INTERCOMPANY MEMORANDUM

CAL CHEM CORPORATION

To: CHE Juniors
Date: Winter Quarter
File: CHE 322L

From: CHE faculty
Laboratory Managers

Subject: Laminar Film Flow Around Long Cylindrical Bubbles

Our environmental group is presently working with a scheme to reduce the sulfur emission from the plant. They need data for mass transfer coefficients so that they can design the appropriate scrubber to meet the emission requirement.

A wetted-wall or falling-film column is a simple device in which a liquid flows in a thin film down the inside surface of a vertical cylindrical tube. The large areas of open surface available for mass transfer in this type of equipment help to compensate for the low mass transfer rates inherent in wetted-wall columns. Since the area for mass transfer is well defined for the wetted-wall, the mass transfer coefficients data can be readily obtained.

The thickness of the liquid film in the wetted-wall column can be measured by a simple experiment in which cylindrical bubbles are created (1). The equipment consists of a vertical, 1.5-m length of clear acrylic tubing connected at the top to a ¾-in. ball valve. The other side of the ball valve is connected to another vertical tube, 1-m in length, of the same material, terminated by another ball valve. In the experiment, a rubber stopper is placed in the bottom opening of the lower tube and then both tubes are completely filled with liquid. With the top ball valve closed and the valve between the two tubes open, one then removes the stopper from the bottom. As liquid flows out of the tubes, an air bubble forms and grows as it rises in the tube. The gas slug velocity may be determined from the time required for the bubble to rise a given height. The air slug is allowed to rise freely until its nose is about midway between the two ball valves. At this time the lower valve is closed as a beaker is simultaneously placed directly under the lower tube exit. The liquid ultimately collected in the beaker is the volume of the liquid film running down the wall between the lower ball valve and tube exit. The liquid film thickness may then be determined from this volume of liquid collected.

References

CAL CHEM CORPORATION

To: CHE Juniors
From: CHE faculty
Laboratory Managers

Date: Winter Quarter
File: CHE 322L

Subject: Operating Characteristics of an Air Lift

Our pilot plant design group is presently working with a liquid system that requires the use of specialized pumping equipment. Water that contains a very small amount of a highly corrosive chemical in solution is to be pumped a few feet vertically and discharged to a recovery vessel. Our engineers feel that an air lift might represent the most durable and inexpensive way to pump this liquid.

Correlations in the literature do not apply for the short air lift we propose. Please investigate the relationship between air injection rate and water discharge rate for several submergence heights. Display your results graphically and compare with the correlation for taller air lifts. Also give us an idea of the efficiency of operation we can expect from an air lift.

To aid you in this study, we have had our technicians fabricate an air lift in the laboratory.
Experiment No. 1

Part A: Laminar Film Flow Around Long Cylindrical Bubbles

The velocity $U$ of a rising cylindrical bubbles shown in Figure 1 is given by\(^{[1]}\)

$$U = 0.345 (gD)^{1/2} \tag{1}$$

where $g$ is the acceleration of gravity. Equation (1) is valid for cylindrical bubbles in liquids of low to moderate viscosity corresponds to the criterion\(^{[2]}\)

$$N_f = (gD^3)^{1/2}/\nu > 300$$

where $N_f$ is the dimensionless inverse viscosity and $\nu$ is the kinematic viscosity of the liquid.

The volumetric balance of gas and liquid flowing through any cross section of the tube requires that

$$Q = \frac{\pi}{4} (D - 2\delta)^2 U \tag{2}$$

where $Q$ is the volumetric flow rate of liquid running down along the tube wall, and $\delta$ is the thickness of the liquid film. At steady state the film thickness $\delta$ will be constant along the lower tube length. If $\delta/D << 1$, $Q$ can be estimated from the flow of liquid running down along a vertical wall. If $w$ is the width of the wall then

$$q = \frac{Q}{w} = \frac{g\delta^3}{3\nu} \tag{3}$$
Since $\delta/D \ll 1$, $w$ is the perimeter of the tube, $\pi D$. Equation (3) is valid for laminar flow when

$$\text{Re}_f = \frac{4q}{\nu} = \frac{U(D - 2\delta)^2}{\nu D} < 1500$$

(4)

where $\text{Re}_f$ is the film Reynolds number.

Equations (2) and (3) can be combined to obtain

$$U = \frac{4gD}{3\nu} \frac{\delta^3}{(D - 2\delta)^2}$$

(5)

The bubble velocity from Eq. (1) is substituted into (5)

$$\frac{\delta^3}{(D - 2\delta)^2} = \frac{\nu}{3.86(gD)^{1/2}}$$

(6)

Eq. (6) is valid only if $N_f > 300$ and $\text{Re}_f < 1500$. The dimensionless film thickness $\xi$ is defined as

$$\xi = \frac{\delta}{D} N_f^{2/3}$$

(7)

A relationship between $\xi$ and the Reynolds number can be obtained by using the definition of $N_f$ and substituting $\delta$ from Eq. (3) into Eq. (7)

$$\xi = 0.909 \text{Re}_f^{1/3}$$

(8)

Data Analysis

1. Measure the bubble velocity $U$ and the liquid film thickness $\delta$ for pure water, 20 wt% glycerol solution, and 40 wt% glycerol solution.

2. Plot the experimental $\delta$ versus $\nu$ and compare with the calculated $\delta$ from Eq. (6). Note: the experimental $\delta$ is determined from the volume of liquid collected, $V$, through

$$V = \frac{\pi}{4} H[D^2 - (D - 2\delta)^2]$$

3. Plot the experimental $\xi$ versus $\text{Re}_f$ and compare with the calculated $\xi$ from Eq. (8). Note: the experimental $\xi$ is determined from the measured $\delta$ and $N_f$. $\text{Re}_f$ is determined from Eq. (4).

4. Derive Eq. (3) above[^3]. Include this derivation in the Appendix of your report.
Table 1. Absolute Viscosities of Aqueous Glycerol Solutions

<table>
<thead>
<tr>
<th>Wt % Glycerol</th>
<th>20°C</th>
<th>25°C</th>
<th>30°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.005</td>
<td>0.893</td>
<td>0.800</td>
</tr>
<tr>
<td>20.0</td>
<td>1.769</td>
<td>1.542</td>
<td>1.360</td>
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<tr>
<td>40.0</td>
<td>3.750</td>
<td>3.181</td>
<td>2.731</td>
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</tbody>
</table>

Table 2. Densities of Aqueous Glycerol Solutions

<table>
<thead>
<tr>
<th>Wt % Glycerol</th>
<th>20°C</th>
<th>25°C</th>
<th>30°C</th>
</tr>
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<tbody>
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<td>40.0</td>
<td>1.09930</td>
<td>1.09710</td>
<td>1.09475</td>
</tr>
</tbody>
</table>

References

3. “Shell” program from Interactive Computer Modules for IBM-PC, Chemical Engineering Department, University of Michigan.
4. Handbook of Chemistry and Physics, CRC Press.
Part B: Operating Characteristics of an Air Lift

![Diagram of an air lift](image)

**Fig. 1** Simplified sketch of an air lift.

The air lift can be used to pump liquid from a well as shown in Fig. 1. Compressed air is injected near the bottom of the well. The air and liquid mixture, being lighter than liquid alone, will rise from the well to the discharge level.

If a mass \( M \) of liquid is raised through a net height \( h_r \) by mass of air \( m_a \), the net work received by the liquid is

\[
W_{\text{received}} = Mgh_r
\]

(1)

The work supplied by the air to pump the amount \( M \) of liquid is equal to the work done by a mass \( m_a \) of air in expanding isothermally to atmospheric pressure \( P_a \).

\[
W_{\text{supplied}} = P_a v_a m_a \ln\left(\frac{P}{P_a}\right)
\]

(2)

where \( v_a \) is the specific volume of air at atmospheric pressure and \( P \) is the pressure of air at the injection point. This pressure would be equal to atmospheric pressure plus the hydrostatic pressure due to the column \( h_s \) of the liquid with density \( \rho \).

\[
P = P_a + h_s \rho g
\]

(3)

The efficiency of the air lift \( \eta \) is therefore given by

\[
\eta = \frac{Mgh_f}{P_a v_a m_a \ln\left[1 + h_s \rho g / P_a\right]}
\]

(4)
The Ingersoll-Rand Company has developed entirely empirical information on air-lift performance. The volume of air at atmospheric pressure required to raise 1 gal of water is given by

\[ V_{\text{air}} = 0.67 \frac{h_r}{C \log_{10}[1 + h_s / 34]} \], where \( C = 245 \) for \( 10 \leq h_r \leq 60 \) (5)

where \( h_r \) and \( h_s \) are in feet and air is at the water temperature.

Data Analysis

1. Plot a graph of \( \eta \) versus \( h_s \), using experimental data and values calculated from Eq. (4).

2. Plot a graph of \( \eta \) versus work input from air \( W_{\text{supplied}} \), using experimental data and calculated values.

3. Plot the mass of air required to raise one gallon of water versus \( h_s \), using experimental data and calculated values from Eq. (5). Determine the value of \( C \) so that equation (5) will fit the experimental data.

References

1. Perry, J. H., Chemical Engineers’ Handbook, McGraw-Hill, 1984, pg. 6-16
2. Coulson J. M. et al., Chemical Engineering, Volume 1, Pergamon, 1993, pg. 300
INTERCOMPANY MEMORANDUM

CAL CHEM CORPORATION

To: CHE Juniors
From: CHE faculty
Laboratory Managers

Subject: Operating Characteristics of a Centrifugal Pump

Date: Winter Quarter
File: CHE 322L

The biomedical supply division of CAL CHEM, BIOMED, wants to use an available centrifugal pump to supply cooling water to a fermentation reactor. BIOMED want us to determine the maximum head developed by the pump and the efficiency of the pump as a function of flow rate. They also want to know if the pump has the general characteristic of a centrifugal pump.

Please set up an experiment so that you can obtain and analyze the appropriate data to provide BIOMED with the information they need. Our technicians are very familiar with the operation of the pump. They will assist you in the process of taking the data you need if necessary.
Experiment No. 2

PUMPS: TYPES, CHARACTERISTICS, OPERATING CURVES

![Diagram of a simple centrifugal pump]

Centrifugal pumps are widely used for transferring liquids of all types. These pumps are available in capacities from 2 GPM to $10^5$ GPM, and for discharge pressures from a few feet to approximately 7000 psi (Ref. 1).

A centrifugal pump typically consists of an impeller rotating within a stationary housing. The impeller usually consists of two flat disks, separated by a number of curved vanes (blades), mounted on a shaft that project outside the housing. Power from an outside source is applied to the shaft, rotating the impeller within the stationary housing. The revolving vanes of the impeller produce a reduction in pressure at the inlet hole or eye of the impeller. Fluid then flows to the eye. From the eye the fluid is flung outwards by centrifugal force into the periphery of the housing and from there to the volute chamber and finally to the pump exit. The overall pressure increase across the pump at low flow rates is approximately (Ref. 2)

$$\Delta p = \rho u_2^2 = \rho \pi^2 D^2 N^2$$

where

- $\rho$ = density of the fluid
- $u_2$ = tangential impeller velocity
- $D$ = diameter of impeller
- $N$ = rotational speed of the impeller

so that the dimensionless group $\Delta p/(\rho D^2 N^2)$ should be roughly constant at low flow rates. Let $c_1D$ be the gap between the disks of the impeller and $c_2u_2$ be the radial velocity outwards; then the volumetric flow rate is given by

$$Q = (\pi D)(c_1D)(c_2u_2) = (\pi D)(c_1D)(c_2\pi DN) = c_1c_2\pi^2 D^3 N$$
where $c_1$ and $c_2$ are constants.

This model proposes that the flow rate is proportional to the tangential velocity of the impeller. Thus the dimensionless group $Q/(ND^3)$ is approximately constant at low flow rates. Even though the assumptions made for the two dimensionless groups $\Delta p/(\rho D^2 N^2)$ and $Q/(ND^3)$ are for low flow rates they are usually adequate to characterize all centrifugal pumps at any flow rate. A plot of $\Delta p/(\pi D^2 N^2)$ versus $Q/(ND^3)$ will have the general shape given by the curve in Fig. 2.

![General characteristic curve for centrifugal pump.](image)

We have a centrifugal pump testing unit in the transport laboratory. This consists of the pump, the DC motor, a dynamometer, meters for D.C. volts and amps, various pressure gauges, a water feed tank and a weigh tank for mass measurement during a measured time interval.

Make a sketch of the pump testing unit, showing all relevant information. This does not have to be exactly to scale, but it should resemble the physical unit.

Our laboratory technician has prepared the following operating procedure for the test unit. This may not be complete, but it gives you something to start with.

**Pump Test Unit Operation**

1.- Set the pump speed as specified. Initially use the highest allowable pump speed. Limit the speed to below 3000 rpm.
2.- The pump speed is read from an AC voltmeter located to the left of the motor. The rpm scale is shown above the voltage scale.
3.- Check the water level in the tank supplying the pump. Record the water temperature.
4.- For a given pump speed, establish the maximum flow rate through the pump by opening wide the valve in the outlet line. Additional runs can be made at lower flow rates by throttling this valve.
5.- Allow sufficient time for steady flow to be established before making the first experimental run.

6.- Record the time for pumping a definite mass of water. When the tank is nearly full, open the quick release valve to drain water back to the feed tank.

7.- Record all of the appropriate pressure readings and all other important information. Remember that in weighing, a tare is often used. Even if the tare is zero it is good practice to mention this.

8.- In this DC motor the armature voltage controls the pump speed. Adjust this voltage (and pump speed) with the large circular dial on the variac on the table.

9.- Record the voltage and amperage for both the armature and field coils. The armature current is measured by the large ammeter provided. The armature voltage is measured by the meter mounted in front of the motor. The voltage and amperage for the field coils are measured by the meters located on a separate box.

10.- Record the torque produced by the motor on the shaft.

The DC motor consists of two basic units, the field, which is the electromagnet with its coils, and the armature, the structure that supports the conductors that cut the magnetic field and carry the exciting current. When current is passed through the armature of a DC motor, a torque is generated by magnetic reaction, and the armature revolves. The power supplied to the motor is then given by

$$P_{\text{motor, in}} = V_a I_a + V_f I_f$$

where $V_a$ and $I_a$ are armature voltage and armature amperage respectively and $V_f$ and $I_f$ are field voltage and field amperage respectively. You should record $V_f$ and $I_f$ even if $V_f I_f$ is much less than $V_a I_a$.

Motor efficiency is defined as

$$\eta_{\text{motor}} = \frac{P_{\text{motor, out}}}{P_{\text{motor, in}}}$$

where

$$P_{\text{motor, out}} = \omega T, \quad \omega$$ is angular velocity and $T$ is the torque applied to the impeller.

Pump efficiency is defined as

$$\eta_{\text{pump}} = \frac{P_{\text{pump, out}}}{P_{\text{pump, in}}}$$

where $P_{\text{pump, in}} = P_{\text{motor, out}}$, and

$$P_{\text{pump, out}} = \dot{m} \Delta p/\rho, \quad \dot{m}$$ is the mass flow rate and $\Delta p$ is the pressure rise across the pump.

The overall efficiency is defined as

$$\eta_{\text{overall}} = \frac{P_{\text{pump, out}}}{P_{\text{motor, in}}}$$
The operating curves which you are to produce should show head, pump efficiency and power ($P_{\text{pump, out}}$) as a function of capacity (flow rate). After you have performed the first run as outlined above, make a series of runs at reduced flow rates down to and including zero rate. You should have at least six runs for the high pump speed. Reduce the pump speed, to about two thirds of the maximum speed, and make another six runs. Make a third series of runs at a still lower pump speed.

After you complete the pump curves determine the flow rate versus the valve position. You will require some kind of marker on the valve handle (a paper clip, a felt pen mark, etc.). Valve position can be measured by turns and fractions of turns. Measure flow rate versus valve position, from closed to wide open. A good control valve would have approximately a linear relation between flow and valve opening. Would the globe valve be a good control valve?

**Analysis of Experimental Data**

For each run:
- Convert mass-time data to gpm.
- Convert pressure readings to feet head (of water).
- Calculate the power (hp) to the pump.
- Calculate the pump, the motor and the overall efficiencies.

**The Report**

Present results graphically showing head, pump efficiency and power versus capacity (flow rate). There is probably too much information for one graph, so experiment with various ways of displaying your results until you find one that is easy for the reader to understand. Present all of your efficiency results in a small table and plot $\Delta p/ (\rho D^2 N^2)$ versus $Q/(ND^3)$.

Answer the following questions:
1. How does one determine the correct direction of rotation for a centrifugal pump?
2. What is "cavitation"? Why should it be avoided? How is it avoided?
3. What is NPSH?
4. What is a positive displacement pump? Why should a positive displacement pump have a relief valve?
5. Most pumps have some kind of packing box or seal. What can you determine about the seal of the Teel pump?

**References**

1. Perry’s Chemical Engineers’ Handbook, Sixth Edition (Section 6-7)
CAL CHEM CORPORATION

To: CHE Juniors
From: CHE faculty
Date: Winter Quarter
File: CHE 322L
Subject: Cooling by Hilsch Tube

Nitrogen
30 kg/hr
3.0 atm
300 K

15 kg/hr
1 atm, 450 K

15 kg/hr
1 atm, 150 K

The Diamond Bar plant has a nitrogen stream at 3.0 atm and 300.0°K that is presently vented to the atmosphere. The management would like to use this stream to satisfy some of the heating or cooling requirements of other processes. Their engineer, James Hank, a Cal Poly graduate, has devised a process that will produce equal amounts of a hot stream at 450°K and a cold stream at 150°K and thus satisfy simultaneously some heating and cooling requirements. Furthermore, James claims that his device will be self-sustaining because no additional heat or work need be supplied to the device. The management would like an explanation of this device.

The principles behind James’ device are believed to be the same as the Hilsch-Ranque Vortex Tube, which is available in our laboratory. Please take the necessary data to verify the claims by James Hank.

Operate the Hilsch tube over the widest possible range of the operating variables. The inlet air pressure to the Hilsch tube can be set by the air regulator. The ratio of the hot air flow rate to the cold air flow rate ($\dot{m}_{hot}/\dot{m}_{cold}$) can be changed by turning the knob at the hot outlet of the Hilsch tube.

You will be measuring temperatures and flow rates. Use thermocouples for the temperature measurement. Ask the technician for the vane anemometer to measure air flow rates. The vane anemometer is a revolution counter with jeweled bearing, actuated by a small windmill. Air velocity is displayed in ft/s or m/s by a selector switch.
Experiment No. 3

COOLING BY HILSCH TUBE

The actual flow rate will be cancelled out in the analysis, so it is not important to have an accurate flow rate. However, the ratio of the hot to cold air flow rates is required in the calculation, the measurements of the hot and cold air velocity must be consistent to yield good results. Air leaving the Hilsch tube flows through a pipe where the anemometer is used to measure air flow rate. The mass flow rate leaving a pipe can be estimated by

\[ \dot{m} = A \rho v \]

where \( A \) is the inside area of the pipe, \( v \) is the air velocity, and \( \rho \) is the air density. The temperature of the air leaving the anemometer must be measured to determine the air density by ideal gas law. This temperature is not the hot temperature \( T_h \) or the cold temperature \( T_c \) used in Eq. (1). \( T_h \) and \( T_c \) must be measured just inside the Hilsch tube.

The laboratory work is relatively easy to complete. Be certain that your data should have at least six different inlet pressures at three different \( \dot{m}_{\text{hot}}/\dot{m}_{\text{cold}} \) for a total of eighteen different conditions. You should make at least six, good experimental runs at a given inlet air pressure and \( \dot{m}_{\text{hot}}/\dot{m}_{\text{cold}} \). Note the time required to reach steady state operation. Compute the standard error in your experimental readings and in your calculated results.

Analysis

This is mostly a “think” experiment that tests your knowledge of thermodynamics. When you perform a thermodynamic analysis of a process remember:
- both the first and second laws must be followed for the process to be theoretically possible,
- always do the first law analysis before the second law analysis.
- if you are analyzing a process and the first law is violated you must stop at that point. Go over the first law calculations and, if possible, discover the error. Examine carefully any assumption that you made. Only after the first law analysis is correct can you perform the second law analysis.

Apply the first law to the Hilsch tube

\[ T_i = hT_h + (1 - h)T_c \]  

(1)
where \( h = \frac{\dot{m}_{\text{hot}}}{\dot{m}_{\text{total}}} \), \( \dot{m}_{\text{total}} = \dot{m}_{\text{hot}} + \dot{m}_{\text{cold}} \), \( T_i \) = inlet air temperature, \( T_c \) = outlet cold air stream, and \( T_h \) = outlet hot air stream.

The maximum temperature spread from the Hilsch vortex tube will be obtained for a reversible operation where the total entropy change is zero (Ref. 1, pg. 123).

\[
\Delta S = hC_p \ln \left( \frac{T_h}{T_i} \right) + (1 - h)C_p \ln \left( \frac{T_c}{T_i} \right) + R \ln \left( \frac{P_i}{P_o} \right) = 0
\]

where \( R \) = gas constant, \( P_i \) = inlet air pressure, \( P_o \) = outlet air pressure, and \( C_p \) = heat capacity of air.

Substitute \( T_h/T_i \) from Eq. (1) into Eq. (2) to obtain

\[
hC_p \ln \left\{ \frac{1 - (1-h)(T_c/T_i)}{h} \right\} + (1 - h)C_p \ln \left( \frac{T_c}{T_i} \right) + R \ln \left( \frac{P_i}{P_o} \right) = 0
\]

Eq. (3) can be solved for \( T_c/T_i \) if \( h \) and \( (P_i/P_o) \) are known.

- Solve Eqs. (1) and (3) for \( T_h/T_c \), \( T_c/T_i \), and \( T_h/T_i \) at \( (P_i/P_o) = 2 \) over the range \( 0.2 \leq h \leq 0.8 \). Plot the results.
- Solve Eqs. (1) and (3) for \( T_h/T_c \), \( T_c/T_i \), and \( T_h/T_i \) at \( h = 0.5 \) over the range \( 1.5 \leq (P_i/P_o) \leq 7.5 \) obtainable from the experiment. Plot the results.

The Hilsch vortex is not a reversible device, hence the maximum temperature spread will be lower than the value predicted by Eqs. (1) and (3). An analytical solution by B. Ahlborn et al focuses on the influence of the bulk kinetic energy on the energetics of each of the two fluid streams from a vortex tube (Ref. 2). The heating in a vortex tube is attributed to conversion of kinetic energy into heat and the cooling is attributed to the reverse process. This model yields the upper limit for the temperature increase on the hot side:

\[
T_h \leq T_i [1 + X(\gamma - 1)/\gamma]
\]

where \( \gamma = C_p/C_v \) and \( X = \frac{P_i - P_o}{P_i} \)

and a lower limit for the temperature reduction on the cold side:

\[
T_c \geq T_i (1 - X)^{(\gamma-1)/\gamma}
\]

Analyze James' device using both the first and second laws of thermodynamics. However, proving a device satisfying these laws gives us no insight into how to design such a device. There are many schemes you could use to evaluate James’ claim that no additional heat and work are required for his device. One possible scheme is described below (Ref.3):

- Step 1: Split the inlet flow of 30 kg/hr into two streams of 15 kg/hr. Both are at 300°K and 3.0 atm.
- Step 2: Compress one stream reversibly and adiabatically to pressure \( P_a \) where the temperature is 450°K. This stream is the hot stream. Determine the outlet pressure \( P_a \) and the
required work $W_a$ from the energy balance. You can assume that the heat capacity of nitrogen is 29.3 J/mol·°K, independent of temperature or pressure.

- Step 3: Expand the other stream reversibly and adiabatically to pressure $P_b$ where the temperature is 150°K. This stream is the cold stream. From the energy balance the work obtained should be equal to -$W_a$. We now have two streams at the correct flow rate and temperature but whose pressures differ from the final desired value. No net heat or work transfer has taken place.

- Step 4: Expand the hot stream reversibly and isothermally to 450°K and 1 atm. Let this work be $W_h$ and the heat flow to the stream be $Q_h$.

- Step 5: Compress the cold stream reversibly and isothermally to 150°K and 1 atm. Let this work be $W_c$ and the heat rejected from the stream be $Q_c$.

- Step 6: A heat engine is required to pump the heat $Q_c$ to the hot stream. Determine the work $W_{engine}$ required. Determine the heat supplied to the hot stream by the heat engine.

- Step 7: Check if there is enough energy to supply heat $Q_h$ to the hot stream.

- Step 8: Evaluate the entropy change associated with step 4, 5, 6, and 7. Compare this value with the entropy change obtained from Eq. (3).

**Report**

1. Present the results of your experimental work in the form of a graph of $T_h/T_c$ vs. $P_i/P_o$ at various $\dot{m}_{hot}/\dot{m}_{total}$.

2. On the same graph, plot $T_h/T_c$ calculated from Eqs. (1), (3) and from Eqs. (4), (5). Could you determine whether $T_h/T_c$ is independent of $\dot{m}_{hot}/\dot{m}_{total}$ as predicted by Eqs. (4), (5).

3. Use the scheme outlined in the analysis section to evaluate James’ device. Draw a diagram to explain the operation of this scheme.

4. Two devices could be used to meet the specifications claimed by James Hank. The first device is the Hilsch-Ranque vortex tube that you evaluated in the lab, the second device consists of an air-driven turbine that extracts work from a flowing gas (Ref.1, pg. 124). Draw diagrams to describe the operation of these devices and state whether one or both of these devices could be used to satisfy the specifications claimed by James Hank.

5. Derive Eq. (2) from the First and the Second laws of Thermodynamics.

**References**

The biomedical supply division of CAL CHEM, BIOMED, obtained a contract to supply dialysate used in hemodialysis. In this operation, heparinized blood flows through a device containing a membrane as shown in Figure 1. The dialysate or exchange fluid flows on the membrane side opposite the blood. The toxic waste products, mostly urea, from the blood diffuse through the membrane to the dialysate. The composition of the dialysate is based on the need to restore the uremic plasma to the normal state without depleting the essential electrolytes in blood plasma (1). The membranes are usually made from such materials as cellulose, cellulose acetate, polyacrylonitrile, and polycarbonate. The membrane surface area is about 1 m². Blood flow rates are in the range of 100 to 300 ml/min and the dialysate flow rate is about twice that of the blood.

BIOMED would like to know the viscosity of the dialysate to calculate the necessary pressure drop across the hemodialyzer for the required dialysate flow rate. Please calibrate our viscometer so that we could accurately measure the dialysate viscosity that is on the order of the viscosity of water.

Reference:
Experiment No. 4

VISCOSITY MEASUREMENT

Introduction

A number of devices are available to measure the viscosity of a Newtonian fluid. However, an accurate determination of viscosity requires a careful analysis of the experimental technique that is used. To study some of the problems involved in viscosity measurement, a Cannon-Fenske and a Stormer viscometer will be considered.

The Cannon-Fenske viscometer is a capillary type viscometer with its main component shown in Figure 1. To make a measurement, suck the liquid past the upper etched line and then release it. The efflux time $t_{\text{efflux}}$ required for the liquid level to pass from the upper to the lower etched line is measured. Since the efflux volume $V_{\text{efflux}}$ between the etched lines is calibrated, the average volume flow rate $Q$ through the capillary is given by

$$Q = \frac{V_{\text{efflux}}}{t_{\text{efflux}}} \quad (1)$$

![Fig. 1 Capillary viscometer.](image-url)
The volumetric flow rate through the uniform capillary described above can also be obtained from application of the steady momentum balance to the laminar flow of fluids. The resulting equation is the Hagen-Poiseuille relationship (Ref. 1).

\[ Q = \frac{\pi R^4 \Delta P}{8 \mu L} \]  
(2)

where

- \( \Delta P \) = total up-stream pressure minus total downstream pressure
- \( R \) = capillary radius
- \( L \) = capillary length
- \( \mu \) = absolute viscosity

If the total pressure change is due to hydrostatic head alone, Eq. (2) can be written as

\[ \nu = \frac{\pi g (\Delta h) R^4}{8 Q L} \]  
(3)

where

- \( g \) = acceleration of gravity
- \( \Delta h \) = upstream hydrostatic head minus downstream hydrostatic head
- \( \nu \) = kinematic viscosity

Using \( Q \) from Eq. (1), the kinematic viscosity can be obtained

\[ \nu = \left[ \frac{\pi g (\Delta h) R^4}{8 L V_{\text{efflux}}} \right] t_{\text{efflux}} = C t_{\text{efflux}} \]  
(4)

where the quantity in square bracket is a known constant \( C \) for a given viscometer.

**Experimental Work**

There is a black, loose-leaf binder containing background information on viscometers and two ASTM (American Society for Testing Materials) procedures for viscometers. D-445 is more general. D-446 is more specific. There is viscosity data for castor oil vs. temperature and for aqueous glycerol vs. concentration. Please look at all articles.

**Use of Cannon-Fenske (C-F) viscometer tubes**

We have a number of Cannon-Fenske viscometer tubes. You must determine the correct size of the C-F tube for each fluid using Table 1. If the recommended size is not used for the viscosity range you should discuss the consequences in the report. The instructions for the Cannon-Fenske viscometer are (from the manual):

1. The viscometer should be cleaned with a suitable solvent and dry, clean, filtered air should be blown through the viscometer to remove any remaining traces of solvent.
2. The instrument should be periodically cleaned with chromic acid to remove any possible traces of organic deposits.
3. If a possibility of lint, dust, or other solid material is present in liquid sample, this may be removed by filtering through a sintered glass filter or fine mesh screen.
4. To introduce sample into the viscometer, invert viscometer and immerse tube “A” into liquid. Apply suction to “J” which causes the sample to rise to etched line “E”. Turn the viscometer to normal position and wipe tube “A” clean.
   - An alternative step is to use a pipette and pour liquid into leg “J”.
5. Insert the viscometer into a holder and place in Constant-Temperature Bath. Allow 10 minutes for viscometer to reach equilibrium at 100°F and 15 minutes at 210°F.

![Cannon-Fenske Routine Viscometer](image)

**Fig. 2** Cannon-Fenske Routine Viscometer

6. Vertical alignment may be accompanied in bath by suspending a plumb bob in tube "J".
7. Apply suction to tube “A” and bring sample into bulb “D”, a short distance above mark “C”.
8. The efflux time is measured by allowing the sample to flow freely through mark “C”, measuring the time for the meniscus to pass from “C” to “E”.
9. To repeat efflux time, repeat steps 7 and 8.

You must determine the viscometer (calibration) constant for every C-F tube. The viscometer constant is determined by using the same procedure for a liquid of known viscosity (at the measured temperature). The following table provides the recommended viscosity ranges and the approximate constant for each size C-F tube.

**Table 1** Recommended Viscosity Ranges for The C-F Viscometers

<table>
<thead>
<tr>
<th>Size</th>
<th>Centistoke/Second Approximate Constant</th>
<th>Centistokes Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.002</td>
<td>0.5 to 2</td>
</tr>
<tr>
<td>50</td>
<td>0.004</td>
<td>0.8 to 4</td>
</tr>
<tr>
<td>75</td>
<td>0.008</td>
<td>1.6 to 8</td>
</tr>
<tr>
<td>100</td>
<td>0.015</td>
<td>3 to 15</td>
</tr>
<tr>
<td>150</td>
<td>0.035</td>
<td>7 to 35</td>
</tr>
<tr>
<td>200</td>
<td>0.1</td>
<td>20 to 100</td>
</tr>
<tr>
<td>300</td>
<td>0.25</td>
<td>50 to 250</td>
</tr>
<tr>
<td>350</td>
<td>0.5</td>
<td>100 to 500</td>
</tr>
</tbody>
</table>
First you must determine the sample size for the C-F tubes. Also determine the viscometer constant, using distilled water, for three tubes of the same size.

Make up a solution of 10.0 w% of aqueous glycerol and determine its viscosity at five different temperatures from about 70. to 150.°F. Use distilled water to calibrate the tube used. Make up a solution of 40.% aqueous glycerol and determine its viscosity at five different temperatures from about 70. to 150.°F. Use the 70.°F, or room temperature viscosity, of the 10.% glycerol for calibrating.

We will be using the oil filled, temperature controlled bath in the transport laboratory. You might need a light to illuminate the bath for ease in making readings. The bath heats quickly but cools slowly ... so plan accordingly. The heating element control has H, M, ML and L(ow) settings. The control has a magnetic drive unit so it is necessary to turn the control knob slowly. You can heat quickly by starting on H(igh), but then you must change to a lower setting so that you don’t overshoot the temperature. Note that there are cooling coils which can be used to cool the bath (if you connect them to the cooling water supply), but you will not be able to cool the bath quickly.

Record the efflux time from three to six times for each viscosity measured. If there is a noticeable difference between the times thus measured, the viscometer may be dirty, the stopwatch may be inaccurate or there may be some other problem. In any case take corrective action. When finished, thoroughly clean the viscometers and turn off the thermostat.

Calculate all viscosities, both kinematic and absolute. Calculate the best average viscosity for each solution and each temperature. You should fit the data obtained for the two solutions to the following equations:

\[
\text{viscosity} = A \exp(-BT)
\]

\[
\text{viscosity} = A \exp(-B/T)
\]

where \(A\) and \(B\) are constants and \(T\) is the absolute temperature in °R.

Also compare the measured viscosity of the 10.% glycerol with the literature value.

**The Stormer Viscometer (Viscosimeter)**

The Stormer viscometer consists of two concentric cylinders, one of which is rotated while the other is held stationary. The fluid viscosity can be determined from the rotational speed and the torque \(\mathcal{T}\) on the cylinder (Ref. 1, pg. 146-149).

\[
\mathcal{T} = C \mu \Omega
\]

where \(\Omega\) is the angular speed. The constant \(C\) can be calibrated by a liquid with known viscosity.
Try to take as much data as possible since you might not have enough time to perform all the experiments recommended in this section.

This version of the Stormer viscometer is easy to operate, produces results quickly and is adaptable to a variety of materials, in contrast to the Cannon-Fenske tube. However it is not as accurate as C-F. See the information folder in the laboratory for the Stormer instructions. What size sample is required?

Set up the viscometer according to the instructions. Calibrate it with distilled water and 40.0% glycerol solution. Measure the viscosity of the 20.0% glycerol solution at room temperature. Bring a sample, of a viscous hydrophilic material, from home for testing. Some examples are catsup, chili sauce, condensed milk, honey, mustard, pancake syrup or a cornstarch solution. Test this sample at room temperature using appropriate loads. Plot revolutions per second versus load. How would you describe the flow properties of your sample?

Look up the viscosities of 40.0% sucrose solutions in a table in one of the references. Prepare a viscosity-temperature plot, over the range from 0 - 100°C, using the Arrhenius type relationship. Determine the two constants for the least squares line.

Report

The Cannon Fenske results are the viscosity versus temperature graphs for the two equations. Which relationship gives the better correlation? Also list the viscometer constants for the three tubes of the same size. How much do they vary? Why do they vary?

Use a table to express the Stormer viscosity results. Calculate the viscosity of the 20.0% glycerol sample using the viscometer constant from both the distilled water and 40.0% glycerol. How do these viscosities compare with each other and with the literature value? The results of the viscosity measurements of the viscous, hydrophilic sample at different loads should be presented using a graph.

The following questions should be answered:

1. Could you calculate the viscometer constant for a C-F tube, using an equation (not using the draining time)?

2. What would you have to do to calculate the viscometer constant for the Stormer viscometer?

3. For the capillary-type viscometers it has been found that, under certain conditions of fluid properties and/or for different viscometer tube, the kinematic viscosity is best related to the efflux time by the following equation:

\[ \nu = C \frac{t_{\text{efflux}}}{B/t_{\text{efflux}}} \]  

where C and B are constants.
   a. Explain the theoretical significance of this equation.
b. Under what experimental conditions will Eq. (5) reduce to Eq. (4).
c. How would one evaluate the constants, B and C, experimentally?

4. In the development of Eq. (4) it was assumed that a fully developed velocity-profile exists in the capillary tube at all times and is directly related to the instantaneous hydrostatic-head across the capillary. Discuss the validity of this assumption. Is this a quasi steady-state assumption?

References

The food supply division of CAL CHEM, POLYFOOD, obtained a contract to supply rations for the army. One condition of the contract is the requirement that the food must contain at least ten Calories per gram. You should note that 1 unit of the capitalized Calorie in food products is equivalent to 1000 calories or 4187 J in engineering notation. POLYFOOD want us to measure accurately the calories contain of the food by burning it completely in a bomb calorimeter. Our technician will help you with the operation of the bomb calorimeter so that you can provide the necessary data for POLYFOOD.
Introduction

The heat of combustion for a substance is defined as the enthalpy change $\Delta H$ for a process in which the substance reacts with oxygen to yield specified products such as $\text{CO}_2$, $\text{H}_2\text{O}$, $\text{SO}_2$. Thus the heat of combustion of benzoic acid at $25^\circ\text{C}$ is $\Delta H$ for the process

$$\text{C}_6\text{H}_5\text{CO}_2\text{H}(s) + 7.5\text{O}_2(g) = 7\text{CO}_2(g) + 3\text{H}_2\text{O}(l) \quad (1)$$

with reactants and products at this temperature. The heat of combustion can be calculated from the temperature rise that results when the combustion reaction occurs in an adiabatic calorimeter shown in Fig. 1. The substance is burned in a steel bomb that is immersed in a can of water fitted with a precise thermometer. The can is placed within an outer water-filled jacket that is maintained at the same temperature as that of the water in the can.

![Diagram of adiabatic combustion calorimeter]

In the actual calorimeter process, the final and initial temperatures are not equal so that the process does not correspond exactly to the conditions specified by Eq. (1). The heat of reaction for substance $i$ at temperature $T$ may be evaluated from

$$\Delta H_i,T = \Delta U_T + \Delta (PV) \quad (2)$$

If the contribution $\Delta(PV)$ is small compared to the internal energy then

$$\Delta H_i,T = \Delta U_T = (C_w m_w + C) \Delta T \quad (3)$$

where

$m_w = \text{mass of water}$

$C_w = \text{heat capacity of water per unit mass}$
\[ C = \text{heat capacity of the calorimeter (bomb and contents, can, etc.)} \]
\[ \Delta T = \text{temperature rise of water after combustion} \]

Eq. (3) does not include the heat contribution from the fuse wire or the contribution from any side reaction.

**Experimental Work**

The Cal Chem. Corporation design group would like to know the heat of combustion of some unknown substances. Use the Parr Oxygen Bomb Adiabatic Calorimeter located in the lab to determine the heat of combustion of an unknown sample. Even though our lab technician has outlined an operational procedure for this calorimeter, this procedure is not detailed enough for the design group. First note that in this adiabatic calorimeter there is one general procedure but two modes of operation; calibration and sample testing.

Calibration, with a known weight of a pure material with known heat of combustion, is necessary to determine accurately the calibration constant. Do only one calibration per laboratory period to conserve the benzoic acid supply. It is interesting to note that this constant can be estimated using a few simple measurements and a knowledge of the instrument.

Sample testing employs the same experimental procedure as calibration. However a different computer program in the calorimeter will calculate the heat of combustion when you input the calibration constant.

Draw a complete sketch of the experimental apparatus.

Please work out a detailed experimental procedure. Note the differences between calibration and sample testing.

You should first calibrate the calorimeter with a benzoic acid sample and then use this calibration constant to determine the heat of combustion of the unknown substance.

Note that there is an instruction manual located in the lab for your information.

Our lab technician's procedure follows:

**Calibration run**

1. Turn on the power switch to start water flowing. This will allow water jacket to adjust the water temperature to the room temperature. Start filling the bucket with the water from the jacket outlet so that the water in the bucket will be at the same temperature as the water jacket and the bomb will fire quicker. Two liters of water is required.
2. Weight about 0.900 to 1.100 grams of benzoic acid into the sample. Record its exact weight in the data book.
3. Place the sample cup into the cup holder attached to the electrode assembly.
4. Measure and cut a piece of fuse wire up to 10 cm long and attach it to the electrode assembly. Make sure that the wire does not touch the sample cup.
5. Put the electrode assembly inside the bomb and attach the screw cap on top of the bomb carefully. Tighten it by hand only.
6. The bomb must be pressurized with oxygen gas. To do so place the slip connector on the oxygen hose which slides over the gas inlet fitting on the bomb head. Put the connector down as far as it will go. Close the valve on the fitting connection and open the oxygen tank valve not more than a quarter turn. Open the filling connection control valve and slowly pressurize the bomb to 30 atm.
7. Release the pressure in the oxygen hose before disconnecting the bomb.
8. Place the bucket into the jacket assembly. Make sure that the three dimples in the bottom of the bucket rests on supporting pins when the bucket is placed in the jacket. The single dimple should always point forward.
9. Place the bomb into the center of the water bucket with a tongs. Attach two ignition wires on the top of the bomb. If the bomb leaks, depressurize and check the gasket. If there is no leak, tuck in the rest of the ignition wires and remove the tongs.
10. Close the calorimeter lid by pulling the holding pin. Check the lid to make sure it is closed then let go of the holding pin. Lower the thermometer down. This thermometer measures the bucket temperature.
11. Turn on the power switch of the controller. Press the start key. The EE light will now flash requesting the identification letter for the bomb and the bucket to be used in the test. Press A.
12. For the calibration run, press STAR CODE (*) and then 15. Enter the sample weight.
13. The PRE light on the panel will glow during this period. Record the temperature at this point, this will be the temperature \( T_i \).
14. Then the FIRE light will flash on the controller for about 50 seconds. If firing is confirmed the controller will go to the post fire mode and the POST light will appear.
15. The bucket temperature will rise during this mode indicating that the heat is transferred from the bomb to the water.
16. When the temperature reaches steady state the controller will make a long beep sound. It will indicate the calorimeter constant. Record this constant as well as the final temperature \( T_f \) of the water bucket.
17. Lift the water bucket thermometer and then open the top cover of the jacket carefully. Remove the bomb from the water bucket.
18. Unscrew the pressure release valve of the bomb and depressurize the bomb slowly.
19. Measure the fuse wire left in the bomb. This will be used to correct for the energy contribution from the fuse wire. Clean the sample cup for the next run.

Sample run

1. For the sample testing, follow the same procedure from steps 1 to 11.
2. For step 12 enter STAR CODE (*) 16 instead of STAR CODE (*) 15.
3. Enter the calorimeter constant from the calibration run and the weight of the unknown.
4. Repeat steps 13 to 19 and record the heat of combustion of the unknown sample.

The proper placement of the fuse wire, relative to the sample in the sample holder, is quite tricky. Ask someone from the previous group to give you some hints.

After developing your detailed calibration procedure, show the modifications necessary for sample testing.
You are to determine the heat of combustion of one or more unknown samples which will be given to you. What can you discern about the sample(s) before analysis?

**REPORT**

**Results** - In a table show all of your heat of combustion results (every run is an individual result) and the other sample information. What do you think the sample(s) might be?

What was the value of the calibration constant? What was your estimated value for the calibration constant?

In order to make your results more useful to the Cal Chem. design group (and to increase your knowledge of thermochemistry), please prepare a table showing heats of combustion (Kcal/gm) and formula for the following organic chemical groups:

<table>
<thead>
<tr>
<th>group 1</th>
<th>group 2</th>
<th>group 3</th>
<th>group 4</th>
<th>group 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>methane</td>
<td>benzene</td>
<td>methanol</td>
<td>formic acid</td>
<td>lactose</td>
</tr>
<tr>
<td>ethane</td>
<td>toluene</td>
<td>ethanol</td>
<td>acetic acid</td>
<td>sucrrose</td>
</tr>
<tr>
<td>propane</td>
<td>o-xylene</td>
<td>1-propanol</td>
<td>succinic acid</td>
<td>cellulose</td>
</tr>
<tr>
<td>i-butane</td>
<td>m-xylene</td>
<td>2-propanol</td>
<td>oleic acid</td>
<td></td>
</tr>
<tr>
<td>n-butane</td>
<td>p-xylene</td>
<td>1-butanol</td>
<td>benzoic acid</td>
<td></td>
</tr>
</tbody>
</table>

**Discussion** - Explain any discrepancy between literature and experimental values of the heat of combustion of the unknown substance(s) (if you are told what the unknown substances are).

Discuss how you would refine your estimated calibration constant calculation to achieve a better value.

For each of the five groups of organic compounds, make a short statement that describes how structure influences the heat of combustion.

Place your detailed operational procedure in the appendix.

Questions and Statements to answer:
1. Estimate the error incurred by not titrating for acid formed during the combustion of the sucrose sample.
2. After studying the design and operation of this apparatus explain clearly why this is an adiabatic operation.
3. How did you estimate the calibration constant?
4. Knowledge of the approximate molecular weight of the unknown sample would surely be very useful. What relatively simple test could you recommend for the approximate molecular weight determination for organic compounds?

**References**


**INTERCOMPANY MEMORANDUM**
Welcome to CAL CHEM corporation. Many organizations that employ engineers have standardized their calculation methods for project progress reports. This is done to facilitate communication within the organization and to provide better control of work quality. Standardization permits routine checking of work as well as the opportunity to switch people to the tasks that are the most urgent.

In your career at Cal Poly you have used a variety of report formats. Some of these were quite structured and others were not. Now that you are a staff member with CAL CHEM Corporation, we would like you to use our style of reporting. The format is designed to facilitate the communication of information to your supervisor (instructor).

We want you to perform a simple experiment, siphon for draining tank, and write a report using our format. Please read our instructions for writing a report carefully and follows them.
Experiment No. 6

SIPHON FOR DRAINING TANK

Introduction

As shown in Fig. 1, a tube of inside radius $R$ and total length $L$ is used for siphoning water from a tank. The discharge from the siphon is at a distance $h$ below the level of water in the tank. If the frictional loss in the tube is neglected, the following formula can be derived to estimate the time taken for $h$ to drop from an initial $H_i$ to a final $H_f$.

![Figure 1. Tank draining by siphon](image)

$$t = \frac{2R_i^2 (H_i^{1/2} - H_f^{1/2})}{R^2 (2g)^{0.5}}$$

(1)

where $R_i$ is the tank inside radius and $g$ is the gravity acceleration.

If you assume laminar flow, taking tube friction to be the only resistance and ignoring kinetic-energy effects, you can derive another formula to estimate the time taken for $h$ to drop from an initial $H_i$ to a final $H_f$.

$$t = \frac{8\mu LR_i^2}{\rho g R^4} \ln\left(\frac{H_i}{H_f}\right)$$

(2)

If the flow is turbulent then for smooth tube

$$t = \frac{7}{3} \left(\frac{R_i}{R}\right)^2 \left(\frac{0.0791 L \mu^{1/4}}{2^{1/4} \rho^{1/4} g^{1/4} R^{5/4}}\right)^{4/7} [H_i^{3/7} - H_f^{3/7}]$$

(3)
**Experimental Work**

The apparatus consists of the followings:

1) a cylindrical tank with a ¼" OD polyethylene tubing; 2) tap water; 3) stop watch; 4) thermometer; 5) graduated cylinders.

**Procedure**

1. Fill the tank with tap water to above the \( H_i \) level in the tank and measure the water temperature.
2. Record tank inside diameter, tube inside diameter, and tube length. Record the distance from the water level \( H_i \) to the tube exit.
3. Allow the liquid to start siphoning from the tank.
4. Start the timing when the water pasts a certain \( H_i \) level in the tank.
5. Stop the timing when the water pasts a certain \( H_f \) level in the tank.
6. Fill water to above the \( H_i \) level in the tank and repeat step 3-6 for at least 8 more times.

**Report**

1. Derive Eqs. (1), (2), and (3) in details. The followings are an outline of the necessary steps to derive Eqs. (2) and (3).

Application of the steady state energy balance

\[
\frac{1}{2} \Delta u^2 + g \Delta z + \frac{\Delta p}{\rho} + 2f_L u_m^2 \frac{L}{D} = 0
\]

gives

\[
\frac{1}{2} u_m^2 - gh + 2f_L u_m^2 \left( \frac{L}{D} \right) = 0
\]

where

- \( u_m \) = average velocity through the pipe.
- \( L \) = length of pipe.
- \( D \) = inside diameter of pipe.
- \( f \) = friction factor
- \( g \) = acceleration of gravity

If the entrance loss at the inlet to the pipe and the kinetic energy of the emerging stream, \( \frac{1}{2} u_m^2 \), are neglected then
where \( R = D/2 \) = pipe inside radius.

Since the entrance loss at the inlet of the pipe is neglected, the friction is just due to the friction loss in the pipe. When laminar flow occurs

\[ f_F = \frac{16}{Re} \]

where

\[ R = \frac{D}{2} \]

\[ \nu = \text{dynamic viscosity} \]

The average velocity for laminar flow is

\[ u_m = \frac{\rho g R^2 h}{8 \mu L} \]

For turbulent flow through smooth tubes, the friction factor is given by

\[ f_F = \frac{0.0791}{Re^{1/4}} ; \quad 4000 < Re < 10^5 \]

The average velocity for turbulent flow is

\[ u_m = \left( \frac{2^{1/4} \rho^{1/4} g R^{5/4}}{0.0791 L \mu^{1/4}} \right)^{4/7} H^{4/7} \]

The change in height \( H \) of the liquid level in the tank is related to the velocity \( u_m \) through a conservation of mass

\[ \pi R_t^2 \frac{dh}{dt} = -\pi R^2 u_m \]

where

\[ R_t = \text{tank radius} \]

\[ t = \text{time} \]

Substitution of the appropriate velocity into the mass balance yields Eq. (2) or Eq. (3).

2. Use error analysis to report the mean draining time and a 95 percent confidence time interval.

3. Estimate the water viscosity using the following equation

\[ \mu = \exp[29.76 - 5.24 \ln(T)] \]
where \( \mu \) is in centipoise and \( T \) is in \(^\circ\text{K}\).

4. For each experimental run, determine an average volumetric flow rate by dividing the volume of water drained with the time recorded. Calculate a mean water velocity in the tube.

5. Calculate the Reynolds number and the draining times from Eqs. (1), (2), and (3).

6. Discuss any difference between the values obtained from Eqs. (1), (2), and (3) and the experimental value. Which formula should provide a better value for the draining time? Why?

References