_{Ai} = 609x_{Ai} = 609 \times 2.0 \times 10^{-5} = 1.218 \times 10^{-2} \text{ atm}$$

The mole fraction of H_2S in the gas phase at the interface is then

$$y_{Ai} = \frac{p_{Ai}}{p} = \frac{0.01218}{1.5} = 0.00812$$

$$(1 - y_A)_{lm} = \frac{(1 - y_A) - (1 - y_{Ai})}{\ln\left(\dfrac{1 - y_A}{1 - y_{Ai}}\right)} \approx \frac{(1 - y_A) + (1 - y_{Ai})}{2} = 0.979$$

$$N_A = \frac{k_c P}{(1 - y_A)_{lm} RT}(y_A - y_{Ai}) = N_A = \frac{k_c}{(1 - y_A)_{lm} RT}(p_A - p_{Ai})$$

$$N_A = \frac{9.576 \times 10^{-4}}{0.979} \frac{(0.05 - .01218)}{(82.06 \times 10^{-3})(273 + 30)} = \mathbf{1.486 \times 10^{-3} \text{ kmol/m}^2 \cdot \text{s}}$$