

Chapter 4

Example 4.4-5

¹A liquid mixture containing 10 mol % n-heptane and 90 mol % n-octane is fed at its boiling point to the top of a stripping tower at 101.32 kPa. Figure E4.4-5 depicts a stripping tower where the feed stream is the saturated liquid and the distillate stream is the saturated vapor. There is no reboiler or condenser in a stripping tower. The bottoms are to contain 98 mol % n-octane. For every 3 mol of feed, 2 mol of vapor is withdrawn as product. Calculate the composition of the vapor and the number of theoretical plates required. The equilibrium data are given below

x	0	0.012	0.039	0.067	0.097	0.284
y	0	0.025	0.078	0.131	0.184	0.459

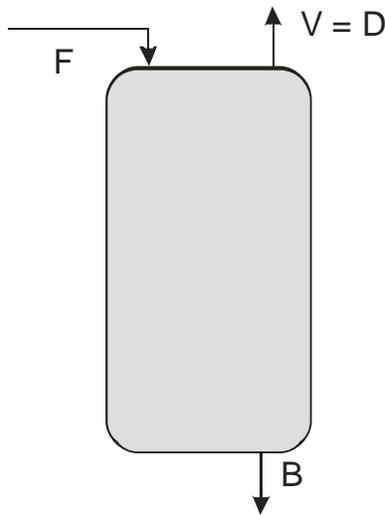


Figure E4.4-5 Stripping tower with feed F and product $D = V$.

Solution

$$F = 3 = D + B = 2 + B \Rightarrow B = 1 \text{ mol/unit time}$$

$$50 = 0.90D + 0.10B \Rightarrow D = 50 \text{ kmol/h}, B = 50 \text{ kmol/h}$$

Operating line

$$yV + x_B B = xL \Rightarrow y = xL/V - x_B B/V = 1.5x - 0.02 \times 1/2 = 1.5x - 0.01$$

Overall balance

$$y_D D + x_B B = x_F F \Rightarrow y_D = x_F F/D - x_B B/D = 0.1 \times 1.5 - 0.02 \times 1/2 = \mathbf{0.14}$$

The number of theoretical plates required can be determined from the following Matlab codes:

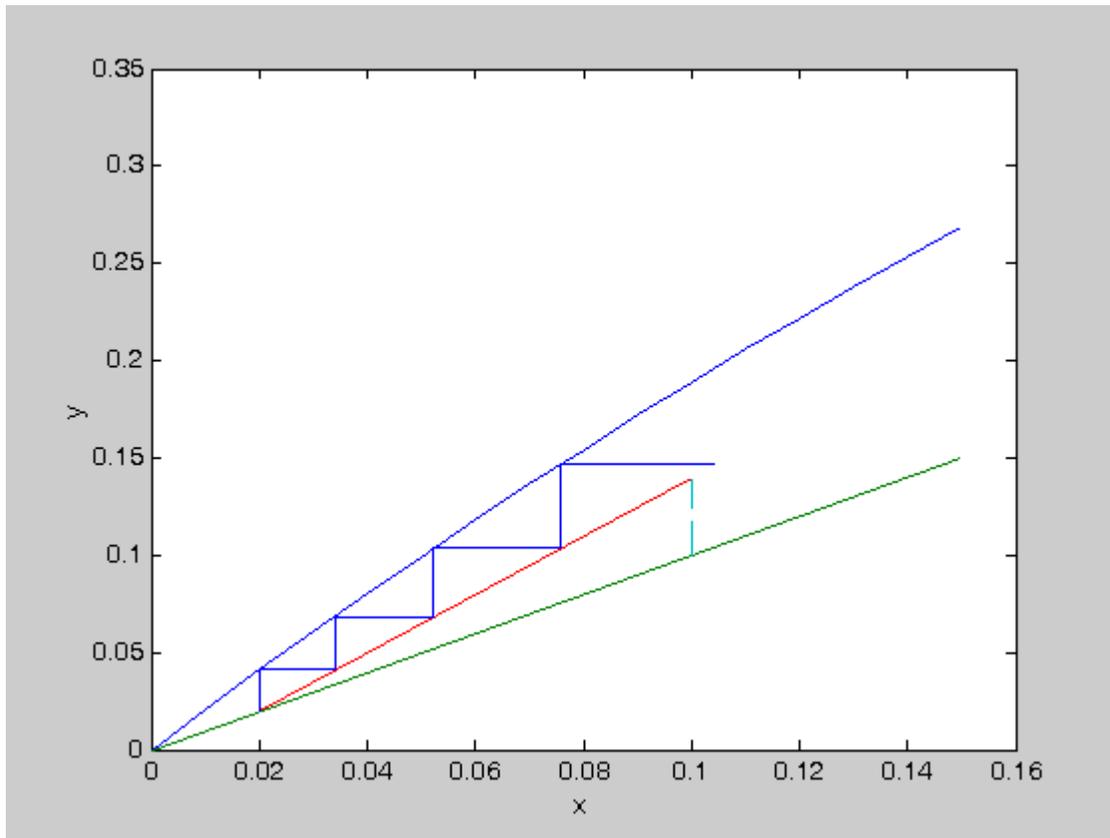
¹ Geankoplis, C.J., Transport Processes and Separation Process Principles, 4th ed., Prentice Hall, 2003, p. 754

```

% Example 4.4-5
clf
xB=0.02;
V=2;L=3; B=1;
% Stripping operating line:  $y = a*x + b$ , where
a=L/V;b=-xB*B/V;
xF=0.10;yF=a*xF+b;
xe=[0 0.012 0.039 0.067 0.097 0.284];
ye=[0 0.025 0.078 0.131 0.184 0.459];
pp=spline(xe,ye);
x=0:0.01:0.15;
y=ppval(pp,x);
plot(x,y,[0 .15],[0 .15],[xB xF],[xB yF],[xF xF],[xF yF],'-')
xlabel('x');ylabel('y')
hold on;
xo=xB;yo=xB;
for i=1:20;
ye=ppval(pp,xo);
line([xo xo],[yo ye])
xe=xo;
xo=(ye-b)/a;
line([xe xo],[ye ye])
if ye>yF, break, end
yo=ye;
end
net=i-1 + (yF-yo)/(ye-yo);
fprintf('# of equilibrium stages = %g\n',net)

>> E4d4d5
# of equilibrium stages = 3.84557

```

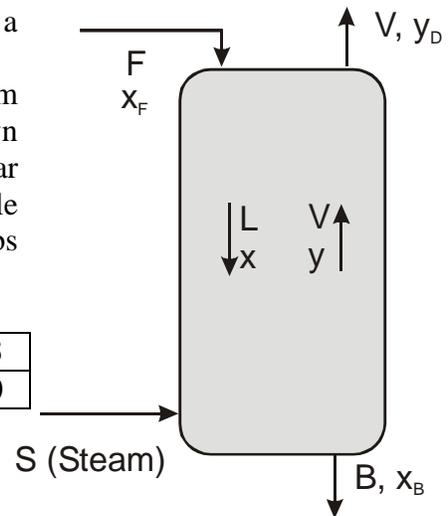


Example 4.4-6 -----

²A liquid feed at the boiling point contains 3.3 mol % ethanol and 96.7 mol % water and enters the top tray of a stripping tower shown.

Saturated steam is injected directly into liquid in the bottom of the tower. The overhead vapor which is withdrawn contains 99% of the alcohol in the feed. Assume equimolar overflow for this problem. Equilibrium data for mole fraction of alcohol are as follows at 101.32 kPa abs pressure.

x	0	0.0080	0.020	0.0296	0.033
y	0	0.0750	0.175	0.250	0.270



(a) For an infinite number of theoretical steps, calculate the minimum moles of steam needed per mole of feed. (Be sure and plot the q -line)

(b) Using twice the minimum moles of steam, calculate the number of theoretical steps needed, the composition of the overhead vapor, and the bottoms composition.

Solution -----

(a) Basis: $F = 100$ mol/h. For equimolar overflow $S = V$ and $B = F = 100$ mol/h

² Geankoplis, C.J., Transport Processes and Separation Process Principles, 4th ed., Prentice Hall, 2003, p. 754

$$F + S = V + B$$

Ethanol balance:

$$x_F F = x_B B + y_D V$$

$$(0.033)(100) = x_B(100) + y_D V$$

From the problem statement: $y_D V = (0.99)(0.033)(100)$ and $x_B(100) = (0.01)(100)(0.033)$

$$x_B = 0.00033$$

For infinite number of trays, y_D is in equilibrium with $x_F = 0.033 \Rightarrow y_D = 0.27$

$$y_D V = (99)(0.033) \Rightarrow V_{\min} = (99)(0.033)/0.27 = 12.1 \text{ mol/h} = S_{\min}$$

Minimum 0.121 mol steam/mol feed

(b) Using twice the minimum moles of steam, calculate the number of theoretical steps needed, the composition of the overhead vapor, and the bottoms composition.

$$S = 2S_{\min} = 24.2 \text{ mol/h}$$

Since the overhead vapor contains 99% of the alcohol in the feed, $x_B = \mathbf{0.00033}$

Ethanol balance over the top part of the tower:

$$100(x_F - x) = 24.2(y_D - y)$$

At the bottom: $y = 0$, $x = x_B = 0.00033 \Rightarrow y_D = (100)(0.033 - 0.00033)/24.2 = \mathbf{0.135}$

Operating line: $100(0.033 - x) = 24.2(0.135 - y)$

The number of theoretical steps can be determined from the following Matlab codes:

```
% Example E4d4d6
clf
xB=0.00033;xF=0.033;yD = 0.135;
% Operating line: 100(0.033 - x) = 24.2(0.135 - y)
xe=[0      0.0080      0.020 0.0296      0.033];
ye=[0      0.0750      0.175 0.250 0.270];
pp=spline(xe,ye);
x=0:0.001:0.033;
y=ppval(pp,x);
yF=ppval(pp,xF);
plot(x,y,[0 .035],[0 .035],[xB xF],[0 yD],[xF xF],[xF yF],'-')
xlabel('x');ylabel('y')
```

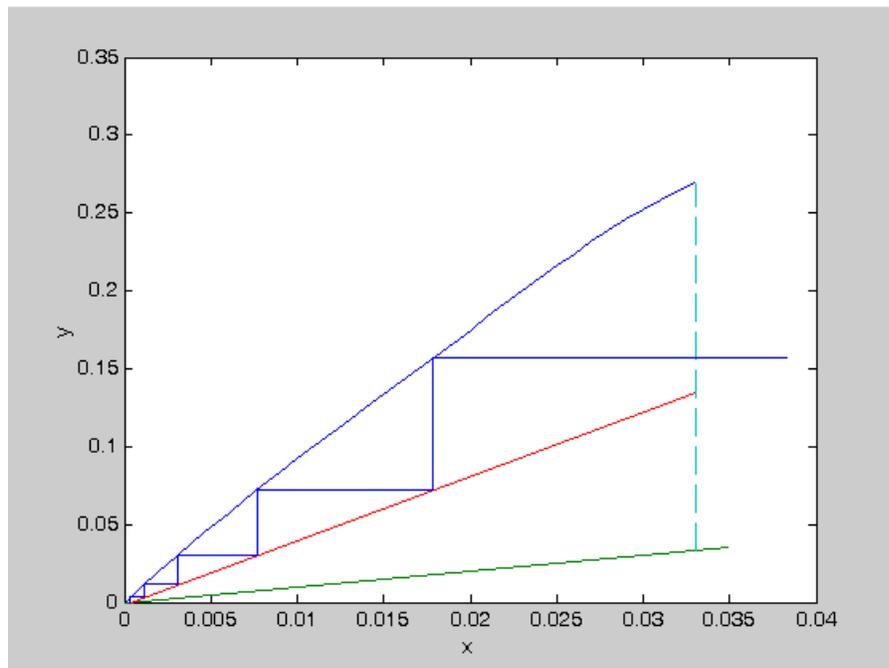
```

hold on;
xo=xB;yo=0;
for i=1:20;
ye=ppval(pp,xo);
line([xo xo],[yo ye])
xe=xo;
xo=xF-.242*(0.135-ye);
line([xe xo],[ye ye])
if ye>yD, break, end
yo=ye;
end
net=i-1 + (yD-yo)/(ye-yo);
fprintf('# of equilibrium stages = %g\n',net)

```

```
>> s5p3
```

of equilibrium stages = 4.73777



Example 4.4-7 -----

³(a) For the cascade shown below, calculate the composition of streams V_4 and L_1 . Assume constant molar overflow, atmospheric pressure, saturated liquid and vapor feeds, and the vapor-liquid equilibrium data given below.

x	0	0.10	0.30	0.50	0.70	0.90	1.000
y	0	0.20	0.50	0.68	0.82	0.94	1.000

³ J. D. Seader and E. J. Henley, Separation Process Principles, Wiley, 1998

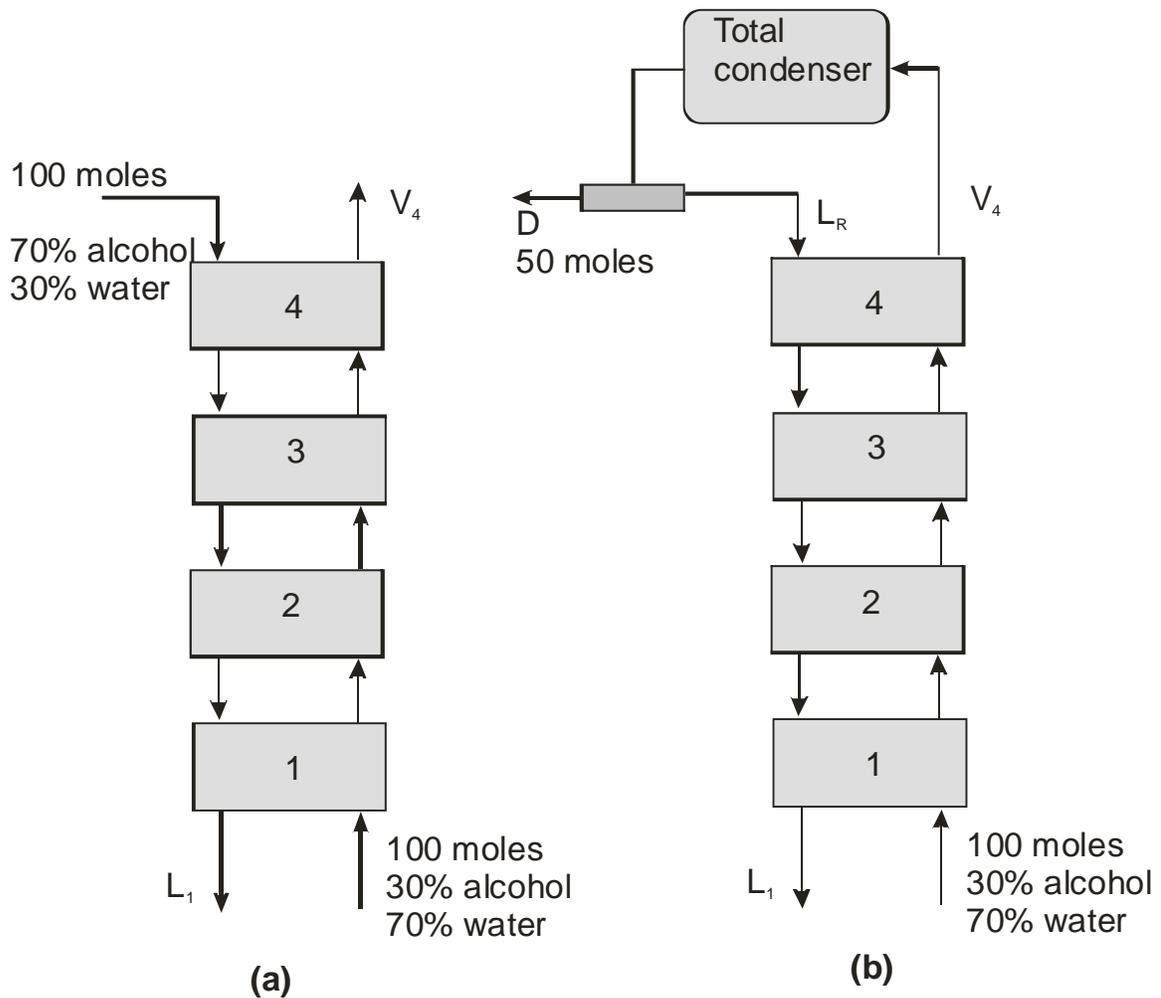


Figure E4.4-7 Vapor-liquid equilibrium cascade.

- (b) Given the feed compositions in cascade (a), how many stages would be required to produce a V_4 containing 85% alcohol?
- (c) For the configuration in cascade (b), with $D = 50$ moles what are the composition of D and L_1 ?
- (d) For the configuration in cascade (b), how many stages are required to produce a D of 50% alcohol?

Solution -----

Alcohol balance over the lower section of the cascade gives:

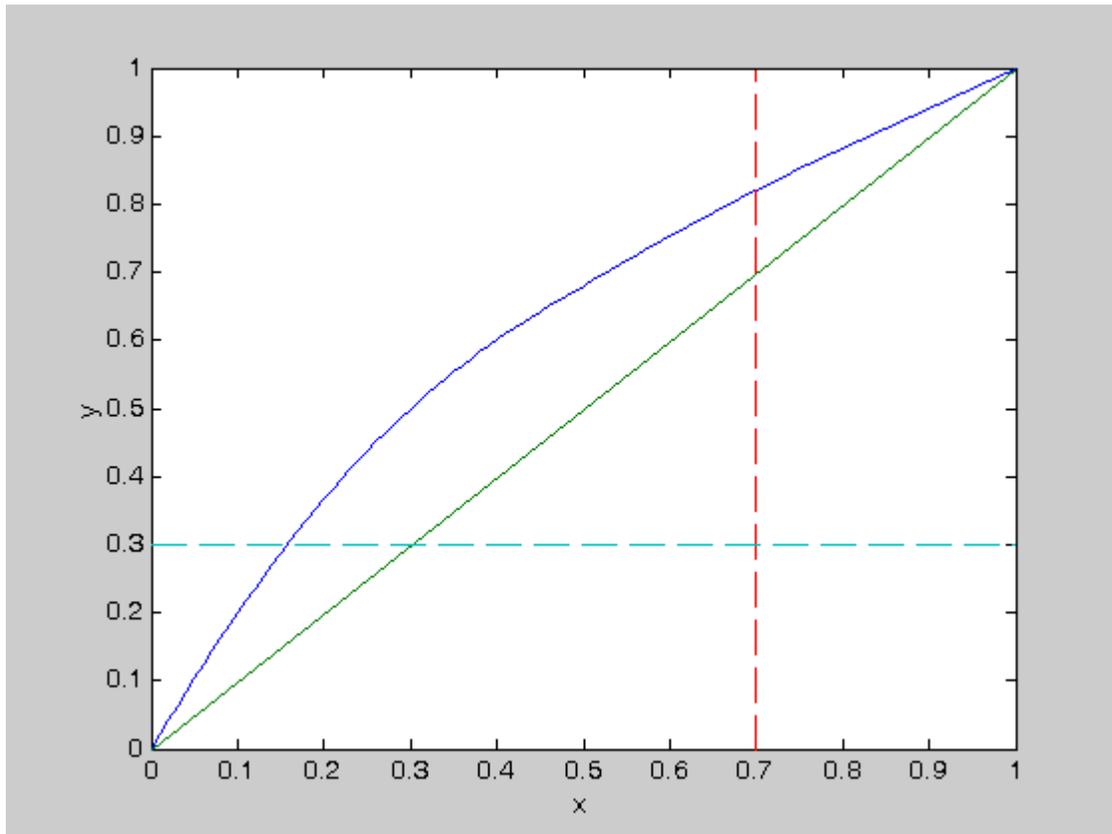
$$Lx_1 + Vy = Lx + Vy_0$$

Since $L = V = 100$ moles, we have

$$y = x + y_0 - x_1 = x + 0.3 - x_1$$

This operating line will terminate on the line $y_0 = 0.3$ and $x_5 = 0.7$ and provide 4 equilibrium stages. Composition of streams V_4 and L_1 can be determined from the following procedure:

Assume a value for x_1 (between 0.15 and 0.30) and step off between the equilibrium curve and the operating line to obtain 4 equilibrium stages. If the composition of x_5 is equal to 0.7 then y_4 is the correct composition of stream V_4 .



The procedure can be accomplished by the following Matlab codes:

```
% Example E4d4d7
clf
x5=0.7;y0 = 0.3;
% Operating line: 100(0.033 - x) = 24.2(0.135 - y)
xe=[0 0.10 0.30 0.50 0.70 0.90 1.000];
ye=[0 0.20 0.50 0.68 0.82 0.94 1.000];
pp=spline(xe,ye);
x=0:0.01:1;
y=ppval(pp,x);
plot(x,y,[0 1],[0 1],[x5 x5],[0 1], '--',[0 1],[y0 y0], '--')
xlabel('x');ylabel('y')
hold on;
x1=input('x1 ( between 0.15 and 0.30) = ');
xo=x1;yo=y0;
for i=1:4;
ye=ppval(pp,xo);
line([xo xo],[yo ye])
xe=xo;
xo=ye+x1-y0;
```

```

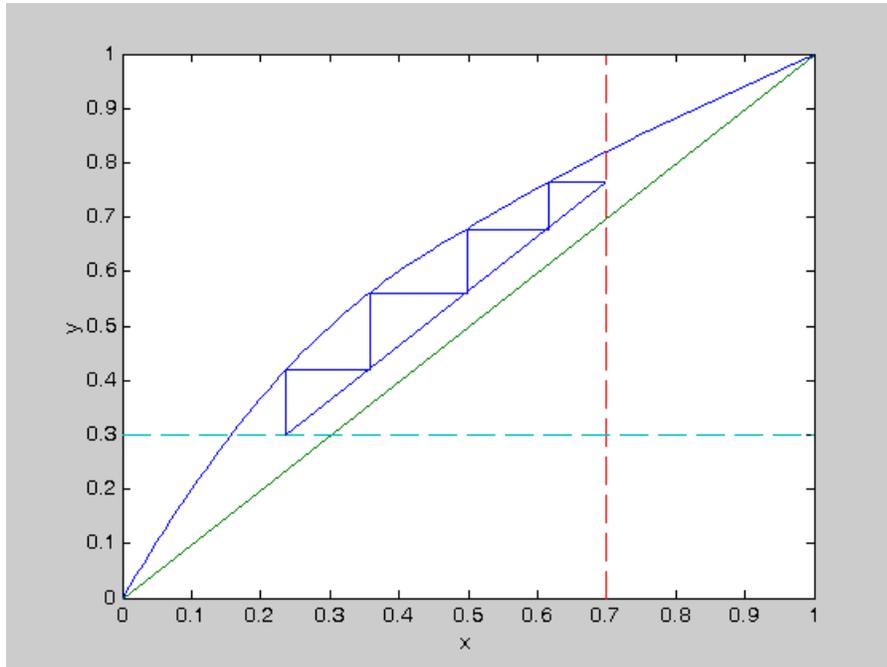
line([xe xo],[ye ye])
yo=ye;
end
fprintf('x5 = %g\n',xo)
line([x1 x5],[y0 yo])

```

```

>> E4d4d7
x1( between 0.15 and 0.30) = .237
x5 = 0.700456

```



(b) Given the feed compositions in cascade (a), how many stages would be required to produce a V_4 containing 85% alcohol?

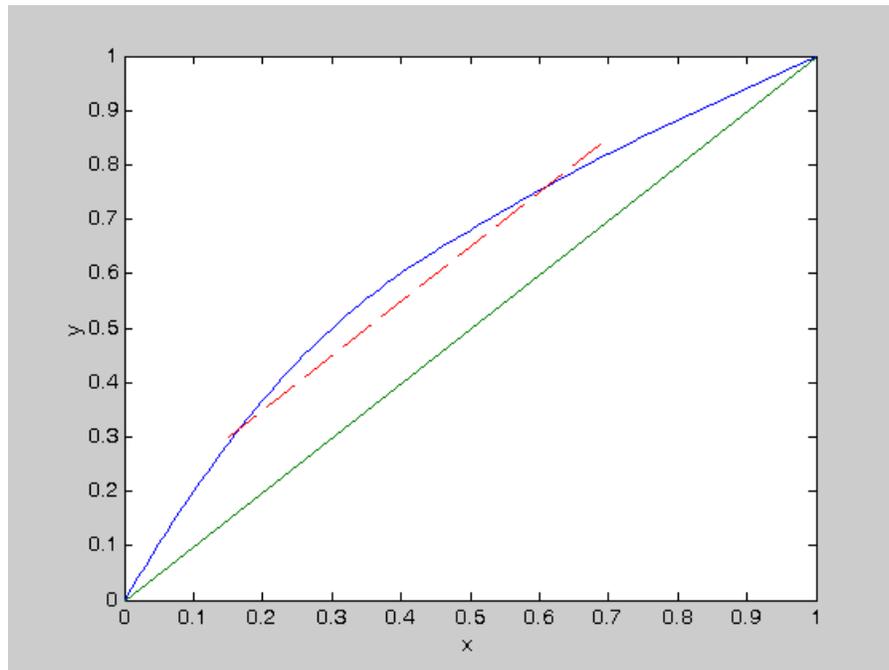
Alcohol balance over the cascade gives:

$$Lx_1 + Vy_4 = Lx_5 + Vy_0$$

Since $L = V = 100$ moles, we have

$$x_1 = x_5 + y_0 - y_4 = 0.7 + 0.3 - 0.85 = 0.15$$

Plot operating line $\{[0.15 \ 0.7], [0.3 \ 0.85]\}$, the operating line intersects the equilibrium curves. A vapor stream containing 85% alcohol cannot be obtained (infinite number of stages required).



(c) For the configuration in cascade (b), with $D = 50$ moles what are the composition of D and L_1 ?

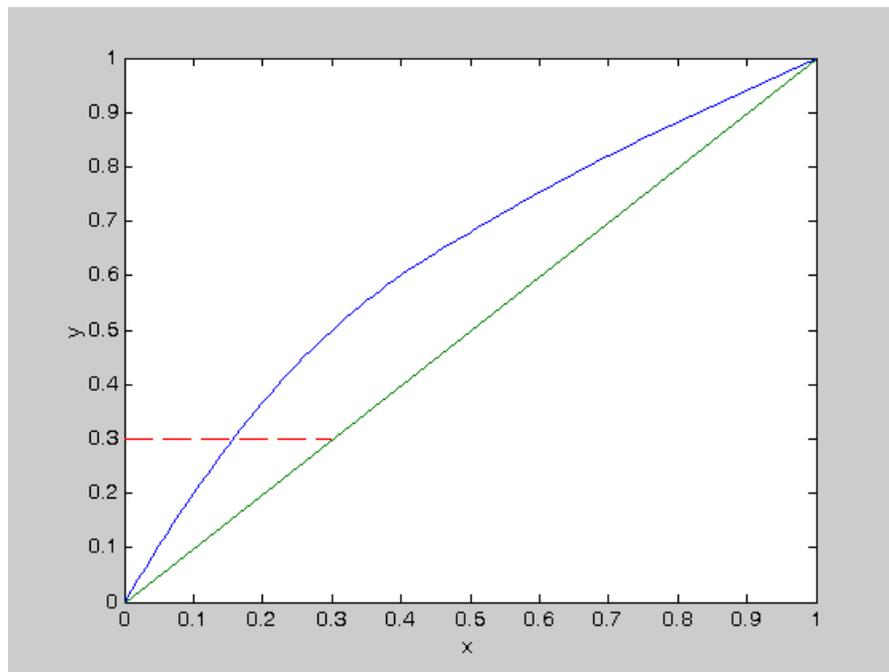
$$Lx + Dx_D = Vy \Rightarrow y = (L/V)x + (D/V)x_D$$

$$L + D = V \Rightarrow L = V - D = 100 - 50 = 50 \text{ moles}$$

$$\text{Operating line: } y = 0.5x + 0.5x_D$$

This operating line will terminate on the line $y_0 = 0.3$ and $y = x$ and provide 4 equilibrium stages. Composition of streams D and L_1 can be determined from the following procedure:

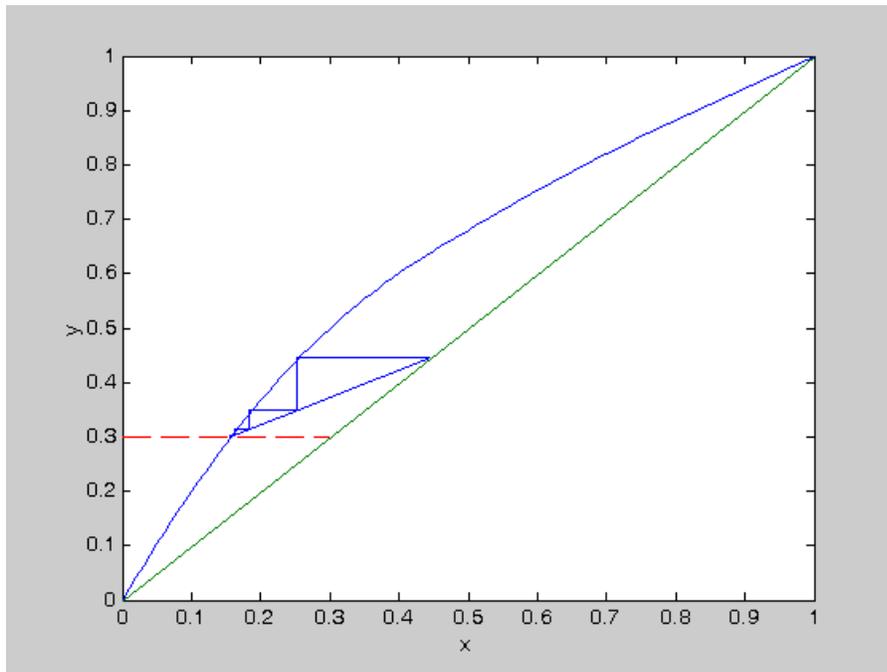
Assume a value for x_D (between 0.15 and 0.30) and step off between the equilibrium curve and the operating line to obtain 4 equilibrium stages. If the composition of y_0 is equal to 0.3 then x_D is the correct composition of stream D .



The procedure can be accomplished by the following Matlab codes:

```
% Operating line: 100(0.033 - x) = 24.2(0.135 - y)
xe=[0 0.10 0.30 0.50 0.70 0.90 1.000];
ye=[0 0.20 0.50 0.68 0.82 0.94 1.000];
pp=spline(xe,ye);
x=0:0.01:1;
y=ppval(pp,x);
plot(x,y,[0 1],[0 1],[0 y0],[y0 y0],'-')
xlabel('x');ylabel('y')
hold on;
xD=input('xD( between 0.3 and 0.95) = ');
pp=spline(ye,x);
yo=xD;xo=xD;
for i=1:4;
xe=ppval(pp,yo);
line([xe xo],[yo yo])
ye=yo;
yo=.5*xe+.5*xD;
line([xe xe],[yo ye])
xo=xe;
end
fprintf('y0 = %g, x1 = %g\n',yo,xo)
line([xo xD],[yo xD])

>> s5p4c
xD( between 0.3 and 0.95) = .445
y0 = 0.300531, x1 = 0.156062
```

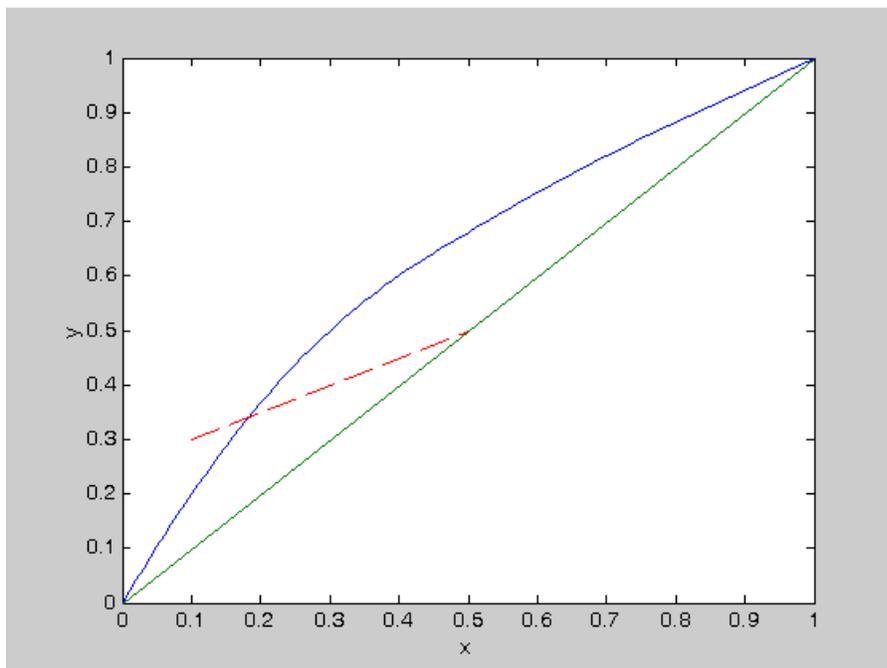


(d) For the configuration in cascade (b), how many stages are required to produce a D of 50% alcohol?

$$y = (L/V)x + (D/V)x_D = 0.5x + 0.25$$

$$\text{At } y = 0.3 \Rightarrow x = 2(0.3 - 0.25) = 0.1$$

Operating line: $[0.1 \ 0.5], [0.3 \ 0.5]$. The operating line intersects equilibrium curve \Rightarrow infinite number of stage.



4.5 Tray Efficiency

For an equilibrium tray, the vapor and liquid leaving the tray are in thermodynamic equilibrium. In a real tray, equilibrium will rarely be attained. The concept of tray efficiency is used to link the performance of a real tray to an equilibrium tray. Tray efficiencies can be defined in several ways:

The overall efficiency, E_o , is given by

$$E_o = N_{\text{equil}}/N_{\text{actual}}$$

The overall efficiency is extremely easy to measure and use; thus, it is the most commonly used efficiency value in the plant. However, the overall efficiency is not representative of column operation because the different compositions on the various trays result in different tray efficiencies. The overall column efficiency⁶ can be estimated from

$$E_o = 0.52782 - 0.27511 \log_{10}(\alpha\mu) + 0.044923 [\log_{10}(\alpha\mu)]^2$$

where α = relative volatility of key components, and μ = viscosity of feed in cP. These properties are determined at the average temperature and pressure of the column.

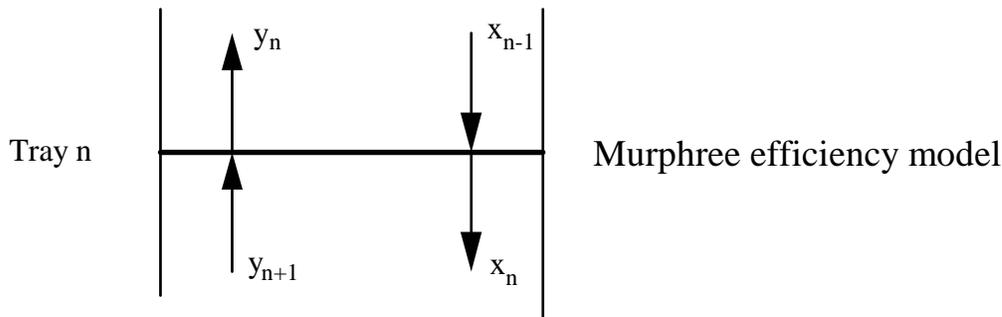
Murphree plate efficiency

The Murphree vapor efficiency is defined as

$$E_{MV} = \frac{y_n - y_{n+1}}{y_n^* - y_{n+1}} = \frac{\text{actual change in vapor}}{\text{change in vapor at equilibrium}}$$

while the Murphree liquid efficiency is defined as

$$E_{ML} = \frac{x_n - x_{n-1}}{x_n^* - x_{n-1}} = \frac{\text{actual change in liquid}}{\text{change in liquid at equilibrium}}$$



Point efficiency

⁶ Wankat, Equilibrium Staged Separation, Elsevier, 1988, pg. 383

If the vapor and liquid compositions are taken at a point on the plate, the above equation gives the local or point efficiency

Estimating Murphree tray efficiencies

The prediction methods available in the open literature are limited either to binary systems or to the efficiency of separation between the key components in a multi-component system. In a binary system, the efficiency obtained for each component must be the same. For a multi-component system; the heavier components will usually exhibit lower efficiencies than the lighter components. The following guide rule can be used to estimate the efficiencies for a multi-component system from binary data:

- If the components are similar, the multi-component efficiencies will be similar to the binary efficiency.
- If the predicted efficiencies for the binary pairs are high, the multi-component efficiency will be high.
- If the resistance to mass transfer is mainly in the liquid phase, the difference between the binary and multi-component efficiencies will be small.
- If the resistance is mainly in the vapor phase, as it normally will be, the difference between the binary and multi-component efficiencies can be substantial.

For mixtures with dissimilar compounds, the efficiency can be very different from the predicted for each binary pair, and laboratory or pilot-plant studies should be made to confirm any predictions.

Van Winkle's correlation can be used to predict plate efficiencies for binary systems. The data used to derive the correlation covered both bubble-cap and sieve plates.

$$E_{MV} = 0.07 Dg^{0.14} Sc^{0.25} Re^{0.08}$$

where

Dg = surface tension number = $(\sigma_L/\mu_L u_v)$,

u_v = superficial vapor velocity,

σ_L = liquid surface tension,

μ_L = liquid viscosity,

Sc = liquid Schmidt number = $(\mu_L/\rho_L D_{LK})$,

ρ_L = liquid density,

D_{LK} = liquid diffusivity, light key component,

Re = Reynolds number = $(h_w u_v \rho_v / \mu_L (FA))$,

h_w = weir height,

ρ_v = vapor density,

(FA) = fractional area = (area of holes or risers)/(total column cross-sectional area)

To evaluate the terms for the plate efficiency (if data are not available)

- Calculate vapor density from the ideal gas equation
- Calculate the density of the liquid mixture as the mole fraction average molar volume
- Calculate liquid mixture viscosity by using the expression

$$\mu_{L,mix} = \left[\sum_{i=1}^n x_i \mu_i^{1/3} \right]^3$$

- Calculate the diffusivity of the liquid light key component by the dilute solution of Wilke and Chang.

Example 4.5-1. -----

Determine the overall efficiency of a distillation column used to separate acetone from water⁵. The average temperature and pressure of the column are 83°C and 1 atm, respectively. The feed is saturated liquid with mole fraction of acetone equal to 0.033 corresponding to equilibrium vapor mole fraction of 0.421.

Solution -----

The overall column efficiency can be estimated from

$$E_o = 0.52782 - 0.27511 \log_{10}(\alpha\mu) + 0.044923 [\log_{10}(\alpha\mu)]^2$$

$$\alpha = \frac{y/x}{(1-y)/(1-x)} = \frac{0.421/0.033}{(1-0.421)/(1-0.033)} = 21.3$$

Viscosities of acetone and water can be obtained from T.K. Prop4 program at 83°C and 1 atm.

$$\mu_{\text{acetone}} = 0.190 \text{ cp}, \mu_{\text{water}} = 0.350 \text{ cp}$$

The mixture viscosity is then

$$\mu_{L,\text{mix}} = \left[\sum_{i=1}^n x_i \mu_i^{1/3} \right]^3 = \left[0.033 \times 0.190^{1/3} + 0.967 \times 0.350^{1/3} \right]^3$$

$$\mu_{\text{mix}} = 0.343 \text{ cp}$$

$$\alpha\mu = 7.31 \Rightarrow \log_{10}(\alpha\mu) = 0.864$$

The overall column efficiency is then

$$E_o = 0.52782 - 0.27511 \log_{10}(\alpha\mu) + 0.044923 [\log_{10}(\alpha\mu)]^2 = \mathbf{0.324}$$

⁵ Sinnott, R.K. Coulson & Richardson's, Chemical Engineering. Boston: Butterworth and Heinemann, 1999.

Example 4.5-2.

Determine the plate efficiency of a distillation column used to separate acetone from water⁵. The plate is at 79°C and 1.14 bar. The cross-sectional area of the column is 0.50 m². The vapor volumetric flow rate is 0.81 m³/s. The following data are provided

ρ_L (kg/m ³)	ρ_v (kg/m ³)	μ_L (kg/m·s)	μ_v (kg/m·s)	h_w (m)	A_h (m ²)
925	1.35	0.34×10^{-3}	10.0×10^{-6}	50×10^{-3}	0.038

Atomic volume of m ³ /kmol	C	H	O
	0.0148	0.0037	0.0074

Solution

We will first calculate the diffusivity of acetone in water at 79°C using Wilke Chang equation:

$$D_L = \frac{1.173 \times 10^{-16} (\phi M_w)^{0.5} T}{\mu V_m^{0.6}}$$

In this equation, $\phi = 2.26$ for water solvent, $M_w = 58$, $T = 273 + 79 = 352$ K, and for acetone (C₃H₆O)

$$V_m = 0.0148 \times 3 + 0.0037 \times 6 + 0.0074 = 0.074 \text{ m}^3/\text{kmol}$$

$$D_L = \frac{1.173 \times 10^{-16} (2.26 \times 58)^{0.5} (352)}{(0.34 \times 10^{-3})(0.074)^{0.6}} = 6.63 \times 10^{-9} \text{ m}^2/\text{s}$$

The plate efficiency can be estimated from

$$E_{MV} = 0.07 Dg^{0.14} Sc^{0.25} Re^{0.08}$$

$$Dg = \text{surface tension number} = (\sigma_L / \mu_L u_v),$$

$$u_v = \text{superficial vapor velocity} = \frac{0.81}{0.50} = 1.62 \text{ m/s}$$

$$\sigma_L = \text{liquid surface tension} = 60 \times 10^{-3} \text{ N/m}$$

$$Dg = \frac{60 \times 10^{-3}}{(0.34 \times 10^{-3})(1.62)} = 109$$

$$Sc = (\mu_L / \rho_L D_{LK}) = \frac{0.34 \times 10^{-3}}{(925)(6.63 \times 10^{-9})} = 55.4$$

⁵ Sinnott, R.K. Coulson & Richardson's, Chemical Engineering. Boston: Butterworth and Heinemann, 1999.

$$Re = \text{Reynolds number} = (h_w u_v \rho_v / \mu_L (FA)),$$

$$FA = \frac{A_h}{A_c} = \frac{0.038}{0.50} = 0.076$$

$$Re = \frac{(50 \times 10^{-3})(1.62)(1.35)}{(0.34 \times 10^{-3})(0.076)} = 4232$$

$$E_{MV} = 0.07 Dg^{0.14} Sc^{0.25} Re^{0.08} = 0.07 (109)^{0.14} (55.4)^{0.25} (4232)^{0.08} = \mathbf{0.72}$$

The efficiency of the plate is 72%.